

In a game a ball is rolled down a slope and along a track until it stops. The distance, in metres, travelled by the ball is modelled by the random variable X with probability density function

$$f(x) = \begin{cases} -k(x-1)(x-3) & 1 \leq x \leq 3, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

- (a) Without calculation, explain why $E(X) = 2$. [1]

- (b)** Show that $k = \frac{3}{4}$. [3]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school writing paper. The lines are evenly spaced and run across the width of the page. There are no margins, text, or other markings on the paper.

(c) Find $\text{Var}(X)$.

[3]

This image shows a single sheet of white paper with ten horizontal dashed lines, typical of primary-ruled notebook paper. The lines are evenly spaced and extend across the width of the page. There is no handwriting or other markings on the paper.

One turn consists of rolling the ball 3 times and noting the largest value of X obtained. If this largest value is greater than 2.5, the player scores a point.

(d) Find the probability that on a particular turn the player scores a point.

[4]

This image shows a full page of white paper with horizontal dashed lines, typical of primary school handwriting practice paper. The lines are evenly spaced and run across the entire width of the page. There are no margins, text, or other markings present.