Jacob has four coins. One of the coins is biased such that when it is thrown the probability of obtaining a head is $\frac{7}{10}$. The other three coins are fair. Jacob throws all four coins once. The number of heads that he obtains is denoted by the random variable $X$. The probability distribution table for $X$ is as follows.

| $x$ | 0 | 1 | 2 | 3 | 4 |
|---|---|---|---|---|---|
| $P(X = x)$ | $\frac{3}{80}$ | $a$ | $b$ | $c$ | $\frac{7}{80}$ |

**(a)** Show that $a = \frac{1}{5}$ and find the values of $b$ and $c$. [4]

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**(b)** Find $E(X)$. [1]

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Jacob throws all four coins together 10 times.

**(c)** Find the probability that he obtains exactly one head on fewer than 3 occasions. [3]

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**(d)** Find the probability that Jacob obtains exactly one head for the first time on the 7th or 8th time that he throws the 4 coins. [2]

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