Computational Fluid Dynamics
Turbulence Models II
Near wall flows
Lecture 10

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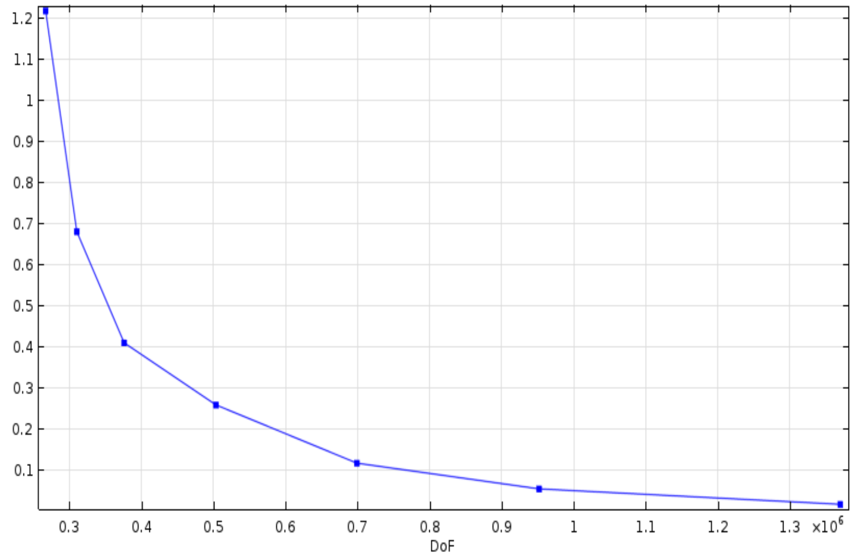
SUMMARY OF LECTURE: TURBULENCE MODELS II

- ❑ Be able to describe how a boundary layer near a wall is divided into sublayers
- ❑ Be able to mathematically describe the velocity profiles of Viscous sublayer and Log-layer
- ❑ Understand how y+ is used to describe boundary layers
- ❑ Understand how viscous stresses and turbulent stresses are related in viscous sublayer and log-layer
- ❑ Understand the origin of damping functions
- ❑ Be aware of near wall improvements of zero-equation models
- ❑ Be able to use and understand the Low Re k-epsilon models
- ❑ Understand how to analyze the near wall behavior of turbulent quantities
- ❑ Be aware of the Wall-function-approach and its strengths and weaknesses

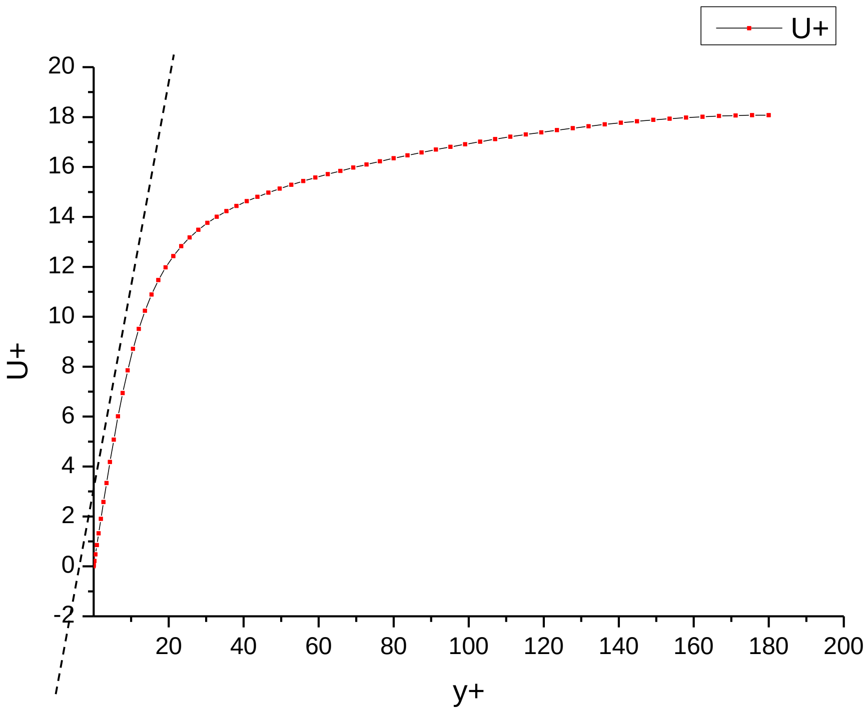
Comments on Mesh convergence and Data export

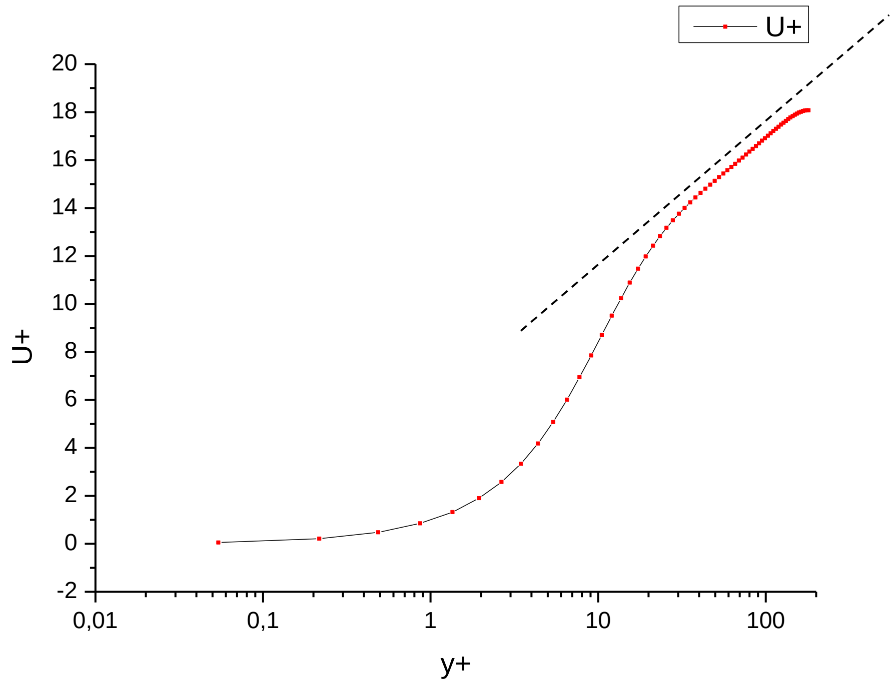
- Mesh convergence
  - Use several Monitor variables
  - Plot values agaist DoF and look for plateus



- Data export
  - Extract reduced data by using cutlines etc
  - Export (try export along Regular grid)
  - Import in Matlab and do analyses and figures

Lab2 Turbulent channel flow: DNS-near wall velocity profile





$$\boxed{U^+ \equiv \frac{U}{u_\tau}} \qquad \boxed{y^+ \equiv \frac{\rho u_\tau y}{\mu}}$$

Friction velocity

$$u_\tau \equiv \sqrt{\tau_w / \rho}$$

τ_w = wall shear stress

Dimensionless universal scaling near wall ➡ Universally valid plots

The standard *k-ε* model v.s. DNS for a Channel flow

Model k-equation

$$\frac{Dk}{Dt} = P_k - \varepsilon + D_k$$

Model ε-equation

$$\frac{D\varepsilon}{Dt} = \frac{\varepsilon}{k}\left(C_{\varepsilon 1}P_k - C_{\varepsilon 2}\varepsilon\right) + D_\varepsilon$$

k-production

$$P_k \equiv \frac{R_{ij}}{\rho}\frac{\partial U_i}{\partial x_j} = \frac{\mu_t}{\rho}S^2$$

k-diffusion

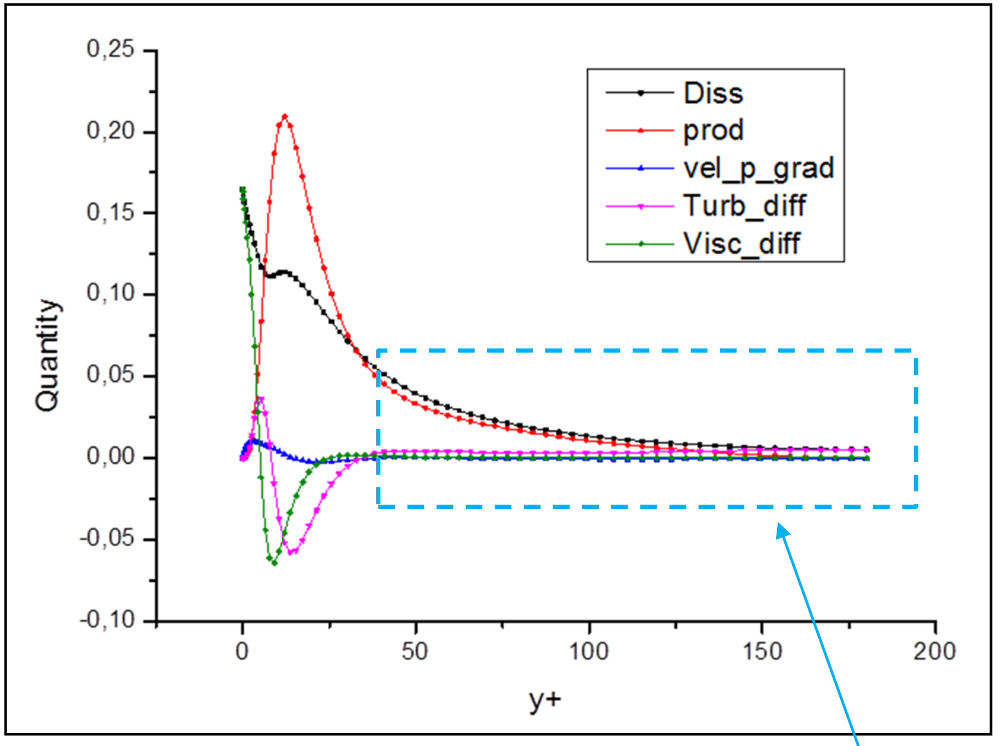
$$D_k \equiv \frac{\partial}{\partial x_j}\left[\frac{1}{\rho}\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]$$

Dissipation

$$\varepsilon \equiv \frac{\mu}{\rho}\left\langle \frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}\right\rangle$$

ε-diffusion

$$D_\varepsilon \equiv \frac{\partial}{\partial x_j}\left[\frac{1}{\rho}\left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right]$$



The standard k-ε model

Turbulent viscosity

$$\mu_t = C_\mu \rho \frac{k^2}{\varepsilon}$$

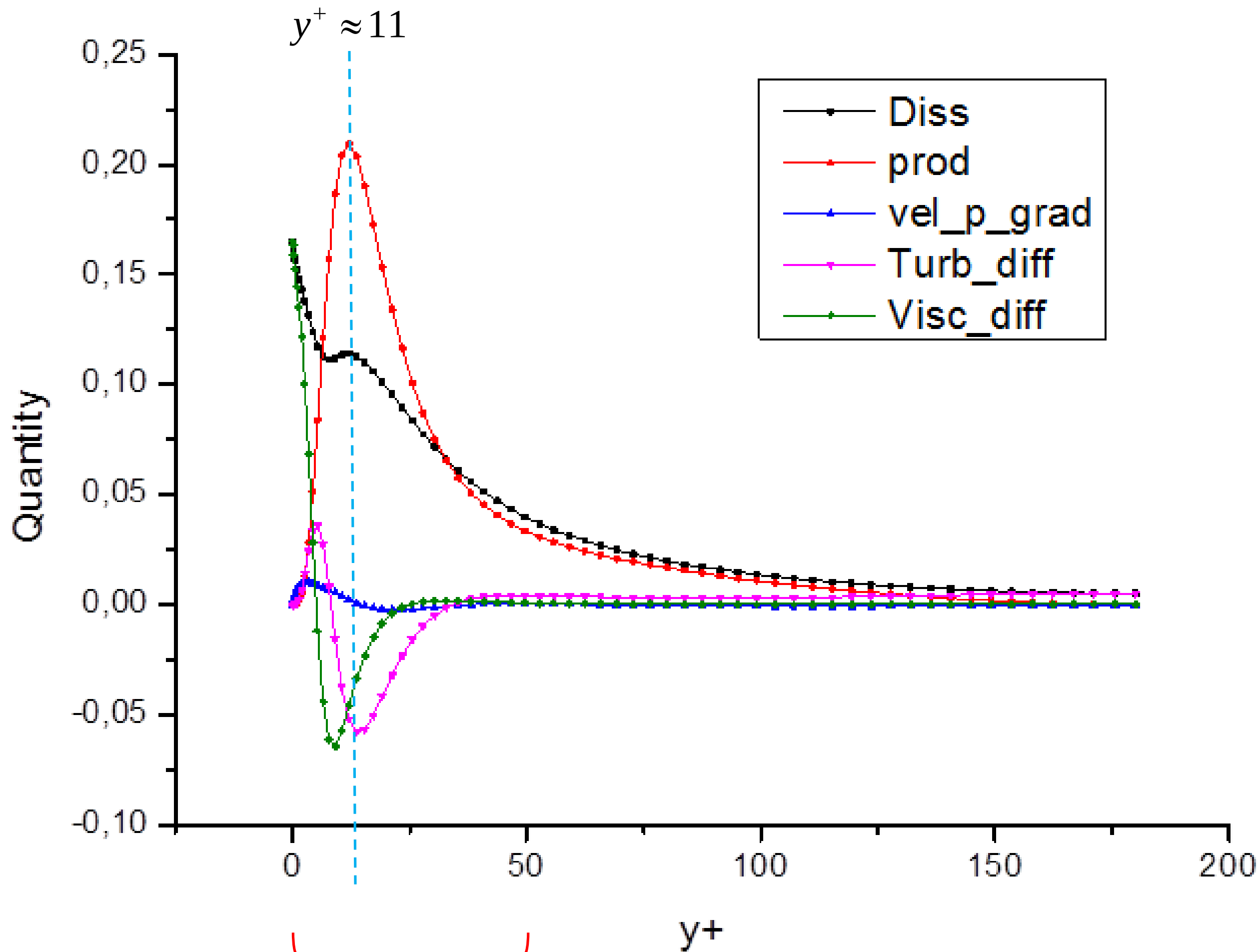
Closure constants (Launder and Sharma)

$$C_\mu = 0.09$$

$$C_{\varepsilon 1} = 1.44 \quad C_{\varepsilon 2} = 1.92$$

$$\sigma_k = 1.0 \quad \sigma_\varepsilon = 1.3$$

Near wall turbulence: DNS-database (Kim etal)



How thick is this region in reality?

$$y^+ \equiv \frac{\rho u_\tau y}{\mu}$$

Friction velocity

$$u_\tau \equiv \sqrt{\tau_w / \rho}$$

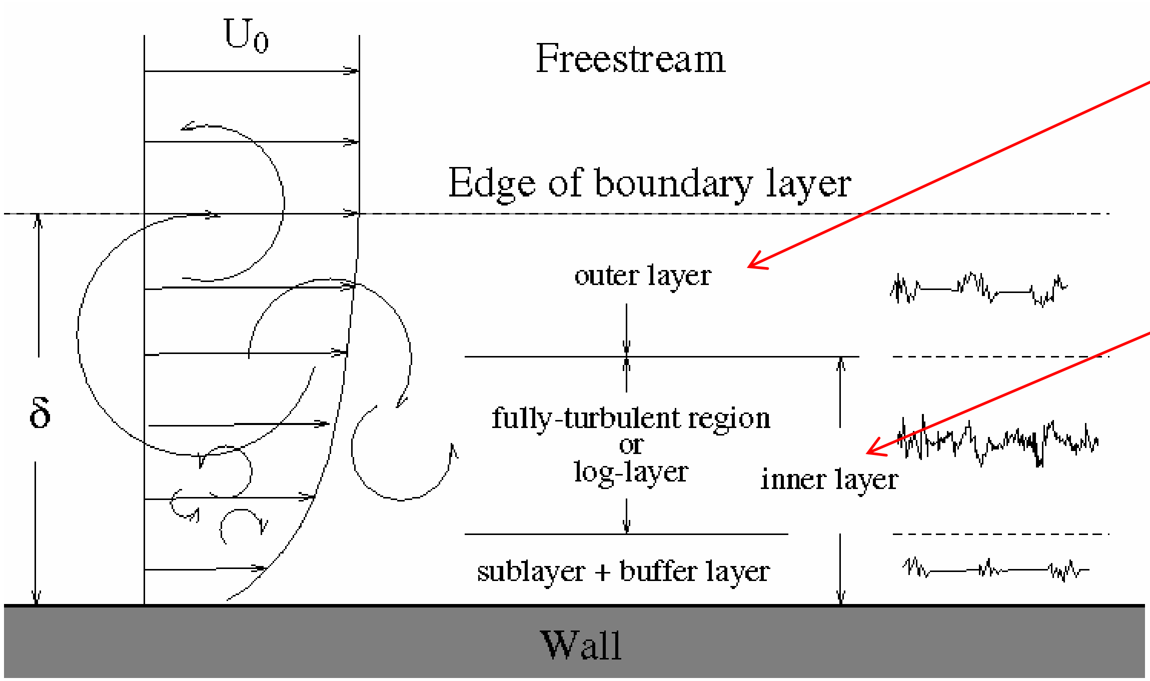
$$50 = \frac{\rho u_\tau}{\mu} y_R$$

$$y_R = 50 \frac{\mu}{\rho u_\tau}$$

Water with $u_\tau \approx 1 cm/s$

$y_R \approx 5mm$

Near wall velocity profile



Eddie size proportional to boundary layer thickness (outer variable)

Eddie size proportional to wall distance (inner variable)

Layers not in scale

From André Bakker lecture notes

Dimensionless wall distance

$$y^+ \equiv \frac{\rho u_\tau y}{\mu}$$

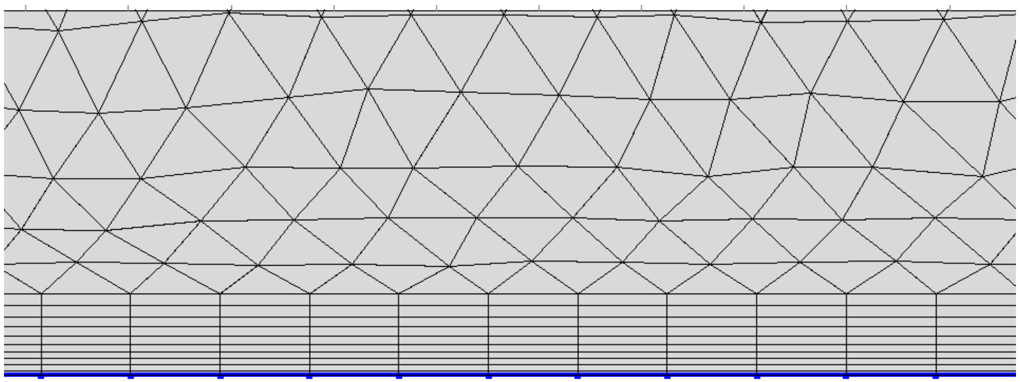
Friction velocity

$$u_\tau \equiv \sqrt{\frac{\tau_w}{\rho}}$$

$$U^+ \equiv \frac{U}{u_\tau}$$

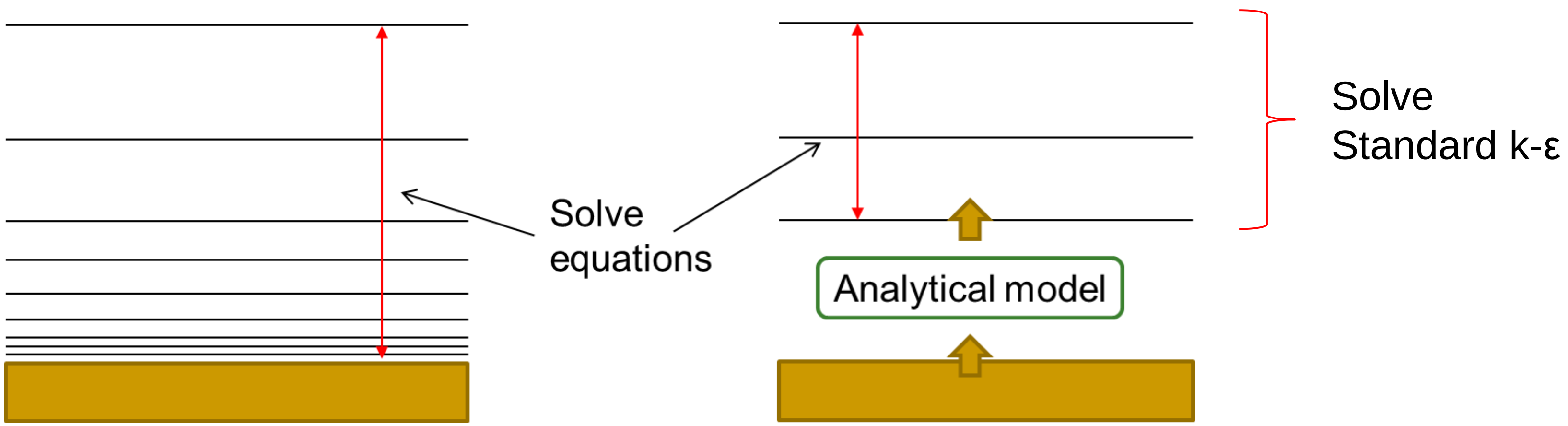
τ_w = wall shear stress

Two numerical approaches: Near-wall modeling and Wall functions



Typical mesh near a wall in Comsol

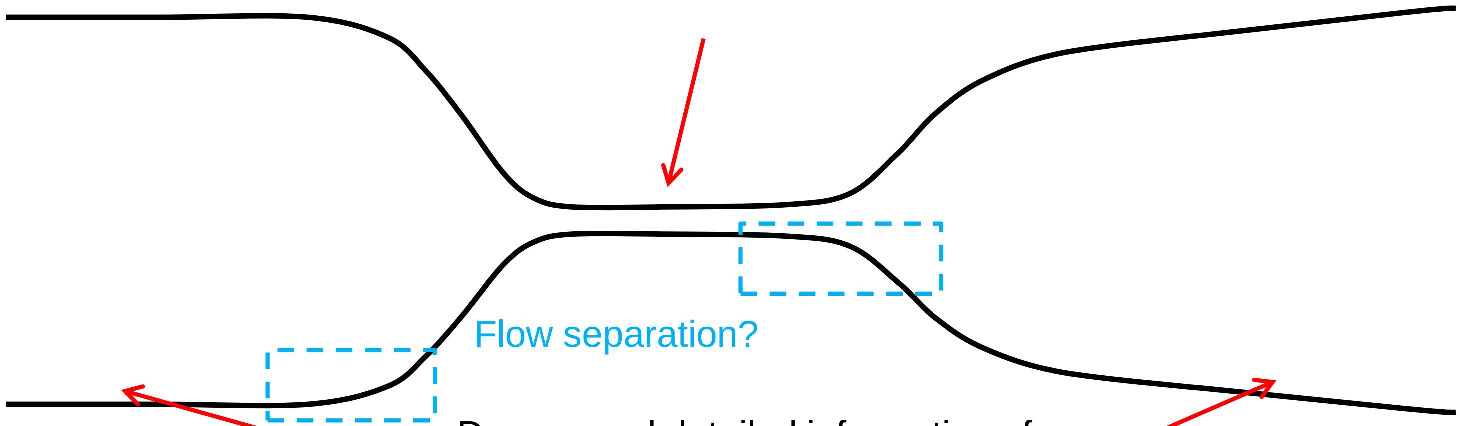
Near-wall modeling v.s. Wall functions



Example: Near wall modeling vs Wall functions in large model

If the simulation geometry are large, the use of wall functions at strategic places reduces the computational time.

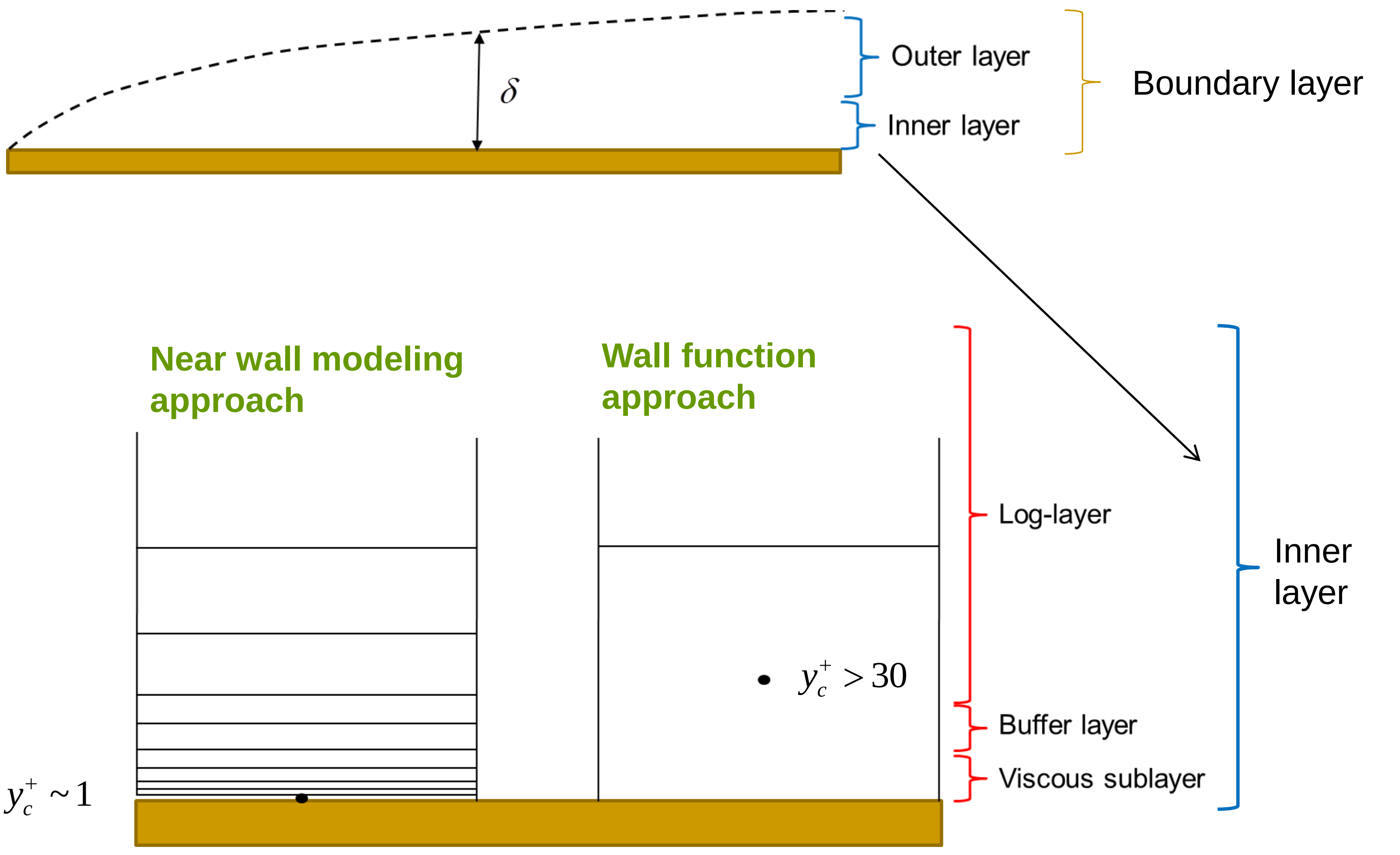
The channel is narrow here and details in the boundary layer flow might be important.
Use wall modeling here (e.g. Low Re-k-ε model)



Do we need detailed information of boundary layer flow here?
If No, apply wall functions in this region.

Many softwares (including Comsol) offers the choice of automatic wall treatment!

Near-wall modeling and Wall functions (cont.)

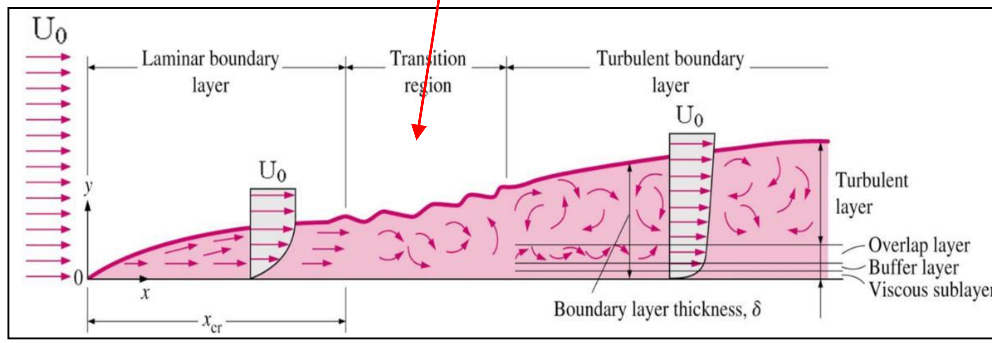


Estimation of the near wall cell size

Example: Turbulent flow over a flat plate

$$\rho = 1.3\,kg/m^3,\ \ \mu = 1.8\cdot 10^{-5}\,Pas$$
$$U = 20\,m/s,\ L = 1\,m$$

Critical value of Reynolds number: $\mathrm{Re}_{critical} \sim 5\cdot 10^5$



1) Calculate Reynolds number:

$$\mathrm{Re} = \frac{\rho U L}{\mu} = 1.4\cdot 10^5$$

$\Rightarrow$ Turbulent flow

2) Determine friction velocity

$$\boxed{y^+ \equiv \frac{\rho u_\tau y}{\mu} \qquad u_\tau \equiv \sqrt{\tau_w/\rho}}$$

Wall shear stress on a flat plate

$$\boxed{\tau_w = c_f\, \tfrac{1}{2}\rho U_0^2}$$

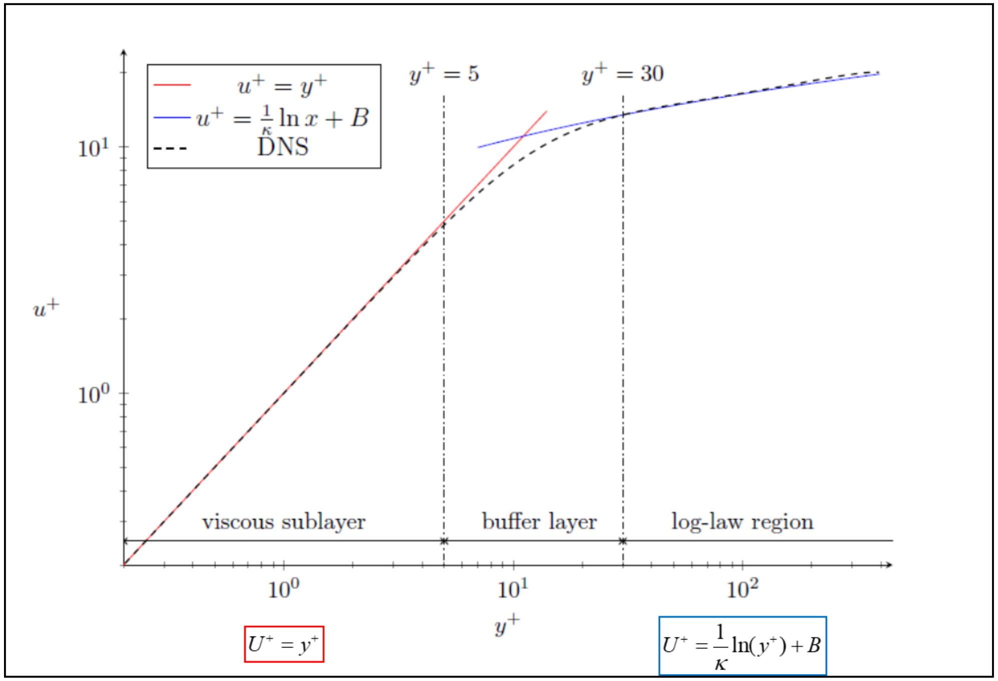
$$\begin{cases} c_f = \dfrac{0.664}{\sqrt{\mathrm{Re}}} & \text{(Laminar, Blasius)} \\ c_f = 0.058\,\mathrm{Re}^{-0.2} & \text{(Turbulent)} \end{cases} \longrightarrow \boxed{u_\tau \approx 0.82\,m/s}$$

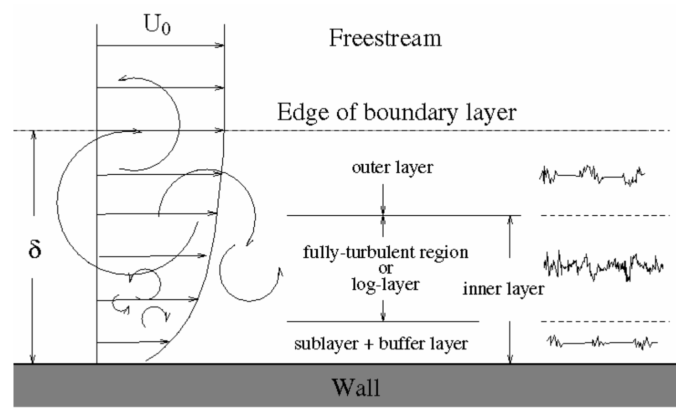
3) Determine height corresponding to chosen y+

$$y = \frac{\mu y^+}{\rho u_\tau} \Rightarrow \begin{array}{l} y_c^+ = 1 \ \rightarrow y_c = 17\,\mu m \\ y_c^+ = 30 \rightarrow y_c = 0.5\,mm \end{array}$$

$y_c =$ Center point coordinate

The structure of a turbulent boundary layer





Layer models

$$U^+ = y^+ \qquad y^+ \le 5$$

$$U^+ = \frac{1}{\kappa}\ln(y^+) + B \qquad y^+ \ge 30$$

Viscous sublayer ← Only viscous stress important

Buffer layer ← Turbulent eddies rapidly damped
Turbulent shear stress reduced

"Law of the wall"
or "log law"

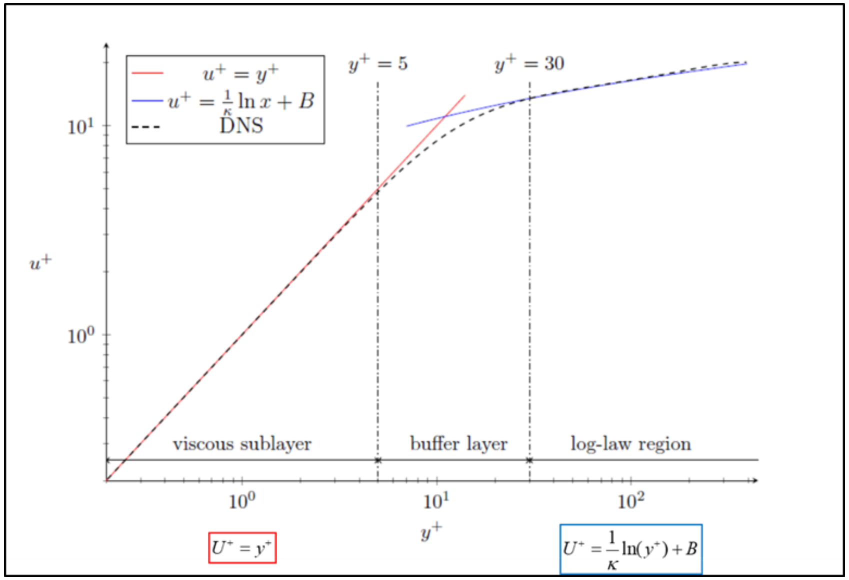
Example calculations

Example 1

At midpoint of buffer layer we have

$$\frac{u_\tau y}{\nu} = 5.5 + \frac{1}{\kappa} \ln\left(\frac{u_\tau y}{\nu}\right)$$

Solving iteratively ➡ $y^+ = \dfrac{u_\tau y}{\nu} \approx 11.2$



Example 2

From project: Channel flow of length 4m, height 0.1m and at Re=6000

$$\left.\begin{aligned} \rho &= 1\,kg/m^3 \\ \mu &= 17 \cdot 10^{-6}\ Pas \\ u_\tau &= 0.04\,m/s \end{aligned}\right\}$$

Upper part of viscous sublayer

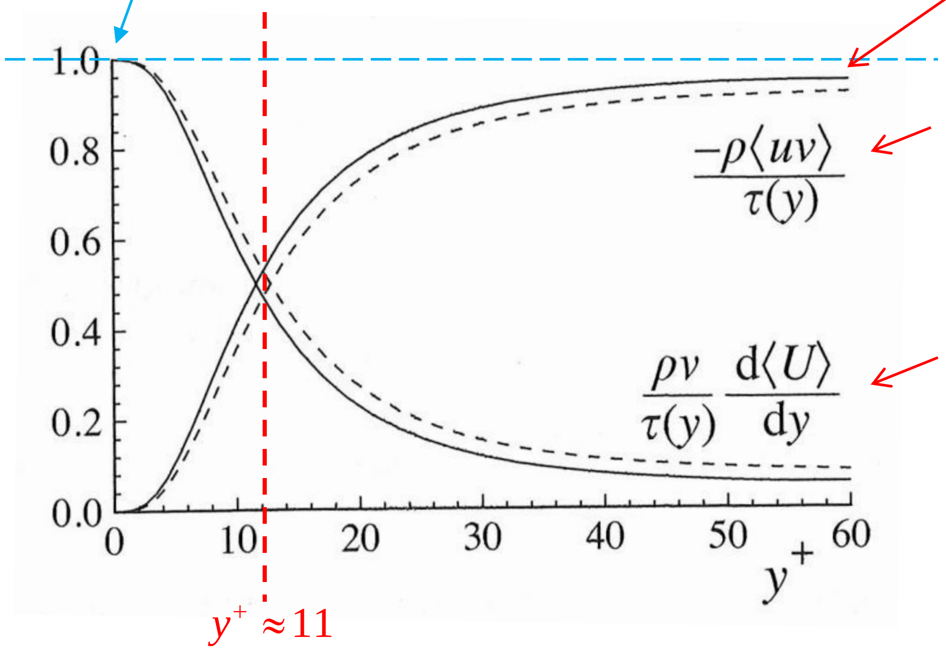
$$y^+ = 5 \;\Rightarrow\; y = \frac{5\mu}{\rho u_\tau} = 2.1mm$$

DNS data: Viscous and turbulent stress near wall

At wall: $\begin{cases} \tau_w = \rho\nu \left. \dfrac{d\langle U\rangle}{dy} \right|_{y=0} \\ \tau = \tau_w \end{cases}$

Solid and dashed lines corresponds to a sensitivity study with two different Re

$-\rho\langle uv\rangle \approx \tau_w$



Relative <u>turbulent</u> stress contribution

$\boxed{R_{ij} \equiv -\langle \rho u_i' u_j' \rangle}$

Relative <u>viscous</u> stress contribution

$\boxed{T_{ij} = \mu\left(U_{i,j} + U_{j,i}\right)}$

Near the wall the turbulent stress contribution goes to zero.

Fig. 7.4. Profiles of the fractional contributions of the viscous and Reynolds stresses to the total stress. DNS data of Kim *et al.* (1987): dashed lines, Re = 5,600; solid lines, Re = 13,750.

Near wall improvment of turbulent viscosity model

$$\boxed{\mu_t = C_\mu \rho \frac{k^2}{\varepsilon}} \longrightarrow C_\mu = \frac{\mu_t}{\rho}\frac{\varepsilon}{k^2}$$

The figure shows that

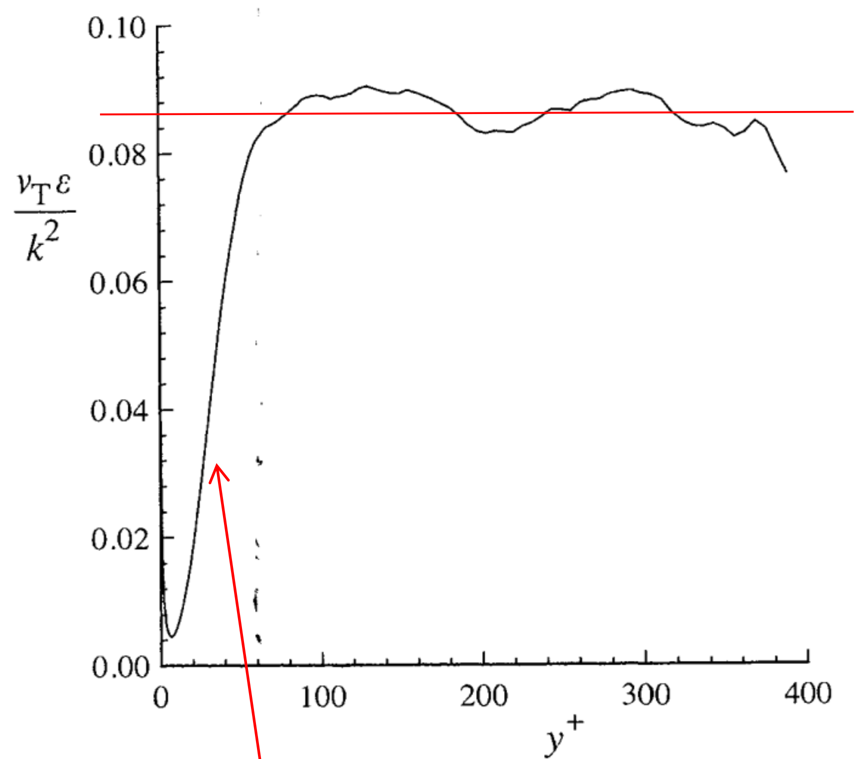
$$C_\mu = \frac{\nu_T \varepsilon}{k^2} \approx 0.09 \ at \ y^+ > 60$$

In order to model the region close to the boundary correctly we introduce a "damping function" f_μ

$$\boxed{\mu_t = f_\mu C_\mu \rho \frac{k^2}{\varepsilon}}$$

$$\Rightarrow \quad \frac{\nu_T \varepsilon}{k^2} = f_\mu C_\mu$$

Channel flow



f_μ should be designed to mimic the behavior near the boundary

Near wall improvment of turbulent viscosity model (cont)

Jones and Launder (1972)

Experiments (and DNS) shows that the turbulent viscosity closure coefficient must decrease near walls.

Exercise

Read, by teacher specified, parts of scientific paper by Jones and Launder.

$$\begin{cases} \mu_t = f_\mu C_\mu \rho \dfrac{k^2}{\tilde{\varepsilon}} \\[2ex] f_\mu = \exp\left(\dfrac{-2.5}{1+\mathrm{Re}_T/50}\right) \end{cases}$$

Small near wall and close to one far away

Damping function

$$\mathrm{Re}_T \equiv \frac{\rho k^2}{\mu \tilde{\varepsilon}}$$

Abe, Kondoh and Nagano (1994)

A part of the the AKN-model used in Comsol CFD-module:

$$f_\mu = \left(1-\exp(-l^*/14)\right)^2 \left[1+5R_t^{-3/4}\exp(-(R_t/200)^2)\right]$$

$$l^* = \frac{\rho u_\varepsilon l_w}{\mu} \qquad R_t = \frac{\rho k^2}{\mu\varepsilon} \qquad u_\varepsilon = \left(\frac{\mu\varepsilon}{\rho}\right)^{1/4}$$

Kolmogorov velocity scale

Zero-equation models: Near wall improvement

1. Prandtl mixing length model

$$\tau = \mu_t \frac{\partial U}{\partial y} \qquad \boxed{\mu_t = \rho l_{mix}{}^2 \frac{\partial U}{\partial y}} \qquad l_m = \alpha\delta \quad \text{Mixing length}$$

Simple near wall modification: $l_m = \begin{cases} \alpha\delta & outer\ layer \\ \kappa y & inner\ layer \end{cases}$

Simplest model
The mixing length has to decrease near wall

Karman constant
$\kappa = 0.41$

2. Van Driest model

Use the Prandtl mixing length model but modify the mixing length according to:

$$\boxed{\begin{aligned} & l_{mix} = \kappa y \left[1 - \exp(-y^+ / A_0)\right] \\ & A_0 = 26 \end{aligned}}$$

These models were implemented in Prandtl exercise (L7)

Standard k-ε model: Near wall modifications

The Low Re k-ε model

Model k-equation

$$\frac{Dk}{Dt} = P_k - \varepsilon + D_k$$

Model ε-equation

$$\frac{D\tilde{\varepsilon}}{Dt} = \frac{\tilde{\varepsilon}}{k}\left(\boxed{f_1}C_{\varepsilon 1}P_k - \boxed{f_2}C_{\varepsilon 2}\tilde{\varepsilon}\right) + D_\varepsilon + \boxed{E}$$

Turbulent viscosity model

$$\mu_t = \boxed{f_\mu}C_\mu \rho \frac{k^2}{\tilde{\varepsilon}}$$

$$D, E,\ f_1,\ f_2\ and\ \ f_\mu\ ?$$

Not diffusion

$$\varepsilon = \tilde{\varepsilon} + \boxed{D}$$

$$\Downarrow$$

$$\tilde{\varepsilon} = 0 \text{ at boundary}$$

k-production

$$P_k \equiv \frac{R_{ij}}{\rho}\frac{\partial U_i}{\partial x_j} = \frac{\mu_t}{\rho}S^2$$

k-diffusion

$$D_k \equiv \frac{\partial}{\partial x_j}\left[\frac{1}{\rho}\left(\mu + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right]$$

Dissipation

$$\varepsilon \equiv \frac{\mu}{\rho}\left\langle \frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}\right\rangle$$

ε-diffusion

$$D_\varepsilon \equiv \frac{\partial}{\partial x_j}\left[\frac{1}{\rho}\left(\mu + \frac{\mu_t}{\sigma_\varepsilon}\right)\frac{\partial \varepsilon}{\partial x_j}\right]$$

$C_\mu = 0.09$

$C_{\varepsilon 1} = 1.44 \qquad C_{\varepsilon 2} = 1.92$

$\sigma_k = 1.0 \qquad \sigma_\varepsilon = 1.3$

Some damping function models

Jones Launder (1972)

$$f_\mu = \exp\left(-\frac{2.5}{1+\mathrm{Re}_T/50}\right)$$

$$f_1 = 1$$

$$f_2 = 1-0.3\exp\left(-\mathrm{Re}_T^2\right)$$

$$\boxed{\begin{aligned} C_{\varepsilon 1} &= 1.55 \\ C_{\varepsilon 2} &= 2.0 \end{aligned}}$$

$$D = 2\nu\left(\frac{\partial\sqrt{k}}{\partial y}\right)^2 \qquad E = 2\nu\nu_T\left(\frac{\partial^2 U}{\partial y^2}\right)^2$$

Lam-Bremhorst (1981)

$$f_\mu = \left[1-\exp\left(-0.0165R_y\right)\right]\left(1+20.5/\mathrm{Re}_T\right)$$

$$f_1 = 1+\left(\frac{0.05}{f_\mu}\right)^3$$

$$C_{\varepsilon 1} = 1.44$$

$$C_{\varepsilon 2} = 1.92$$

$$f_2 = 1-\exp\left(-\mathrm{Re}_T^2\right)$$

$$D = 0 \qquad E = 0$$

$$\mathrm{Re}_T \equiv \frac{\rho k^2}{\mu\varepsilon}$$

$$\mathrm{R}_y \equiv \frac{\rho\sqrt{k}\,y}{\mu}$$

Launder Sharma (1974)

$$f_\mu = \exp\left(-\frac{3.4}{\left(1+\mathrm{Re}_T/50\right)^2}\right)$$

$$f_1 = 1$$

$$f_2 = 1-0.3\exp\left(-\mathrm{Re}_T^2\right)$$

$$\boxed{\begin{aligned} C_{\varepsilon 1} &= 1.44 \\ C_{\varepsilon 2} &= 1.92 \end{aligned}}$$

$$D = 2\nu\left(\frac{\partial\sqrt{k}}{\partial y}\right)^2 \qquad E = 2\nu\nu_T\left(\frac{\partial^2 U}{\partial y^2}\right)^2$$

Chien (1982)

$$f_\mu = 1-\exp\left(-0.0115y^+\right)$$

$$f_1 = 1$$

$$f_2 = 1-0.22\exp\left(-\mathrm{Re}_T^2/36\right)$$

$$C_{\varepsilon 1} = 1.35$$

$$C_{\varepsilon 2} = 1.8$$

$$D = 2\nu\frac{k}{y^2} \qquad E = -\frac{2\nu\varepsilon}{y^2}\exp\left(-0.5y^+\right)$$

Construction of damping functions

$$\mu_t = f_\mu C_\mu \rho \frac{k^2}{\varepsilon}$$

AKN (1994)

$$f_\mu = \left(1 - e^{-l^*/14}\right)^2 \left[1 + 5\,\mathrm{Re}_T^{-3/4}\, e^{-(\mathrm{Re}_T/200)^2}\right]$$

$$f_1 = 1$$

$$f_2 = \left(1 - e^{-l^*/3.1}\right)^2 \left[1 - 0.3\, e^{-(\mathrm{Re}_T/6.5)^2}\right]$$

$$l^* = \frac{\rho u_\varepsilon l_w}{\mu} \qquad \mathrm{Re}_T \equiv \frac{\rho k^2}{\mu \varepsilon} \qquad u_\varepsilon = \left(\frac{\mu \varepsilon}{\rho}\right)^{1/4}$$

$$C_\mu = 0.09,\ \sigma_k = 1.4,\ \sigma_\varepsilon = 1.4$$

$$C_{\varepsilon 1} = 1.5,\ C_{\varepsilon 2} = 1.9$$

“A new turbulence model for predicting fluid flow and heat transfer in separating and reattaching flows”, K. Abe, and T. Kondoh:

> The most important feature of the present k–ε model is the introduction of the Kolmogorov velocity scale, $u_\varepsilon = (\nu\varepsilon)^{1/4}$, instead of the friction velocity u_τ, to account for the near-wall and low-Reynolds-number effects in both attached and detached flows [9]. This model can reproduce the correct near-wall asymptotic relations of turbulence, i.e. $k \propto y^2$, $\varepsilon \propto y^0$, $\nu_t \propto y^3$ and $-\overline{uv} \propto y^3$ for $y \to 0$.

Why is it good idea to remove friction velocity from damping models?

$$u_\tau \equiv \sqrt{\frac{\tau_w}{\rho}}$$

What do they mean by ”correct near-wall asymptotic relations of turbulence”?

$$k \sim y^2 \qquad \varepsilon \sim y^0 \qquad \nu_T \sim y^3 \qquad -\langle u'v' \rangle \sim y^2$$

Near wall analysis of turbulence models in a 2D boundary layer

Near wall

1.
$$u' = a_1 + b_1 y + c_1 y^2 + \ldots$$
$$v' = a_2 + b_2 y + c_2 y^2 + \ldots$$
$$w' = a_3 + b_3 y + c_3 y^2 + \ldots$$

No-slip and no penetration at y=0

$$u' = a_1 = 0$$
$$v' = a_2 = 0$$
$$w' = a_3 = 0$$

2.
$$u_{i,i} = \frac{\partial u'}{dx} + \frac{\partial v'}{dy} + \frac{\partial w'}{dz} = 0$$

=0

$$\left(\frac{\partial v'}{\partial y}\right)_{y=0} = b_2 = 0$$

$$u' = b_1 y + c_1 y^2 + \ldots$$
$$v' = c_2 y^2 + \ldots$$
$$w' = b_3 y + c_3 y^2 + \ldots$$

This lecture

Analysis of interesting quantities

3.
$$k \equiv \frac{1}{2}\langle u_i' u_i' \rangle \qquad \varepsilon \equiv \frac{\mu}{\rho}\left\langle \frac{\partial u_i'}{\partial x_j}\frac{\partial u_i'}{\partial x_j}\right\rangle \qquad P_k \equiv R_{ij}\frac{\partial U_i}{\partial x_j} \qquad R_{ij} \equiv \rho\langle u_i' u_j' \rangle$$

Example: Kinetic energy analysis near wall

$\langle u'^2 \rangle$ v.s. $\langle v'^2 \rangle$ v.s. $\langle w'^2 \rangle$

$$u' = b_1 y + c_1 y^2 + \ldots$$
$$v' = c_2 y^2 + \ldots$$
$$w' = b_3 y + c_3 y^2 + \ldots$$

$$u'u' = b_1^2 y^2 + 2b_1 c_1 y^3 + c_1^2 y^4 + \ldots = b_1^2 y^2 + O(y^3)$$

$$\langle u'^2 \rangle \sim y^2$$

$$v'v' = c_2^2 y^4 + 2c_2 d_2 y^5 + d_2^2 y^6 + \ldots = c_2^2 y^4 + O(y^5)$$

$$\langle v'^2 \rangle \sim y^4$$

This y-dependency near wall should be reproduced by a good turbulence model

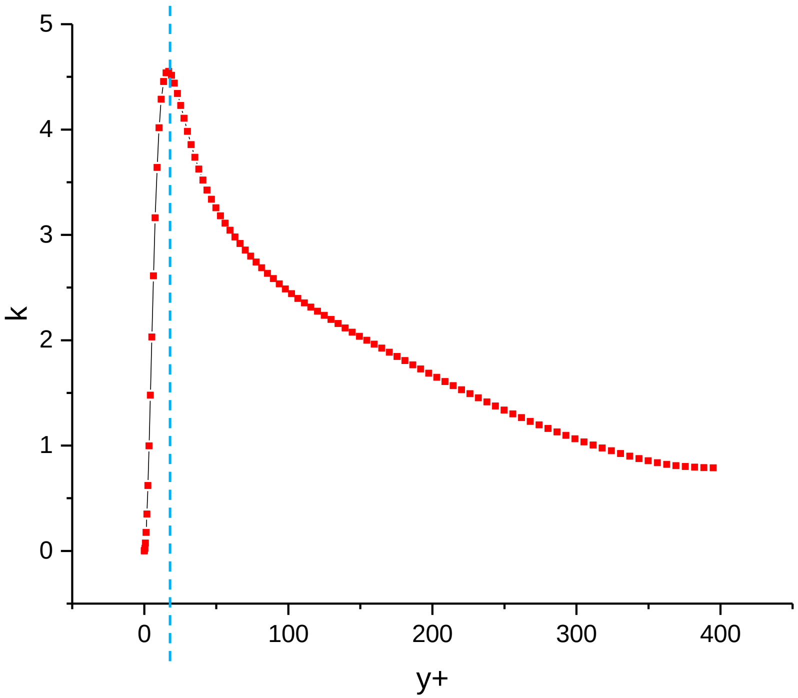
$$k \equiv \frac{1}{2} \langle u_i' u_i' \rangle$$

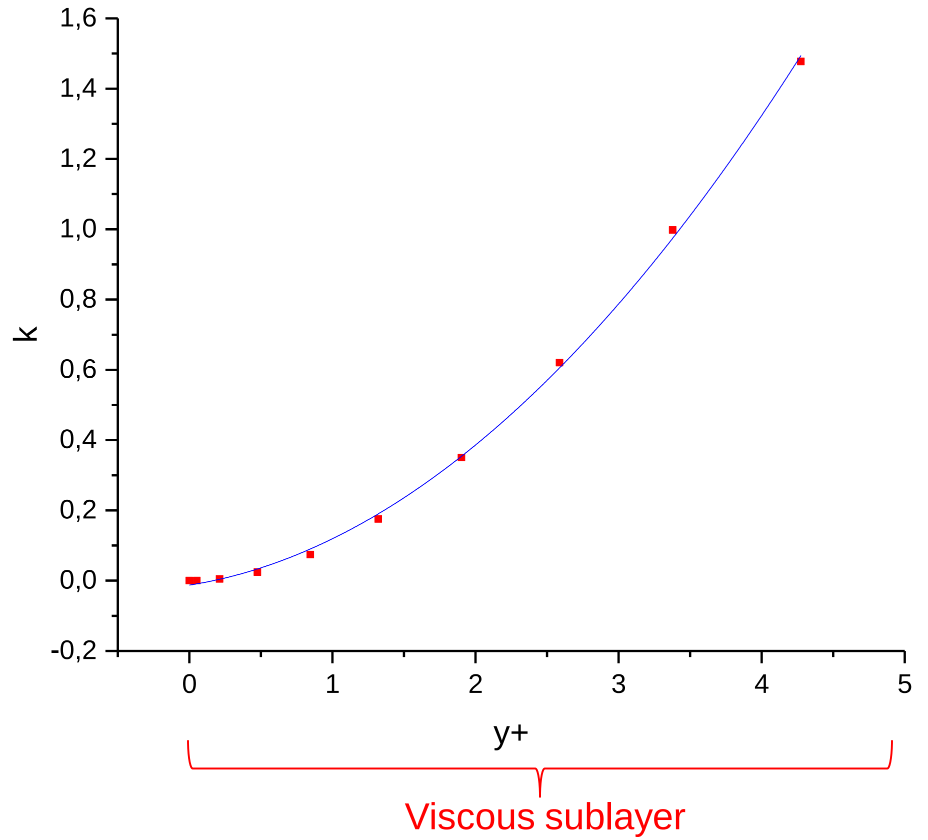
$$u_i' u_i' = u'^2 + v'^2 + w'^2 = b_1^2 y^2 + c_1^2 y^4 + b_3^2 y^2 + \ldots = \left(b_1^2 + b_3^2 \right) y^2 + O(y^3) \Rightarrow k \sim y^2$$

Example (cont.)

DNS Database (Kim etal)

$y^+ \approx 20$





$$y^+ \equiv \frac{\rho u_\tau y}{\mu} \qquad u_\tau \equiv \sqrt{\tau_w / \rho} \qquad k \sim y^2 \Rightarrow k \sim y^{+2}$$

End of lecture