Computational Fluid Dynamics

Introduction to Turbulence I

Lecture 6

Krister Wiklund
Department of Physics
Umeå University

SUMMARY OF LECTURE: INTRODUCTION TO TURBULENCE

- Be aware of general properties of turbulent flow
- Be aware of some instabilities important to turbulent flow
- Be able to describe the general characteristics of grid turbulence in a windtunnel
- Understand the concept of Energy cascade
- Be aware of the inverse cascade for 2D-turbulence
- Be aware Kolmogorov's famous "-5/3-law"
- Be aware of the concept of Vortex stretching and its connection to the energy cascade

Movie: Airfoil NACA4412 Re=4E5

A first picture of turbulence

- **Turbulence** = Blobs (eddies) of intense vorticity
- Stretching drives blobs into vortex tubes
- Each tube generate a velocity field around itself that makes it propel forward
- Each tube is affected by all surrounding tubes by their velocity fields

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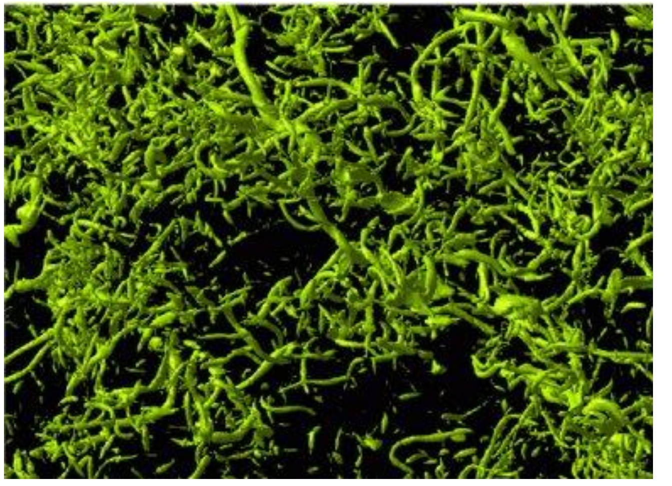
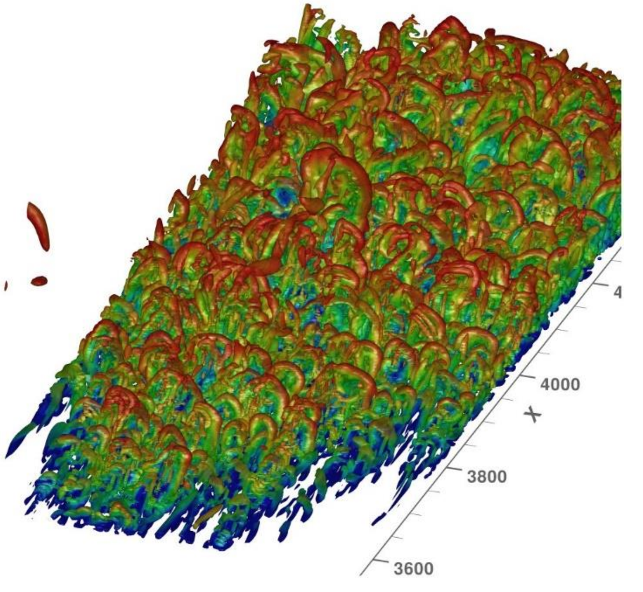
- Motion of vortex tube is due to two sources:
  1. Self propelling
  2. Surrounding vortex tubes
- Many vortices make the total flow chaotic

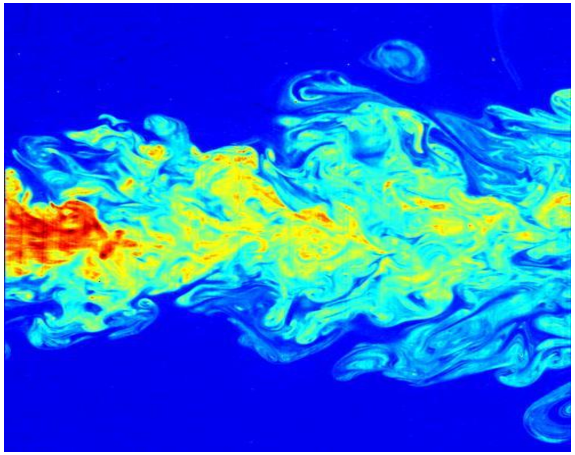




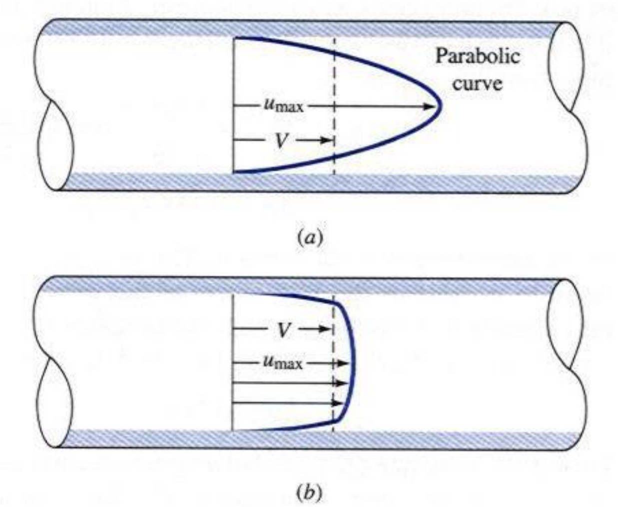
How do we visualize vortex tubes?

We need some criteria to choose what is inside and outside the tube.

General properties of turbulent flows



- Fast irregular and chaotic 3D-motion
- Fast dissipation of energy through viscous shear stress, to sustain flow an energy source is required
- Highly diffusive. Rapid mixing increases momentum transfer
- Laminar/turbulent transition monitored by Reynolds number



$$\mathrm{Re} = \frac{inertia\ force}{viscous\ force} = \frac{\rho U L}{\mu}$$

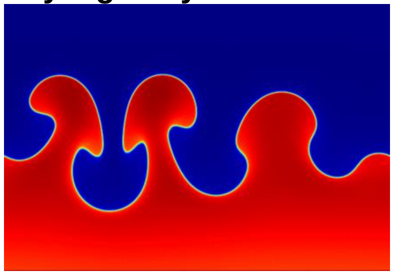
Instabilities: From laminar flow to turbulence

- Small disturbance to a laminar flow
- Transition process
- Linear and non-linear effects

COMSOL Demo
- Rayleigh-Taylor
- Kelvin-Helmholtz

Some flow instabilities
- *Rayleigh-Taylor* (density difference)
- *Kelvin-Helmholtz* (shear velocity)
- *Tollmien-Schlichting* (viscosity driven)

Rayleigh-Taylor



Density difference

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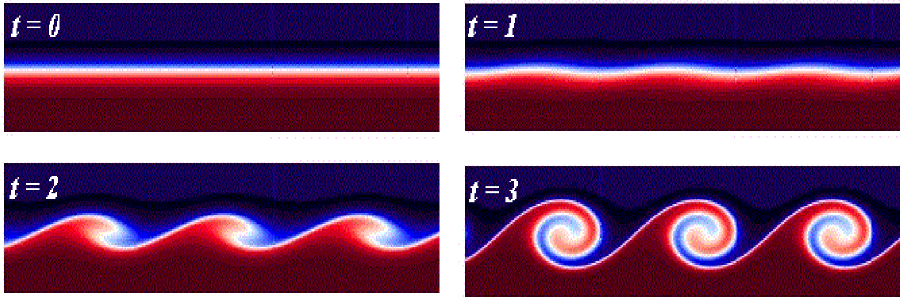
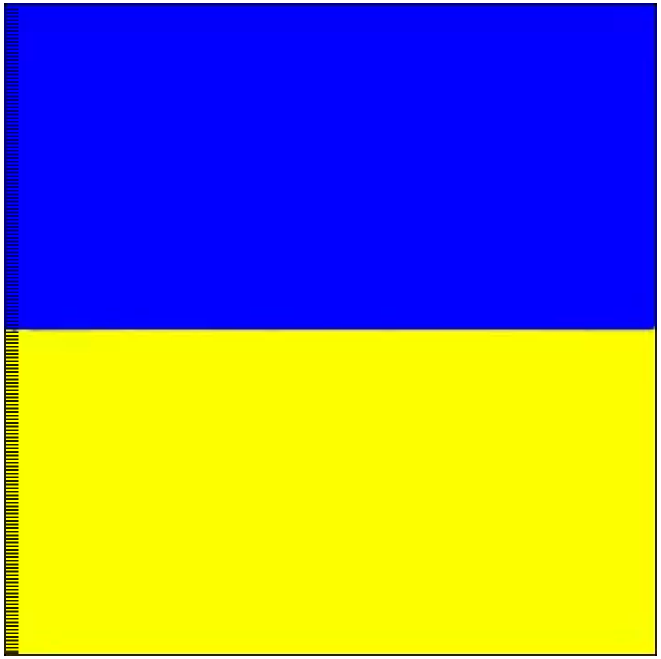
Two-fluid approach

Kelvin-Helmholtz

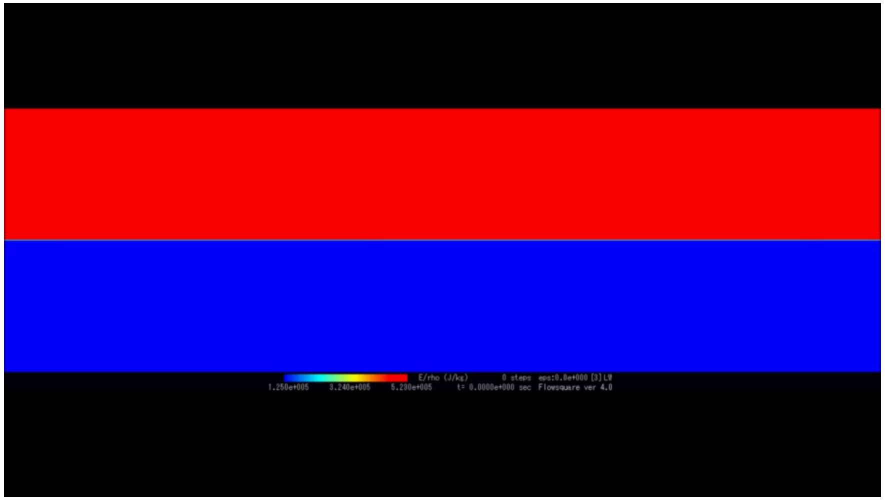


Examples of unstable perturbations and their time evolution

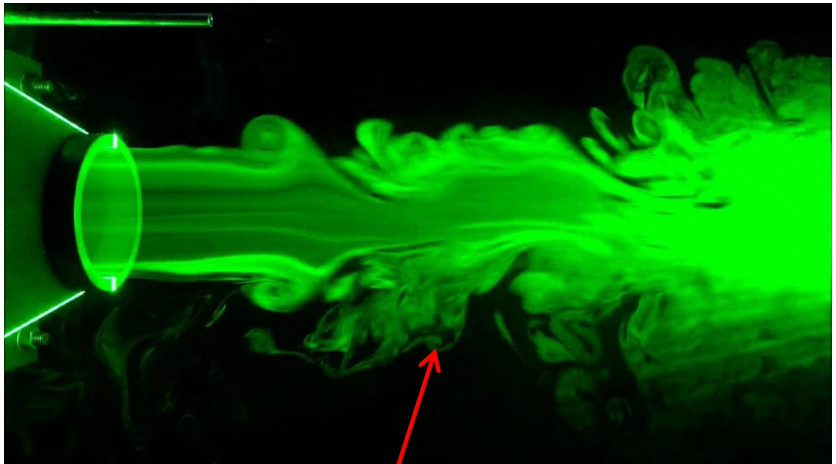
Rayleigh-Taylor Instability



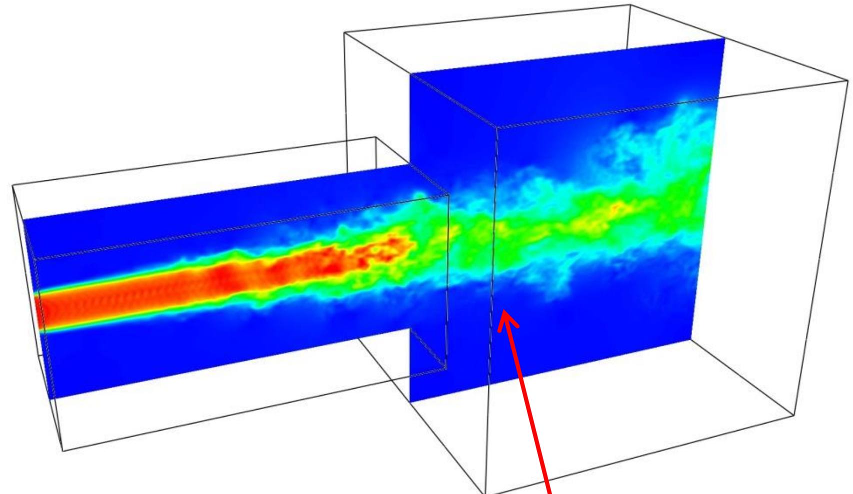
Kelvin-Helmholtz Instability



Example: Important benchmark case, the turbulent jet



Kelvin-Helmholtz instabilities



- No solid boundaries
- Self-similar far downstream
- Lot of experimental data

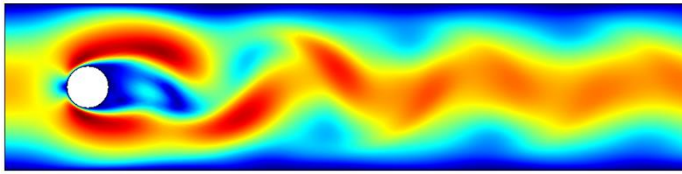
The turbulent jet is an example of free turbulence, i.e. turbulence far from boundaries.
At the exit and before that, turbulence is continously created near the wall due to high velocity gradients.

Two cases

- Free turbulence
- Near wall turbulence

Free turbulence in windtunnels

Flow around each wire in the mesh



From Davidson

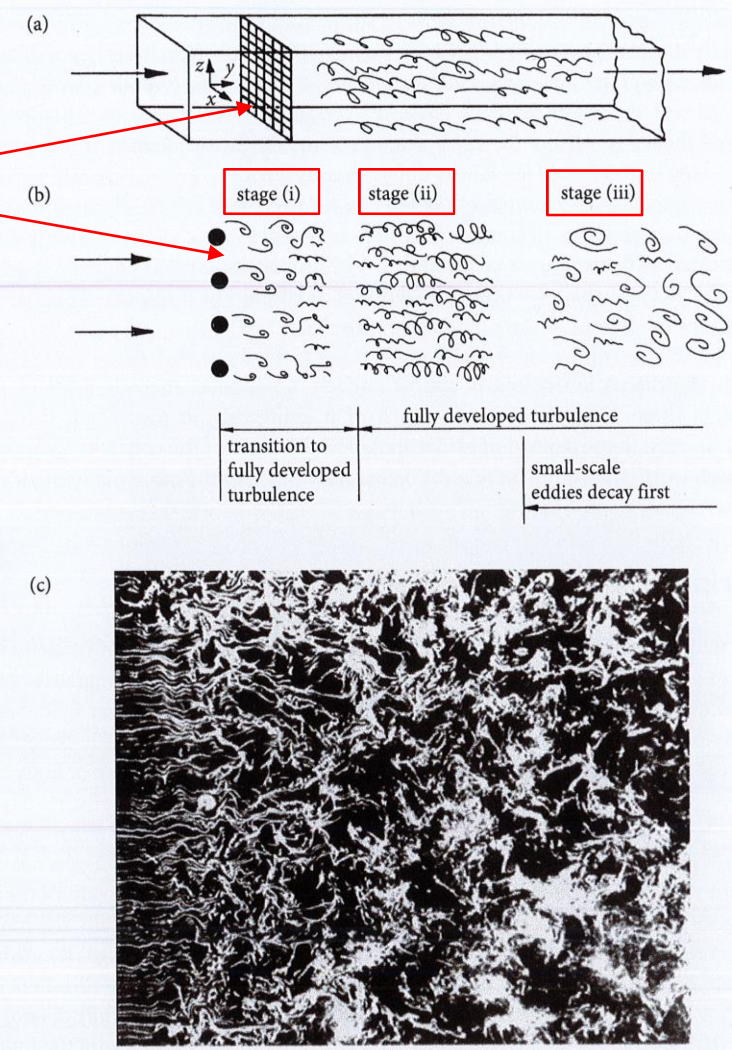


Figure 3.3 (a) The generation of grid turbulence. (b) Schematic of the various stages of development of grid turbulence. The rate of development of the turbulence in the downstream direction is exaggerated. (c) Flow visualization of grid turbulence using smoke.

Stage (i)
- Vortices are created by the mesh and travel downstream
- The vortices interacts, and finer and finer structures are created

Stage (ii)
- At some distance downstream the flow consists of structures of all scales, from initial vortex size down to microscale structures
- This is a fully developed turbulence

Stage (iii)
- At this distance no new large vortices are created by the mesh
- The smallest scale structures dissipates as heat faster than large ones and we are left with mostly large structures
- The turbulence intensity is decaying

Free turbulence in windtunnels (cont.)

$KE =$ Kinetic energy

$l =$ Initial vortex size

$\eta =$ Kolmogorov length

$k = \dfrac{2\pi}{\lambda} =$ Wave number

The initial vortex size is roughly the size of the mesh wires and represents the length scale at where the energy input to turbulence take place.

At the Kolmogorov length scale kinetic energy is converted to heat via friction, and represents the smallest scale in turbulence.

Very important to know if we want to design a numerical simulation of a turbulent flow…

From Davidson

Action of nonlinear term in Navier-Stokes

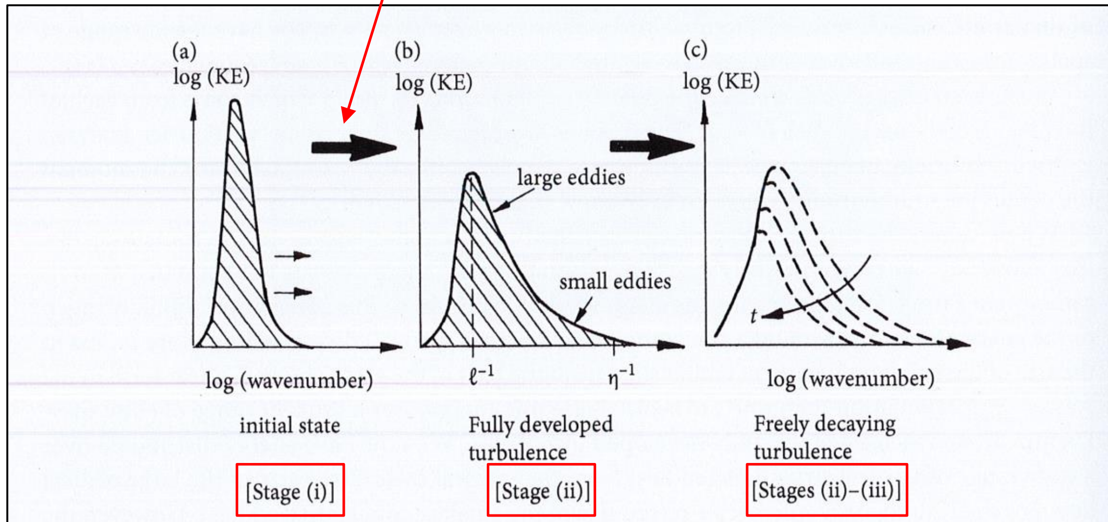


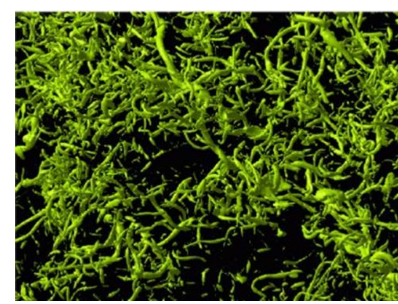
Figure 3.5 The variation of energy with eddy size at different times in the decay of grid turbulence.

Redistribution of energy

Kolmogorov scales reached

Dissipation of energy

Kolmogorov scale eddies



Free turbulence in windtunnels (cont.): Energy and Enstrophy

If we make super detailed velocity measurements in the windtunnel we can determine the kinetic energy and enstrophy for eddies of different sizes (Vol):

Kinetic energy

$$KE \equiv \int_{Vol} \tfrac{1}{2} u^2 dr$$

Enstrophy

$$Z \equiv \int_{Vol} \tfrac{1}{2} \omega^2 dr$$

From Davidson

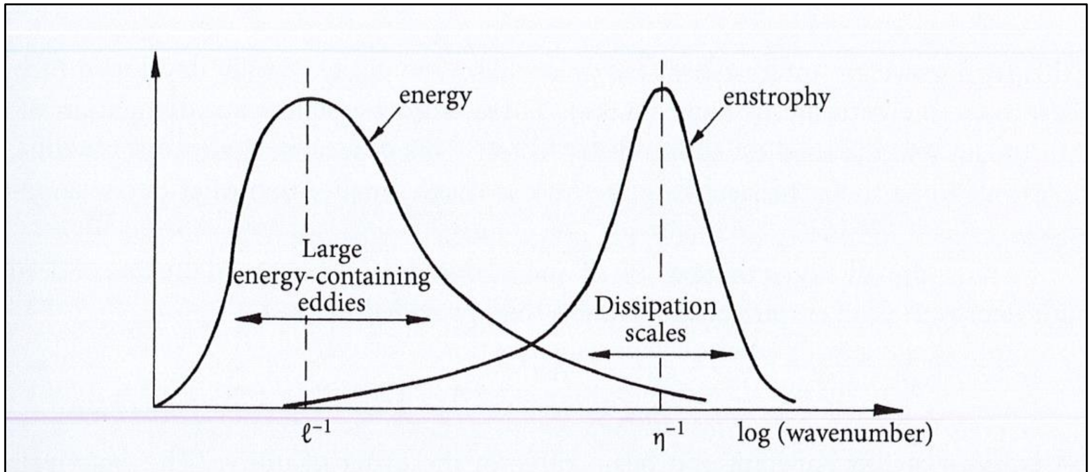


Figure 3.6 The distribution of energy and enstrophy in fully developed turbulence.

The Kinetic energy is distributed around the injection scale, and the Enstrophy is distributed around the Kolmogorov scale, the scale where most dissipation take place.

This distribution make sense since we at previous lecture derived:

$$\boxed{\frac{D}{Dt} KE = -2\nu Z}$$

High Enstrophy generate fast decay of Kinetic energy

Large eddies contain most Kinetic energy since the small eddies are randomly oriented and the velocity field generated by them tend to cancel on average.

The small eddies are tubes of intense vorticity and naturally contribute more to Enstrophy than the slowly rotating large eddies.

Experimental measurements in windtunnels

From Davidson

Measurement by a velocity-probe following the flow along the windtunnel

Many frequencies due to interaction with local surrounding

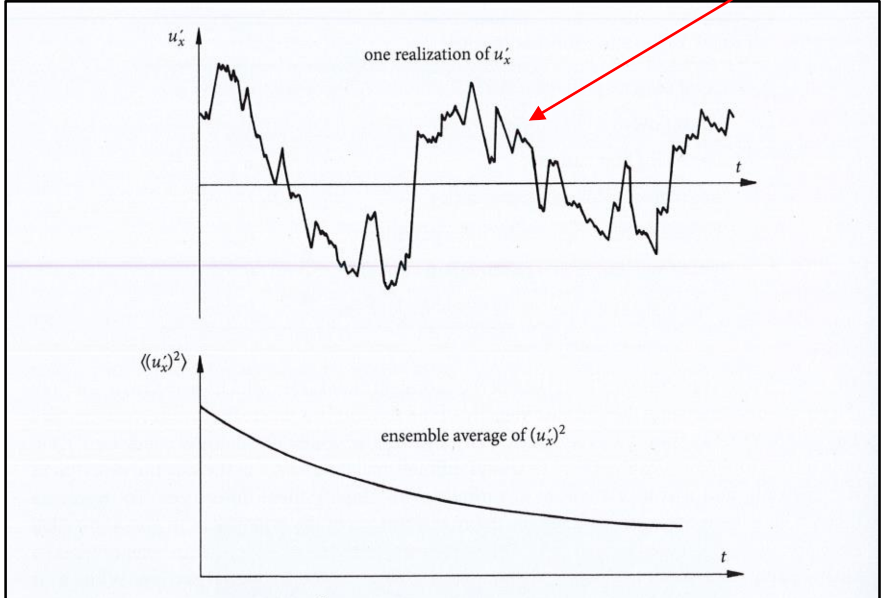


Figure 3.4 $u_x'(t)$, and hence $(u_x')^2(t)$, is a highly irregular function of time in any one realization. However, the ensemble average of $(u_x')^2$ is a smooth function of time.

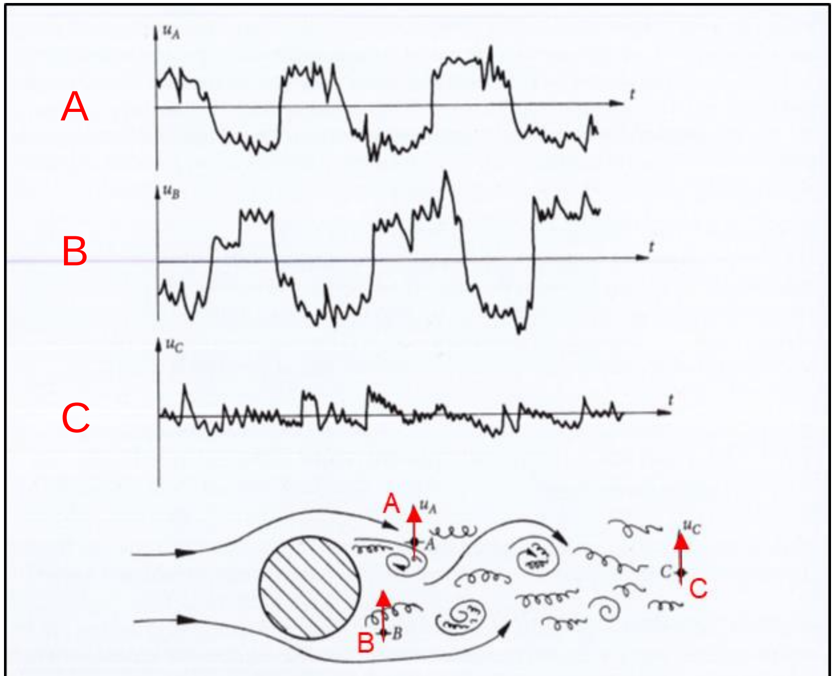


Figure 3.12 Traces of velocity against time. Points A and B are statistically correlated while A and C are only weakly correlated.

Individual measurements are random, but the statistics of the flow over many measurements is smooth.

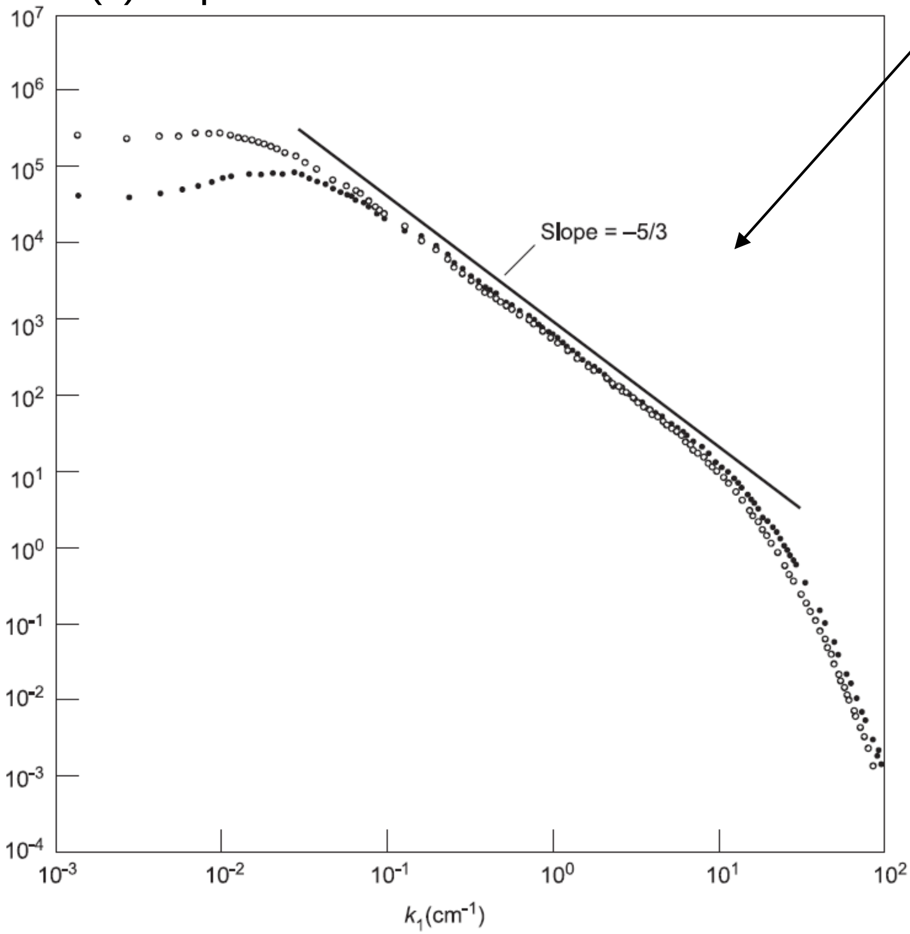
$$\langle u_A u_B \rangle_T \neq 0$$ Correlated

$$\langle u_A u_c \rangle_T \sim 0, \ \langle u_B u_c \rangle_T \sim 0$$ Uncorrelated

Statistical approaches are important and the use of averaging is a common method in turbulence. More about this in the next lecture…

Experimental measurements in windtunnels (cont.): Kinetic energy

E(k) Experimental data

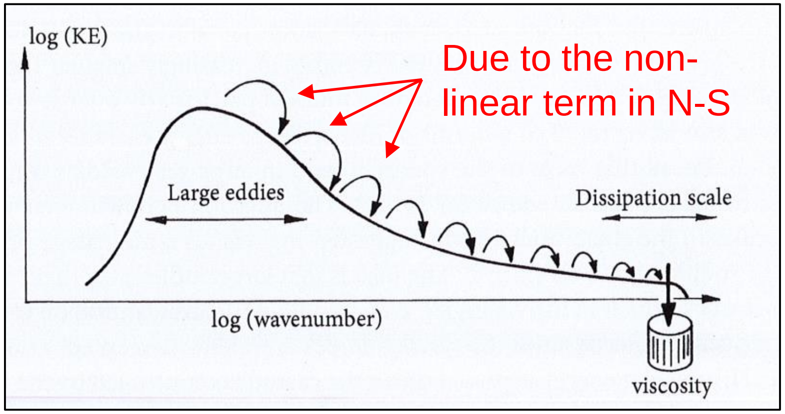


Large scale

Small scale

Why do we have this universal distribution of energy?

The physical process: Energy cascade



$k \sim (length\,scale)^{-1}$

1) Large eddies extract energy from mean flow

2) Due to the non-linear term in N-S the large eddies breaks into smaller ones until all kinetic energy is dissipated into heat

The Energy cascade concept

Laminar flow

Large scale vortices

↓ Viscosity

Heat

Turbulent flow

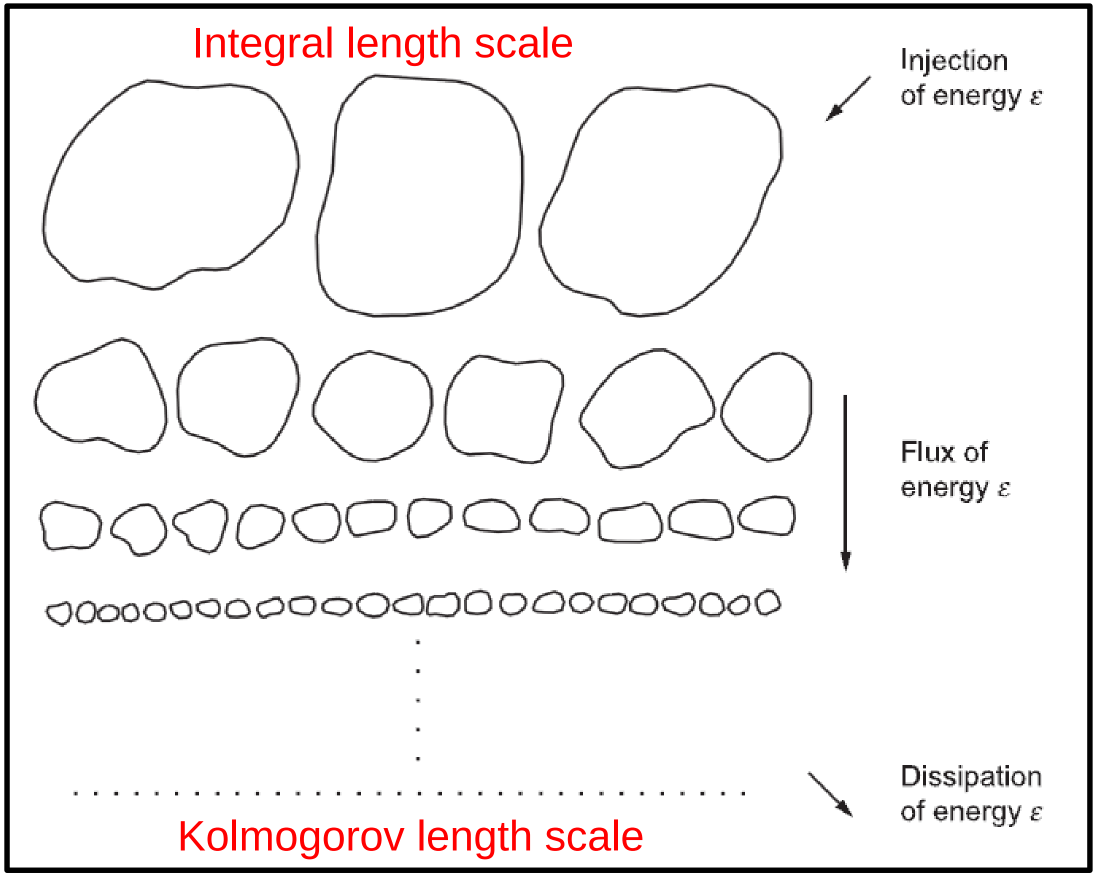
Large scale vortices

↓

Smaller vortices

↓ Viscosity

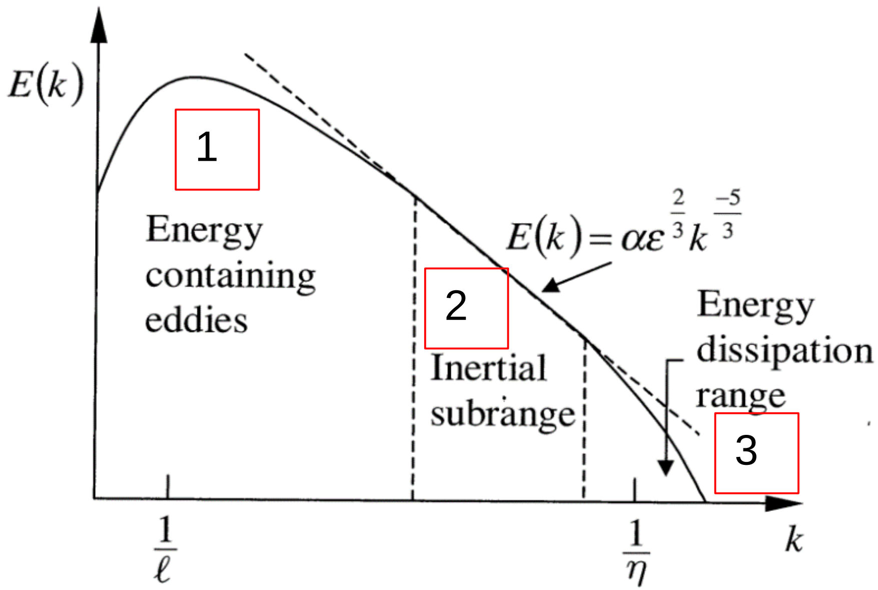
Heat



Largest vortices typically scale with the flow geometry:

- Pipe diameter
- BL thickness

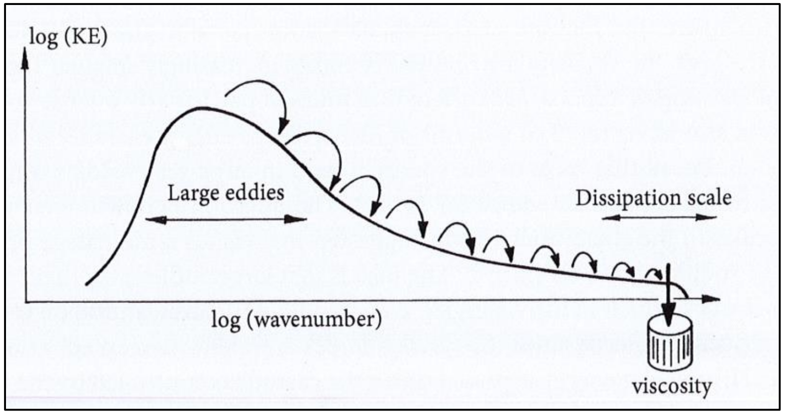
Summary of the energy cascade concept: Kolmogorovs "-5/3 law"



1. Large eddies carries most of the energy. These eddies interact with the mean flow and extract energy from it,

2. Inertial subrange. Require high Re and fully developed turbulence. Flow assumed isotropic (the same in all directions). Eddies are independent of large and small scale eddies in 1. and 3.

3. Dissipation range. Small eddies, assumed isotropic, dissipate energy to heat.



- The term "Inertial subrange" is connected with importance of inertial term in Navier-Stokes

- The energy spectrum dependency on dissipation rate ε and length scale L can be found by dimension analysis to be

$$E(k) = \alpha\varepsilon^{\frac{2}{3}} k^{-\frac{5}{3}}$$

Most important cascade mechanism: Vortex stretching and tilting

$$\boxed{\frac{D\boldsymbol{\omega}}{Dt} = (\boldsymbol{\omega}\cdot\nabla)\mathbf{u} + \frac{\mu}{\rho}\nabla^2\boldsymbol{\omega}}$$

Vorticity equation

$$\left(\boldsymbol{\omega} \equiv \nabla\times\mathbf{u}\right)$$

Note: Alternative form

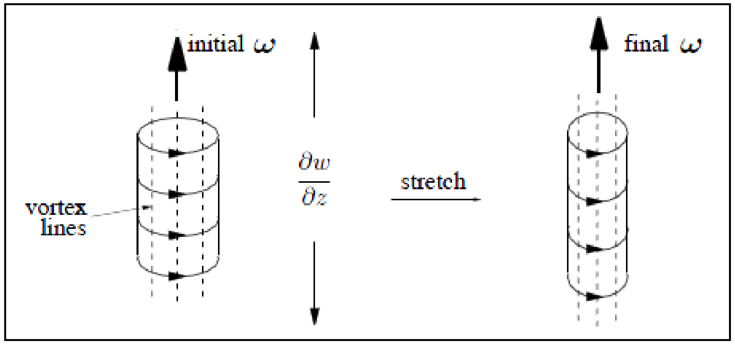
$$\rho\frac{\partial\boldsymbol{\omega}}{\partial t} = \rho\nabla\times\left(\mathbf{V}\times\boldsymbol{\omega}\right) + \mu\nabla^2\boldsymbol{\omega}$$

Inviscid case

$$\frac{D\boldsymbol{\omega}}{Dt} = (\boldsymbol{\omega}\cdot\nabla)\mathbf{u}$$

Ex. Initially vertical vortex tube

$$\frac{D\boldsymbol{\omega}}{Dt} = (\boldsymbol{\omega}\cdot\nabla)\mathbf{u} = \underbrace{\omega\frac{\partial u}{\partial z}\mathbf{e}_x + \omega\frac{\partial v}{\partial z}\mathbf{e}_y}_{\text{Tilting}} + \underbrace{\omega\frac{\partial w}{\partial z}\mathbf{e}_z}_{\text{Stretching}}$$



From lecture notes by David Marshall

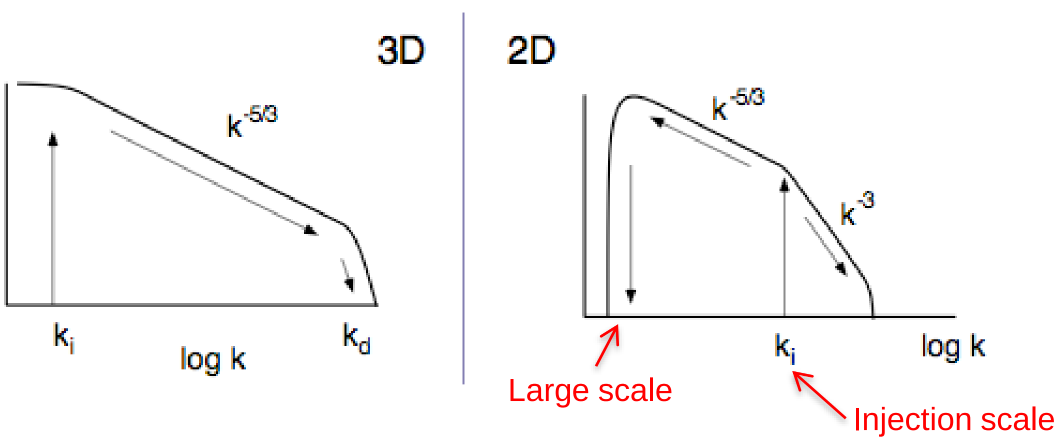
2D-turbulence: The inverse cascade

2D flow: $\boldsymbol{\omega} = \omega(x, y)\hat{\mathbf{z}}, \ \mathbf{u} = \mathbf{u}(x, y)$

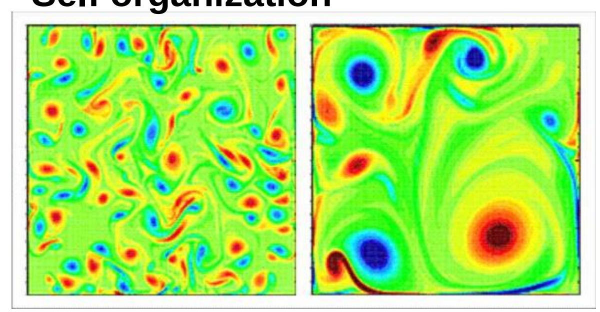
$$\boxed{(\boldsymbol{\omega}\cdot\nabla)\mathbf{u} = 0 \quad \Rightarrow \quad \frac{D\boldsymbol{\omega}}{Dt} = \frac{\mu}{\rho}\nabla^2\boldsymbol{\omega}}$$

No stretching or tilting

- If no viscosity & no forcing, the fluid particles conserve its vorticity
- 2D experiment: Soap film flow
- Jupiter zonal flows



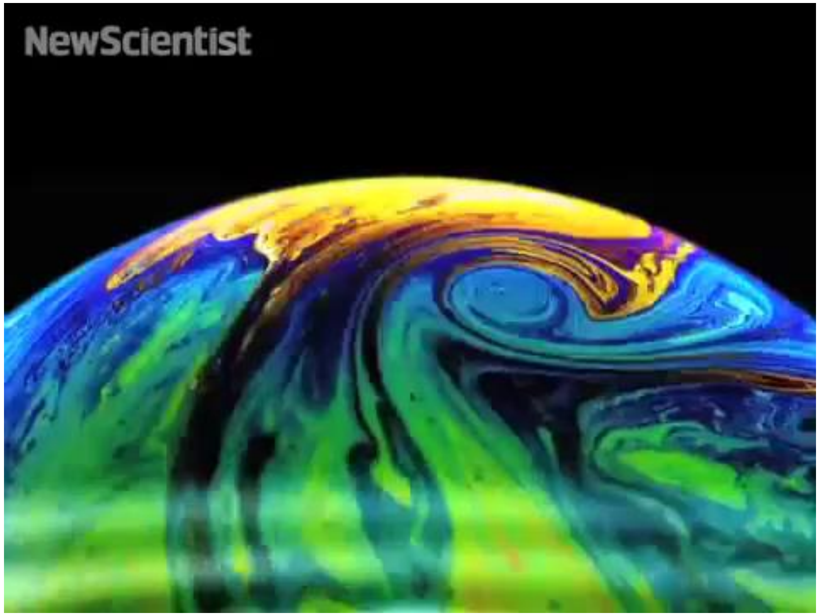
Self organization

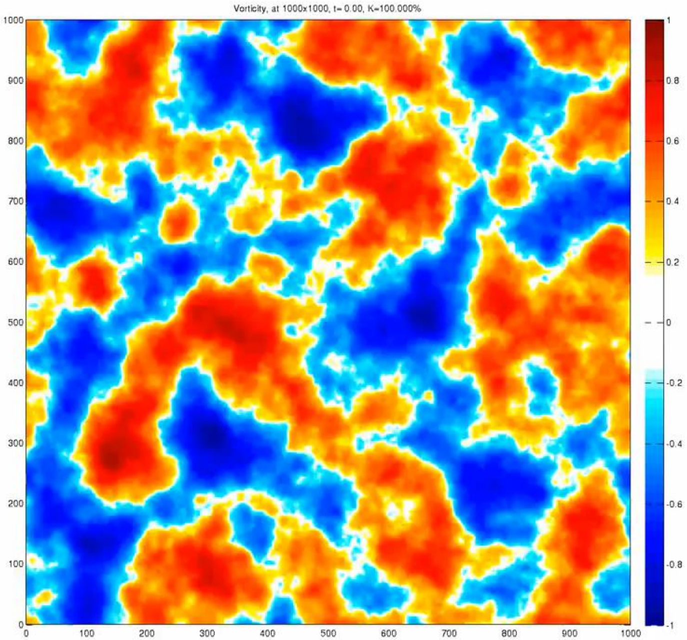


From lecture "Two dimensional turbulence" by A. S. Lanotte 2007

Movies:

1) Soap bubble dynamics
2) 2D decaying turbulence





2D-turbulence (cont.)

Why does the kinetic energy end up in large scale structures in 2D-turbulence?

From Lecture 4:

$$\boxed{\frac{D}{Dt}KE = -2\nu Z}$$

$$\begin{cases} KE \equiv \int_{Vol} \tfrac{1}{2} u^2 dr & \text{Kinetic energy} \\ Z \equiv \int_{Vol} \tfrac{1}{2} \omega^2 dr & \text{Enstrophy} \end{cases}$$

2D: No Vortex stretching

$$\boxed{\frac{DZ}{Dt} = \int_{Vol} \left[\boldsymbol{\omega} \cdot \underbrace{\left(\boldsymbol{\omega} \cdot \nabla\right)\mathbf{u}}_{=0} \right] dr - \nu \int_{Vol} \left(\nabla \times \boldsymbol{\omega}\right)^2 dr}$$

In 2D the Enstrophy is limited by its initial value, it can only decrease

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Consider the case of high to super high Re (think lowering to super low viscosity**)**

- In 3D: If viscosity is decreased, vortex stretching can increase Enstrophy and compensate for the decreased viscosity
- In 2D: When viscosity decreases, vortex stretching is not available and Enstrophy must decrease

$$\boxed{\frac{D}{Dt}KE = -2\nu Z}$$

➡ KE will keep decreasing

➡ During short time spans KE is nearly conserved

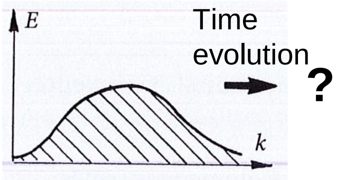
From Davidson

In 2D-turbulence *KE* is nearly conserved and *Z* is decreasing.

By using Fourier-transformation it can be shown that

$$KE = \int_0^\infty E(k)dk \qquad Z = \int_0^\infty k^2 E(k)dk$$

E(k) is the distribution of energy for different length scales *1/k*



1) If Z is decreasing in time then $E(k)$ must change in time (since k does not depend on t).

2) We know that *KE*, the integral of $E(k)$, does not change in time.

3) The integrand of Z is $k^2E(k)$, so to decrease Z in time $E(k)$ can re-distribute in time in a way that more of it ends up at small k.

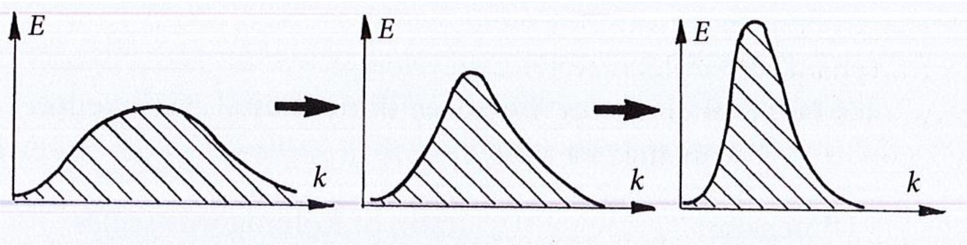


Figure 10.6 Evolution of the energy spectrum.

➡ Energy moves to small k (large vortices), but the area under the curve does not change (KE is conserved).

Kolmogorov theory 1941

K1. Kolmogorov's hypothesis of local isotropy
At sufficiently high Reynolds numbers, the **small-scale** turbulent motions are statistically <u>isotropic</u>.

The same in all directions

K2. Kolmogorov's first similarity hypothesis
In every turbulent flow at sufficiently high Reynolds number, the description of the **small scale** motions have a <u>universal form</u> that is uniquely determined by the *energy dissipation rate* and *viscosity*

Details later…

K3. Kolmogorov's second similarity hypothesis
In every turbulent flow at sufficiently high Reynolds number, the description of the motions of **intermediate scale** have a universal form that is uniquely determined by the *energy dissipation rate* only.

Give us "-5/3 law"

End of lecture