Matematika 3
Quiz 2

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Quiz 2

Material 10

1. What is the difference between vectors and vector spaces?

   Vectors is an entity that has magnitude or length and direction. Vector space is a set of vectors that can be added and multiplied by a scalar.

2. Determine the size of the vector $\vec{PQ} = 5i + 8j + 3k$ and $\vec{RS} = 9i + 4j + 5k$

$$\begin{aligned} \vec{PQ} &= 5i + 8j + 3k \\ |\vec{PQ}| &= \sqrt{5^2 + 8^2 + 3^2} \\ &= \sqrt{25 + 64 + 9} \\ &= \sqrt{98} \\ &= 7\sqrt{2} \end{aligned}$$

$$\begin{aligned} \vec{RS} &= 9i + 4j + 5k \\ |\vec{RS}| &= \sqrt{9^2 + 4^2 + 5^2} \\ &= \sqrt{81 + 16 + 25} \\ &= \sqrt{102} \end{aligned}$$

3. Known $\vec{u} = (1, 2, 3, 5)$ and $\vec{v} = (4, 7, 6, 2)$ determine the length of each vector

$$\begin{aligned} \vec{u} &= (1, 2, 3, 5) \\ ||\vec{u}|| &= \sqrt{1^2 + 2^2 + 3^2 + 5^2} \\ &= \sqrt{39} \end{aligned}$$

$$\begin{aligned} \vec{v} &= (4, 7, 6, 2) \\ ||\vec{v}|| &= \sqrt{4^2 + 7^2 + 6^2 + 2^2} \\ &= \sqrt{105} \end{aligned}$$

4. Known $\vec{u} = (2, 1, 3, 1)$ and $\vec{v} = (3, 3, 1, 2)$ determine the distance between the two vectors.

$$\begin{aligned}
\vec{u} &= (2, 1, 3, 1) \\
\vec{v} &= (3, 3, 1, 2) \\
||\vec{u} - \vec{v}|| &= \sqrt{(2-3)^2 + (1-3)^2 + (3-1)^2 + (1-2)^2} \\
&= \sqrt{1 + 4 + 4 + 1} \\
&= \sqrt{10}
\end{aligned}$$

Material 12

5. Determine the direction cosine $[l, m, n]$ from vector $\vec{r} = 2i - 2j + 7k$

$$\begin{aligned}
r &= \sqrt{2^2 - 2^2 + 7^2} \\
&= \sqrt{4 + 4 + 49} \\
&= \sqrt{57}
\end{aligned}$$

$$\begin{aligned}
l &= \frac{2}{\sqrt{57}} \\
m &= \frac{-2}{\sqrt{57}} \\
n &= \frac{7}{\sqrt{57}}
\end{aligned}$$

6. What is the scalar product of two vectors if a=3, b=5 and $\theta = 45$?

$$\begin{aligned}
\vec{a} &= 3 \\
\vec{b} &= 5 \\
\theta &= 45 \\
\vec{a} \cdot \vec{b} &= ab \cos\theta \\
&= 3 \times 5 \times \cos 45 \\
&= 15 \times \frac{\sqrt{2}}{2} \\
&= \frac{15\sqrt{2}}{2}
\end{aligned}$$

7. What is the vector product of $p \times q = \begin{bmatrix} i & j & k \\ 3 & 4 & 2 \\ 1 & 5 & 2 \end{bmatrix}$

$$\begin{aligned} p \times q &= \begin{bmatrix} i & j & k \\ 3 & 4 & 2 \\ 1 & 5 & 2 \end{bmatrix} \\ &= i\begin{vmatrix} 4 & 2 \\ 5 & 2 \end{vmatrix} - j\begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} + k\begin{vmatrix} 3 & 4 \\ 1 & 5 \end{vmatrix} \\ &= i(8-10) - j(6-2) + k(15-4) \\ &= -2i - 4j + 11k \end{aligned}$$

8. If a = 3i + 2j + 2k and b = 2i +1j + 4k. Determine

   (a) $a.b$

   (b) $a \times b$

   (a)

$$\begin{aligned} a &= 3i + 2j + 2k \\ b &= 2i + 1j + 4k \\ a \cdot b &= (3 \times 2) + (2 \times 1) + (2 \times 4) \\ &= 6 + 2 + 8 \\ &= 16 \end{aligned}$$

   (b)

$$\begin{aligned} a \times b &= \begin{bmatrix} i & j & k \\ 3 & 2 & 2 \\ 2 & 1 & 4 \end{bmatrix} \\ &= i\begin{vmatrix} 2 & 2 \\ 1 & 4 \end{vmatrix} - j\begin{vmatrix} 3 & 2 \\ 2 & 4 \end{vmatrix} + k\begin{vmatrix} 3 & 2 \\ 2 & 1 \end{vmatrix} \\ &= i(8-2) - j(12-4) + k(3-4) \\ &= 6i - 8j - k \end{aligned}$$

Material 12

9. Determine the angle between vectors:

$$p = 3i + 2j + 2k$$
$$q = 3i - 2j + 2k$$

(a) First determine the direction cosine $[l, m, n]$ (for p) and $[l', m', n']$ (for q).

$$\begin{aligned} p &= 3i + 2j + 2k \\ ||p|| &= \sqrt{3^2 + 2^2 + 2^2} \\ &= \sqrt{9 + 4 + 4} \\ &= \sqrt{17} \\ l &= \frac{3}{\sqrt{17}} \\ m &= \frac{2}{\sqrt{17}} \\ n &= \frac{2}{\sqrt{17}} \end{aligned}$$

$$\begin{aligned} q &= 3i - 2j + 2k \\ ||q|| &= \sqrt{3^2 + (-2)^2 + 2^2} \\ &= \sqrt{9 + 4 + 4} \\ &= \sqrt{17} \\ l' &= \frac{3}{\sqrt{17}} \\ m' &= \frac{-2}{\sqrt{17}} \\ n' &= \frac{2}{\sqrt{17}} \end{aligned}$$

(b) Then look for $\cos\theta = l.l' + m.m' + n.n'$

$$\begin{aligned} \cos\theta &= l.l' + m.m' + n.n' \\ &= \frac{3}{\sqrt{17}} \times \frac{3}{\sqrt{17}} + \frac{2}{\sqrt{17}} \times \frac{-2}{\sqrt{17}} + \frac{2}{\sqrt{17}} \times \frac{2}{\sqrt{17}} \\ &= \frac{9}{17} - \frac{4}{17} + \frac{4}{17} \\ &= \frac{9}{17} \end{aligned}$$

10. What is vector direction ratio?

    Vector direction ratio is the ratio of the direction of a vector to the direction of the x, y, and z axes.