Time Series Analysis & Recurrent Neural Networks

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Exercise 1

To be uploaded before the exercise group on Wednesday, April 28th, 2021

Task 1: Weather
The file "dailyweather.csv" contains daily measurements of the temperature, humidity, and air pressure on the balcony of Manuel's parents during the last five years.

1. Are any of the time series correlated with each other?
2. Plot the first return-map of the time series of the temperature. Do you notice any trend? What if you plot the return map for days that are a month apart?
3. Do you notice a trend in the temperature, e.g. can you observe the climate changing over time?

Task 2: Detrending and autocorrelation
The file 'investment.xls'contains scaled quarterly United States private investment per capita rates over the years 1948-1989.

1. By using linear regression, remove the trend from the datasets (Hint: There are in-built functions/packages in matlab and python that can do this for you).
2. Examine (loosely) whether the time series with the linear trend removed is stationary. Is the time series (of the original series before regression) of first differences stationary? How about the time series of second-order differences?
3. Compute the autocorrelation function of the detrended time series. Can you find periodic business cycles (corresponding to peaks in the autocorrelation function)? [Note: write the autocorrelation function yourself]

Task 3: AR models

1. Create your own AR time series of length $T = 200$ and order $p = 4$, with the following coefficients given: $a_0 = 0, a_1 = -.8, a_2 = 0, a_3 = 0$, and $a_4 = .4$, with $\epsilon_t \sim N(0, 1)$, i.e. the noise process drawn from a standard normal distribution, and with the initial value of the time series being $x_0 = 0$.
2. Plot the time series in time as well as the first return-map. What do you notice?

Task 4: Random walks and stationarity
Consider a random walk process with drift a_0:

$$x_t = a_0 + x_{t-1} + \epsilon_t, \tag{I}$$

where $x_0 = 0$ and ϵ_t is white noise with $\epsilon_t \sim N(0, 1)$.

1. Show that you can rewrite the model as $x_t = t\delta + \sum_{k=1}^{t} \epsilon_k$.
2. Determine the mean and autocovariance function of x_t.
3. Explain why the process is not stationary. What about the same process without drift?
4. How could you make this process stationary? (Hint: Think of exercise 2. For more details on random walks, see f.e. the Shumway-Stoffer book).