In a game a ball is rolled down a slope and along a track until it stops. The distance, in metres, travelled by the ball is modelled by the random variable X with probability density function

$$\mathrm{f}(x) = \begin{cases} -k(x-1)(x-3) & 1 \leqslant x \leqslant 3, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a constant.

(a) Without calculation, explain why $\mathrm{E}(X) = 2$. [1]

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(b) Show that $k = \frac{3}{4}$. [3]

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(c) Find Var(X). [3]

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One turn consists of rolling the ball 3 times and noting the largest value of X obtained. If this largest value is greater than 2.5, the player scores a point.

(d) Find the probability that on a particular turn the player scores a point. [4]

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