

# Time Series Analysis & Recurrent Neural Networks

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## Exercise 6

To be uploaded before the exercise group on June 2nd, 2021

### Task 1. Poisson latent variable models.

In 'ex6file.mat' (or according \*.xls files), you will find variables  $\mathbf{U} = \{\mathbf{u}_t\}$  (named 'u'),  $t = 1 \dots T$ , as well as parameters  $\mathbf{A}$ ,  $\mathbf{B}$ ,  $\Sigma$ ,  $\boldsymbol{\eta}_0$ , and  $\Gamma$  obtained from the following model (see also Ch7 eqns. 7.85):

$$\begin{aligned} c_t^{(i)} | \mathbf{z}_t &\sim \text{Poisson}[\lambda_t^{(i)} \Delta t] \quad \text{with} \quad \lambda_t^{(i)} = \exp(\log[\eta_i^{(0)}] + \boldsymbol{\eta}_i^{(1)} \mathbf{z}_t), \\ \mathbf{z}_t &= \mathbf{A} \mathbf{z}_{t-1} + \mathbf{B} \mathbf{u}_t + \boldsymbol{\epsilon}_t, \quad \text{with} \quad \boldsymbol{\epsilon}_t \sim N(\mathbf{0}, \Sigma), \\ \mathbf{z}_1 &\sim N(\boldsymbol{\mu}_0, \Sigma). \end{aligned}$$

Note that matrix  $\Gamma$  collects the  $i = 1 \dots N$  vectors  $\boldsymbol{\eta}_i^{(1)}$  in its rows, i.e.  $\Gamma_{i,:} = \boldsymbol{\eta}_i^{(1)}$ , and  $\boldsymbol{\eta}_0$  is a vector collecting the  $\eta_{0i}$ .

1. Create time series of  $M \times T$  ( $M = 2$ ,  $T = 100$ ) dimensional latent states  $\mathbf{Z} = \{\mathbf{z}_t\}$  (named 'z'), and  $N \times T$  dimensional observations  $\mathbf{C} = \{\mathbf{c}_t\}$  (named 'c') from these variables and parameter settings, and plot them.
2. What is the joint data log-likelihood  $\log p(\{\mathbf{c}_t, \mathbf{z}_t\} | \theta)$  of your generated time series?

### Task 2. Fixed points, stability, and bifurcations.

Consider the univariate nonlinear map

$$x_{t+1} = f(x_t, w, \theta) = w \cdot \sigma(x_t) + \theta, \quad \text{with} \quad \sigma(x) = \frac{1}{1+e^{-x}}.$$

1. For  $w = 7$  and  $\theta = -2.5$ , find the fixed points of the system. Visualize these in a graph. Are they stable?
2. For  $w = 7$ , plot the bifurcation graph as a function of  $\theta \in [-10, 0]$ . Include both stable and unstable objects. How does the system change its dynamical properties as  $\theta$  is varied within this range?

### Task 3. Nonlinear systems, oscillations, and chaos.

Consider the 'Ricker map', with parameter  $r \in \mathbb{R}$ , and variable  $x_t \in \mathbb{R}$ :

$$x_{t+1} = r x_t e^{-x_t},$$

1. What are the fixed point(s) of this map? How many are there?
2. Explore the behavior of the map for a few values  $r \in [\exp(1), \dots, \exp(4)]$  (covering the extremes of this interval), and comment on the dynamics.