Time Series Analysis & Recurrent Neural Networks

lecturer: Daniel Durstewitz
tutors: Manuel Brenner, Daniel Kramer, Leonard Bereska
SS2021

Exercise 4

To be uploaded before the exercise group on May 19th, 2021

Task 1. Granger Causality

1. Create a 2-variate AR(2) time series $x_t = \begin{pmatrix} x_{1,t} \\ x_{2,t} \end{pmatrix}$ with $T = 1000$ time steps and the following parameters: $a_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $A_1 = \begin{pmatrix} 0.2 & 0 \\ -0.2 & 0.1 \end{pmatrix}$, and $A_2 = \begin{pmatrix} 0.1 & 0 \\ -0.1 & 0.1 \end{pmatrix}$, diagonal Gaussian noise matrix $\Sigma = \begin{pmatrix} 0.01 & 0 \\ 0 & 0.01 \end{pmatrix}$, and initial condition $x_0 = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$.

2. Given your knowledge of model parameters above, does x_1 Granger-cause x_2, or does x_2 Granger-cause x_1 according to Granger's definition? Why?

3. In a similar spirit to exercise 1 on the previous sheet, we want to find out if adding parameters to the model increases its explanatory power in a statistically significant way. Assuming that the order of the model ($p = 2$) is known, use the log-likelihood-ratio test statistic to confirm your results from 2 (see eq. 7.35 in script; command `fcdf` in MatLab, and `scipy.stats.f.cdf` in Python give the cumulative F-distribution).

Task 2. M-Step in a linear Gaussian state space model

Consider a linear Gaussian state space model,

$$\begin{aligned} z_t &= Az_{t-1} + \epsilon, & \epsilon &\sim N(0, \Sigma) \\ x_t &= Bz_t + \eta, & \eta &\sim N(0, \Gamma) \end{aligned}$$

In the lecture we derived the M-step to determine the transition matrix A. Derive the M-step for the latent state noise Σ by maximizing the expected log-likelihood $\mathrm{E}[\log p(X, Z)]$, with respect to Σ, where $X = \{x_t \mid t \in 1 \ldots T\}$ and $Z = \{z_t \mid t \in 1 \ldots T\}$ are the sets of all latent states and observations from time 1 to T.
(Hints: First identify the part in the ELBO that explicitly depends on the parameters and ignore the rest. Multiplying both sides by Σ can be useful at one point in the derivation. Further plugging in A as derived in the lecture will help you simplify the final result. The matrix cookbook by Petersen and Pedersen (2012)
`https://www.math.uwaterloo.ca/~hwolkowi/matrixcookbook.pdf`
provides a helpful summary for matrix algebra).