2.1 Rows vs. Columns

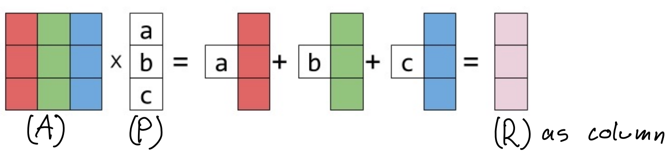
In this exercise, we deal with linear operations $\mathbf{\Pi_P}$ that map an input matrix $\mathbf{A}$ – also called the *argument* of the operation – to an output matrix $\mathbf{R}$, using a parameter matrix $\mathbf{P}$ and matrix-multiplication. We think that operation $\mathbf{\Pi_P}$ acts on $\mathbf{A}$, resulting in matrix $\mathbf{R}$ as the outcome of the operation. Depending on whether the operation acts on the rows of $\mathbf{A}$, or on the columns of $\mathbf{A}$, operation $\mathbf{\Pi_P}$ amounts to a left-multiplication of $\mathbf{A}$ with $\mathbf{P}$, or to a right-multiplication of $\mathbf{A}$ with $\mathbf{P}$ – your first task is to refresh your understanding of which multiplication is needed for which case. Which of the following statements is correct and why?

1. Given $\mathbf{\Pi_P}$ is defined as mapping $\mathbf{A}$ to $\mathbf{R} = \mathbf{PA}$ (*left-multiplication* of $\mathbf{A}$ with $\mathbf{P}$), operation $\mathbf{\Pi_P}$ acts on the *rows* of $\mathbf{A}$, resulting in (the *rows* of) $\mathbf{R}$.
2. Given $\mathbf{\Pi_P}$ is defined as mapping $\mathbf{A}$ to $\mathbf{R} = \mathbf{AP}$ (*right-multiplication* of $\mathbf{A}$ with $\mathbf{P}$), operation $\mathbf{\Pi_P}$ acts on the *rows* of $\mathbf{A}$, resulting in (the *rows* of) $\mathbf{R}$.
3. Given $\mathbf{\Pi_P}$ is defined as mapping $\mathbf{A}$ to $\mathbf{R} = \mathbf{PA}$ (*left-multiplication* of $\mathbf{A}$ with $\mathbf{P}$), operation $\mathbf{\Pi_P}$ acts on the *columns* of $\mathbf{A}$, resulting in (the *columns* of) $\mathbf{R}$.
4. Given $\mathbf{\Pi_P}$ is defined as mapping $\mathbf{A}$ to $\mathbf{R} = \mathbf{AP}$ (*right-multiplication* of $\mathbf{A}$ with $\mathbf{P}$), operation $\mathbf{\Pi_P}$ acts on the *columns* of $\mathbf{A}$, resulting in (the *columns* of) $\mathbf{R}$.

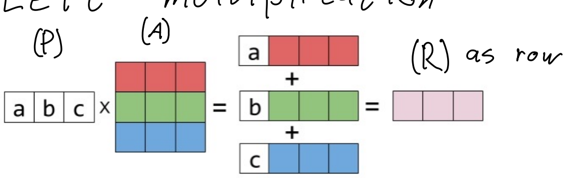
Next, we do some concrete examples. For each one of them, you need to first find whether $\mathbf{\Pi_P}$ leads to a left- or to a right-multiplication of $\mathbf{A}$ with $\mathbf{P}$. Then, you need to find $\mathbf{P}$ (with all its numeric values).

1. Specify $\mathbf{\Pi_P}$, acting on $\mathbf{A} \in \mathbb{R}^{4\times k}$, such that it produces $\mathbf{R}$ (with 3 rows) according to the following:
   (a) the first row of $\mathbf{R}$ should be the second row of $\mathbf{A}$
   (b) second row of $\mathbf{R}$: difference between the first two rows of $\mathbf{A}$ ($1^{\text{st}}-2^{\text{nd}}$)
   (c) row #3 of $\mathbf{R}$: sum of all rows of $\mathbf{A}$
2. Specify $\mathbf{\Pi_P}$, acting on $\mathbf{A} \in \mathbb{R}^{n\times 4}$, such that it produces $\mathbf{R}$ (with 4 columns) according to the following:
   (a) the first two columns of $\mathbf{R}$ should be the first two columns of $\mathbf{A}$, but swapped (#1 $\leftrightarrow$ #2)
   (b) third column of $\mathbf{R}$: average of all columns of $\mathbf{A}$
   (c) fourth column of $\mathbf{R}$: sum of all columns of $\mathbf{A}$, but each column multiplied by its number (index)

Right multiplication



Left multiplication



1. True , see illustration
2. False
3. False
4. True

$$A = \begin{bmatrix} a_1 \cdots & a_k \\ b_1 \cdots & b_k \\ c_1 \cdots & c_k \\ d_1 \cdots & d_k \end{bmatrix}$$

1. Right multiplication

2.2 LU Decomposition

In this exercise, we take advantage of the LU-decomposition of a matrix $\mathbf{M}$ to solve a system of linear equations. You are given:

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 2 \\ 2 & 4 & 2 \\ 5 & 0 & 1 \end{pmatrix}$$

1. Compute the LU-decomposition of $\mathbf{M}$. Doing so, make sure that all values along the main diagonal of matrix $\mathbf{L}$ are equal to 1.

2. Now consider the system $\mathbf{M}\,\mathbf{x} = \mathbf{b}$. As seen in the lecture, we can rewrite this as $\mathbf{L}\,\mathbf{U}\,\mathbf{x} = \mathbf{b}$, given the LU-decomposition of $\mathbf{M}$. Assuming $\mathbf{U}\,\mathbf{x} = \mathbf{y}$ and $\mathbf{L}\,\mathbf{y} = \mathbf{b}$, you're now asked to solve $\mathbf{M}\,\mathbf{x} = \mathbf{b}$ for the following right-hand sides:

$$\mathbf{b} = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 6 \\ 8 \\ 12 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 2 \\ 2 \\ 5 \end{pmatrix}$$

3. Explain why using the LU-decomposition is beneficial when solving this exercise.

1.

$$A = \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & 2 \\ 5 & 0 & 1 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -2 \\ 0 & -5 & -9 \end{bmatrix} \begin{matrix} R_2 = R_2 - 2R_1 \\ R_3 = R_3 - 5R_1 \end{matrix} \quad \begin{matrix} L_{2,1} = 2 \\ L_{3,1} = 5 \end{matrix}$$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -2 \\ 0 & 0 & -14 \end{bmatrix} \quad R_3 = R_3 - (-2.5) R_2 \quad L_{3,2} = -2{,}5$$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & -2{,}5 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -2 \\ 0 & 0 & -14 \end{bmatrix} = LU$$

2.

$$LUx = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & -2{,}5 & -14 \end{bmatrix} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -2 \\ 0 & 0 & -14 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} b \\ b \\ b \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -14 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 5 & -2{,}5 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$y_1 = 0 \qquad = b_1$$
$$y_2 = 2y_1 + y_2 \qquad = b_2$$
$$y_3 = 5y_1 - 2{,}5\,y_2 + y_3 \qquad = b_3$$

I

$b_1 = 1 \qquad y_1 = 1$

$b_2 = 2 \qquad y_2 = 2 - 2y_1 = 0$

$b_3 = 3 \qquad y_3 = 3 - 5y_1 + 2{,}5\,y_2 = -2$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -2 \\ 0 & 0 & -14 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$$

$-14x_3 = -2 \Rightarrow x_3 = 1/7$

$2x_2 - 2x_3 = 0 \Rightarrow x_2 = x_3 = 1/7$

$x_1 + x_2 + 2x_3 = 1 \Rightarrow x_1 = 4/7$

II

$b_1 = 6 \qquad y_1 = 6$

$b_2 = 8 \qquad y_2 = 8 - 2y_1 = -4$

$b_3 = 12 \qquad y_3 = 12 - 5y_1 + 2{,}5y_2 = -28$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -2 \\ 0 & 0 & -14 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -4 \\ -28 \end{bmatrix}$$

$-14x_3 = -28 \Rightarrow x_3 = 2$

$-2x_3 + 2x_2 = -4 \Rightarrow x_2 = 0$

$2x_3 + x_2 + x_1 = 6 \Rightarrow x_1 = 2$

III

$b_1 = 2 \qquad y_1 = 2$

$b_2 = 2 \qquad y_2 = 2 - 2y_1 = -2$

$b_3 = 5 \qquad y_3 = 5 - 5y_1 + 2{,}5y_2 = -10$

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -2 \\ 0 & 0 & -14 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ -10 \end{bmatrix}$$

$-14x_3 = -10 \Rightarrow x_3 = 5/7$

$-2x_3 + 2x_2 = -2 \Rightarrow x_2 = -2/7$

$2x_3 + x_2 + x_1 = 2 \Rightarrow x_1 = 6/7$

3 Using LU decomposition is beneficial because after the initial decomposition we can place the variables into the equations to get the result for any b

Suppose we have the system of equations

$$AX = B.$$

The motivation for an LU decomposition is based on the observation that systems of equations involving triangular coefficient matrices are easier to deal with. Indeed, the whole point of Gaussian elimination is to replace the coefficient matrix with one that is triangular. The LU decomposition is another approach designed to exploit triangular systems.
We suppose that we can write

$$A = LU$$

where L is a lower triangular matrix and U is an upper triangular matrix. Our aim is to find L and U and once we have done so we have found an LU decomposition of A.

2.3 Change of Basis

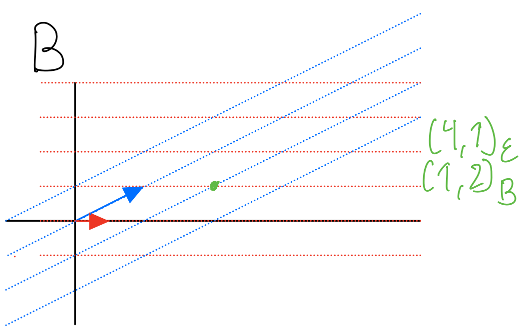
You are given the following matrices:

$$\mathbf{B} = (\mathbf{b}_1\ \mathbf{b}_2) = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}, \quad \mathbf{C} = (\mathbf{c}_1\ \mathbf{c}_2) = \begin{pmatrix} 1 & 3 \\ -1 & 2 \end{pmatrix}$$

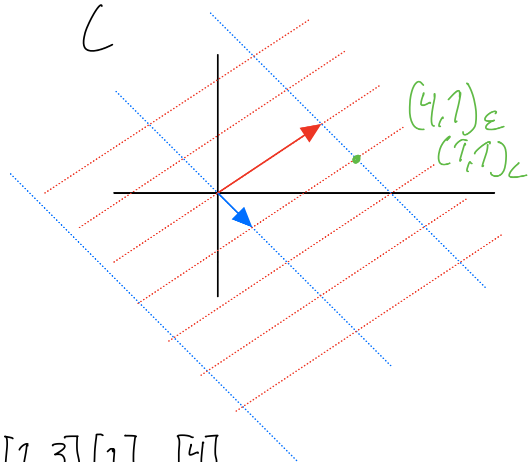
The columns of these matrices give the bases $\mathcal{B}$ and $\mathcal{C}$, respectively. Draw a small figure to find the point $\mathbf{p} = \binom{4}{1}_{\mathcal{E}}$ in $\mathcal{B}$-coordinates, i.e., $[\mathbf{p}]_{\mathcal{B}}$. Verify that this is correct by calculation. Then find the $2{\times}2$-matrix $\mathbf{T}$ that transforms $\mathcal{B}$-coordinates into $\mathcal{C}$-coordinates according to $[\mathbf{x}]_{\mathcal{C}} = \mathbf{T}[\mathbf{x}]_{\mathcal{B}}$. Use this to find $[\mathbf{p}]_{\mathcal{C}}$ and verify this result using the figure.

$B = (b_1\ \ b_2) = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} \qquad C = (c_1\ c_2) = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix} \qquad p = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

B



C



$B\,p = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix} \qquad C\,p = \begin{bmatrix} 1 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$

$A_{B\to C} = C^{-1} \cdot B = \begin{bmatrix} 2/5 & -3/5 \\ 1/5 & 1/5 \end{bmatrix} \cdot \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0{,}2 & 0{,}4 \\ 0{,}6 & 0{,}2 \end{bmatrix}$

$p_C = A_{B\to C} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix}_B = \begin{bmatrix} 0{,}2 & 0{,}4 \\ 0{,}6 & 0{,}2 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

4. Singularity & Solutions

	m, n	Solutions	Singular	Rank	Full/Deficient
A_1	$m > n$	Infinite		2	Deficient
A_2	$m = n$	One	False	3	Full
A_3	$m < n$	None		1	Deficient
A_4	$m = n$	None	True	2	Deficient
A_5	$m < n$	Infinite		2	Full
A_6	$m > n$	One		3	Deficient

$$A_1 = \left[\begin{array}{ccc|c} 0 & 0{,}6 & 0{,}8 & -0{,}8 \\ 1 & 2 & -2 & 4 \\ -0{,}5 & -1 & 1 & -2 \\ 0 & -3 & -4 & 4 \end{array}\right] \underset{=}{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & -4{,}7 & 6{,}7 \\ 0 & 1 & 1{,}3 & -1{,}3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$A_2 = \left[\begin{array}{ccc|c} 1 & 2 & -2 & 4 \\ 14 & -4 & 3 & 25 \\ 0 & -3 & -4 & 4 \end{array}\right] \underset{=}{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right], \quad \det(A_2) = 221$$

$$A_3 = \left[\begin{array}{ccc|c} 0 & 3 & 4 & -4 \\ 0 & -3 & -4 & -7 \end{array}\right] \underset{=}{\text{rref}} \left[\begin{array}{ccc|c} 0 & 1 & 1.3 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right]$$

$$A_4 = \left[\begin{array}{ccc|c} -1 & -2 & 2 & -4 \\ 0 & 3 & 4 & -4 \\ 0{,}5 & 1 & -1 & 0{,}5 \end{array}\right] \underset{=}{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & -4{,}7 & 0 \\ 0 & 1 & -1{,}3 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right], \quad \det(A_4) = 0$$

$$A_5 = \left[\begin{array}{ccc|c} 0 & -3 & -4 & -7 \\ 14 & -4 & 3 & 13 \end{array}\right] \underset{=}{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0{,}6 & 1{,}6 \\ 0 & 1 & 1{,}3 & 2{,}3 \end{array}\right]$$

$$A_6 = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 3 & 4 & 4 \\ -14 & 4 & -3 & -3 \\ -1 & -2 & 2 & 2 \end{array}\right] \underset{=}{\text{rref}} \left[\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

	None	One	Infinite
$m < n$	yes	no	yes
$m = n$	yes	yes	yes
$m > n$	no	yes	yes

, Singular = full-rank

$0 = 1$?

2.5 Consider a cube **C** in 3D space, defined by the following eight points:

$$\mathbf{p}_1 = \begin{pmatrix}0\\2\\0\end{pmatrix}, \mathbf{p}_2 = \begin{pmatrix}1\\2\\0\end{pmatrix}, \mathbf{p}_3 = \begin{pmatrix}1\\2\\1\end{pmatrix}, \mathbf{p}_4 = \begin{pmatrix}0\\2\\1\end{pmatrix}, \mathbf{p}_5 = \begin{pmatrix}0\\3\\0\end{pmatrix}, \mathbf{p}_6 = \begin{pmatrix}1\\3\\0\end{pmatrix}, \mathbf{p}_7 = \begin{pmatrix}1\\3\\1\end{pmatrix}, \mathbf{p}_8 = \begin{pmatrix}0\\3\\1\end{pmatrix}$$

Assume also plane **P**, spanned by vectors

$$\mathbf{v_1} = \begin{pmatrix}4\\0\\-4\end{pmatrix}, \mathbf{v_2} = \begin{pmatrix}8\\8\\8\end{pmatrix}$$

Your task is to do the following:

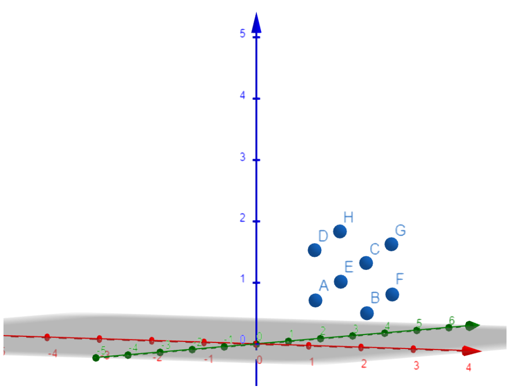
1. Calculate the projection matrix $\mathbf{P_p}$ that projects orthogonally from 3D space onto the 2D subspace defined by the plane **P**.
2. Use $\mathbf{P_p}$ to calculate the orthogonal projection of the cube **C** onto the plane **P**.
3. *Optional: Find out which of the eight corners of the cube project into inner points (fully within the shadow), when compared to the others that are projected to boundary points (at the outer "rim" of the shadow).*

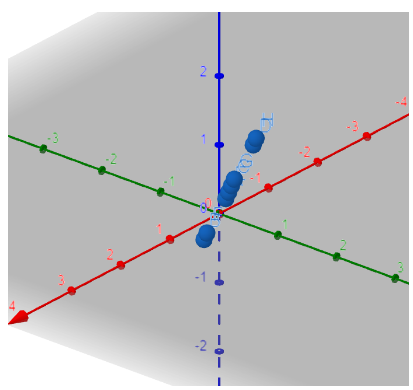
$V_1 \cdot V_2 = \begin{bmatrix}4\\0\\-4\end{bmatrix} \cdot \begin{bmatrix}8\\8\\8\end{bmatrix} = 0$ V_1 and V_2 are lineary independant

$\hat{V}_1 = \begin{bmatrix}0,7071\\0\\-0,7071\end{bmatrix}$ $\hat{V}_2 = \begin{bmatrix}0,5774\\0,5774\\0,5774\end{bmatrix}$ $A = [\hat{V}_2 \ \ \hat{V}_2]$ $P_p = AA^T = \begin{bmatrix}0,8 & 0,3 & -0,2\\0,3 & 0,3 & 0,3\\-0,2 & 0,3 & 0,8\end{bmatrix}$

$P_p P_1 = \begin{bmatrix}0,7\\0,7\\0,7\end{bmatrix}$ $P_p P_2 = \begin{bmatrix}1,5\\1\\0,5\end{bmatrix}$ $P_p P_3 = \begin{bmatrix}1,3\\1,3\\1,3\end{bmatrix}$ $P_p P_4 = \begin{bmatrix}0,5\\1\\1,5\end{bmatrix}$ $P_p P_5 = \begin{bmatrix}1\\1\\1\end{bmatrix}$

$P_p P_6 = \begin{bmatrix}1,8\\1,3\\0,8\end{bmatrix}$ $P_p P_7 = \begin{bmatrix}1,6\\1,6\\1,6\end{bmatrix}$ $P_p P_8 = \begin{bmatrix}0,8\\1,3\\1,8\end{bmatrix}$





P_3 and P_5 are within the shadow

2.6 Interpreting what the SVD tells us about a mapping

Assume a function $\mathbf{f}(x) : \mathbb{R} \to \mathbb{R}^3$ that places samples along/within an arbitrary geometric shape, e.g., a spiral, cylinder, ellipsoid, etc., which is oriented along the negative z-axis. Subsequently, all samples are transformed using $\mathbf{T}$ with its *singular value decomposition* of

$$\mathbf{U} = \begin{pmatrix} \cos(\frac{\pi}{4}) & 0 & \sin(\frac{\pi}{4}) \\ 0 & 1 & 0 \\ -\sin(\frac{\pi}{4}) & 0 & \cos(\frac{\pi}{4}) \end{pmatrix}, \; \mathbf{S} = \begin{pmatrix} k & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \text{ with } k > 1, \text{ and } \mathbf{V} = \begin{pmatrix} \cos(-\frac{\pi}{4}) & 0 & -\sin(-\frac{\pi}{4}) \\ 0 & 1 & 0 \\ \sin(-\frac{\pi}{4}) & 0 & \cos(-\frac{\pi}{4}) \end{pmatrix}.$$

Now find out the following:

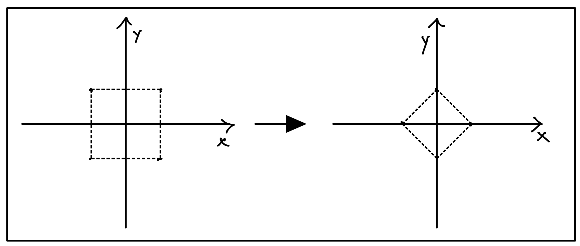
1. Interpret $\mathbf{U}$, $\mathbf{S}$, and $\mathbf{V}$ and precisely describe their transformations. Make sure you can predict the final result as well as the geometric shape after each individual transformation.

How would the repeated application of $\mathbf{T}$ change the result, if $\mathbf{U}$ and $\mathbf{V}$ were unchanged, but

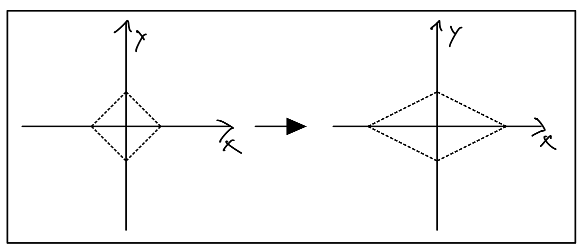
2. $\mathbf{S}$ would be $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{k} \end{pmatrix}$ or $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{k} & 0 \\ 0 & 0 & \frac{1}{k} \end{pmatrix}$ with $k > 1$ instead?

1.

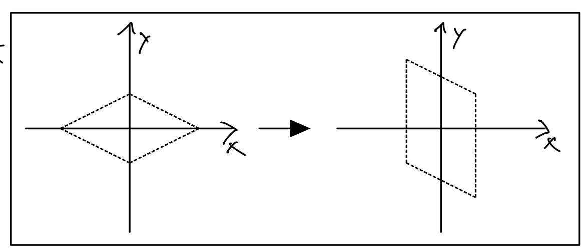
U corresponds to 45° rotation around y-axis



S is "stretching" points in the x-axis of base U. k is how much stretching



V is rotating back to the original basis and keeping the stretch from S



2.

$$S = \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1/k \end{bmatrix} \\ \begin{matrix} x & \;\; y & \;\; z \end{matrix} \end{array}$$

S squeezes z

$$S = \begin{array}{c} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/k & 0 \\ 0 & 0 & 1/k \end{bmatrix} \\ \begin{matrix} x & \;\; y & \;\; z \end{matrix} \end{array}$$

S squeezes y and z

2.7 Eigenvectors & Eigenvalues

For matrix

$$\mathbf{A} = \begin{pmatrix} 8 & 2 & 2 \\ 6 & 4 & 6 \\ 4 & 5 & 3 \end{pmatrix}$$

decide which of the following vectors, if any, are eigenvectors of $\mathbf{A}$.
In case you identify an eigenvector, find the corresponding eigenvalue.

$$\mathbf{v}_1 = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}, \ \mathbf{v}_2 = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}, \ \mathbf{v}_3 = \begin{pmatrix} -1 \\ 1 \\ 1 \end{pmatrix}, \ \mathbf{v}_4 = \begin{pmatrix} 3 \\ -5 \\ 1 \end{pmatrix}, \ \mathbf{v}_5 = \begin{pmatrix} 0 \\ 5 \\ -5 \end{pmatrix}$$

$$Av_1 = \begin{bmatrix} 8 & 2 & 2 \\ 6 & 4 & 6 \\ 4 & 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 8+4+6 \\ 6+8+18 \\ 4+10+9 \end{bmatrix} = \begin{bmatrix} 18 \\ 32 \\ 23 \end{bmatrix}$$ Not eigenvector

$$Av_2 = \begin{bmatrix} 8 & 2 & 2 \\ 6 & 4 & 6 \\ 4 & 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} -2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -16+0+2 \\ 0+0+6 \\ 4+0+3 \end{bmatrix} = \begin{bmatrix} -14 \\ 6 \\ 7 \end{bmatrix}$$ Not eigenvector

$$Av_3 = \begin{bmatrix} 8 & 2 & 2 \\ 6 & 4 & 6 \\ 4 & 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -8+2+2 \\ -6+4+6 \\ -4+5+3 \end{bmatrix} = \begin{bmatrix} -4 \\ 4 \\ 4 \end{bmatrix}$$ Eigenvector $\lambda = 4$

$$Av_4 = \begin{bmatrix} 8 & 2 & 2 \\ 6 & 4 & 6 \\ 4 & 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix} = \begin{bmatrix} 24-10+2 \\ 18-20+6 \\ 12-25+3 \end{bmatrix} = \begin{bmatrix} 16 \\ 4 \\ -10 \end{bmatrix}$$ Not eigenvector

$$Av_5 = \begin{bmatrix} 8 & 2 & 2 \\ 6 & 4 & 6 \\ 4 & 5 & 3 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 5 \\ -5 \end{bmatrix} = \begin{bmatrix} 0+10-10 \\ 0+20-30 \\ 0+25-15 \end{bmatrix} = \begin{bmatrix} 0 \\ -10 \\ 10 \end{bmatrix}$$ Eigenvector $\lambda = 2$