Matematika 3
Vector Dot Product

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1 Exercise

Jika $a = 5i + 4j + 2k, b = 4i - 5j + 3k,$ dan $c = 2i - j - 2k$

1. Nilai $a.b$ dan cosinus arah dari hasil kali vektor $a \times b$

$$
\begin{aligned}
a.b &= \begin{bmatrix} i & j & k \\ 5i & 4j & 2k \\ 4i & -5j & 3k \end{bmatrix} \\
a.b &= (5i \times 4i) + (4j \times -5j) + (2k \times 3k) \\
a.b &= 20i^2 - 20j^2 + 6k^2 \\
a.b &= 20(1) - 20(1) + 6(1) \\
a.b &= 6
\end{aligned}
$$

$$
\begin{aligned}
|a| &= \sqrt{5^2 + 4^2 + 2^2} \\
|a| &= \sqrt{25 + 16 + 4} \\
|a| &= \sqrt{45} \\
|a| &= 3\sqrt{5}
\end{aligned}
$$

$$
\begin{aligned}
|b| &= \sqrt{4^2 + (-5)^2 + 3^2} \\
|b| &= \sqrt{16 + 25 + 9} \\
|b| &= \sqrt{50} \\
|b| &= 5\sqrt{2}
\end{aligned}
$$

$$
\begin{aligned}
cos\theta &= \frac{5}{3\sqrt{5}}.\frac{4}{5\sqrt{2}} + \frac{4}{3\sqrt{5}}.\frac{-5}{5\sqrt{2}} + \frac{2}{3\sqrt{5}}.\frac{3}{5\sqrt{2}} \\
&= \frac{20}{15\sqrt{10}} + \frac{-20}{15\sqrt{10}} + \frac{6}{15\sqrt{10}} \\
&= \frac{20 - 20 + 6}{15\sqrt{10}} \\
&= \frac{6}{15\sqrt{10}} \\
&= \frac{2}{5\sqrt{10}} \\
&= \frac{2}{5\sqrt{10}} \times \frac{\sqrt{10}}{\sqrt{10}} \\
&= \frac{2\sqrt{10}}{50} \\
&= \frac{\sqrt{10}}{25}
\end{aligned}
$$

$$
\begin{aligned}
\theta &= cos^{-1}(\frac{\sqrt{10}}{25}) \\
&= cos^{-1}(0.1265) \\
&= 82.76^{\circ}
\end{aligned}
$$

2. Ukuran dan cosinus dari hasil kali vektor $a \times b$ dan juga sudut dimana hasil kali vektor membentuk sudut dengan vektor c

$$
\begin{aligned}
a \times b &= \begin{bmatrix} i & j & k \\ 5i & 4j & 2k \\ 4i & -5j & 3k \end{bmatrix} \\
a \times b &= (4j \times 3k - 2k \times -5j)i - (5i \times 3k - 2k \times 4i)j + (5i \times -5j - 4j \times 4i)k \\
a \times b &= (12 + 10)i - (15 - 8)j + (-25 - 16)k \\
a \times b &= 22i - 7j - 41k \\
cos\theta &= \frac{a \times b.c}{|a \times b|.|c|} \\
&= \frac{(22i - 7j - 41k).(2i - j - 2k)}{\sqrt{22^2 + 7^2 + (-41)^2}.\sqrt{2^2 + (-1)^2 + (-2)^2}} \\
&= \frac{44 + 7 + 82}{\sqrt{22^2 + 7^2 + (-41)^2}.\sqrt{2^2 + (-1)^2 + (-2)^2}} \\
&= \frac{133}{\sqrt{22^2 + 7^2 + (-41)^2}.\sqrt{2^2 + (-1)^2 + (-2)^2}} \\
&= \frac{133}{\sqrt{484 + 49 + 1681}.\sqrt{4 + 1 + 4}} \\
&= \frac{133}{\sqrt{2214}.\sqrt{9}} \\
&= \frac{133}{3\sqrt{246}.3} \\
&= \frac{133}{9\sqrt{246}} \\
&= 0.9421
\end{aligned}
$$

$$
\begin{aligned}
\theta &= cos^{-1}(0.9421) \\
&= 19.57^{\circ}
\end{aligned}
$$

Corrector: Yanuar Thaif Chalil Candra
Value: 100