Time Series Analysis & Recurrent Neural Networks

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Exercise 3

To be uploaded before the exercise group on May 12th, 2021

Task 1. Univariate AR models
In file ex3file1.mat (ex3file1.xls), you will find four time-series obtained from human fMRI recordings from the dorsolateral prefrontal cortex (DLPFC) and the parietal cortex (Parietal), obtained during a working memory task. For task 1, consider the first time series of DLPFC (termed 'DLPFC1').

1. Compute the log-likelihood of an AR(4) model (e.g, by employing lecture script 1, eq. 7.34). Please write down explicitly.
2. Plot the residuals of the model in a histogram. What do they look like? What do you expect?
3. Compute the log-likelihood of an AR(n) model, with n ranging from 1...5. How does the likelihood change when you increase the order of the model?
4. Bonus exercise: Determine the optimal order p of the AR model by computing the log-likelihood-ratio test statistic. Start with Wilk's D (see also 'ASMN_Ch1_Durstewitz.pdf', eq. 1.35, and/or lecture script 1, eq. 7.35), which simplifies here to

$$D = -2[\log \Sigma_p - \log \Sigma_{p-1}]$$

   Plug D into a χ^2 distribution with appropriate degrees of freedom (Matlab function chi2cdf.m, or scipy.stats.chi2.cdf() in Python). Start with the comparison between an AR(2) vs. AR(1) model, and then keep on computing log-likelihood-ratio test statistics for models of consecutive orders until the difference between models is no longer significant. Do this up to order $p = 5$.

Task 2. Multivariate (vector) AR (=VAR) processes
For this task, use all four time series contained in the data file ('DLPFC1', 'DLPFC2', 'Parietal1', 'Parietal2').

1. Estimate a VAR(1) model by performing multivariate regression on the 4-variate time series. What do the coefficients in matrix A tell you about the coupling between the DLPFC and parietal cortex? Is the resulting VAR(1) model stationary or not?

Task 3. AR Poisson processes

1. Create your own second ordeer Poisson time series with T=1000 time steps and the following given parameters: $A_1 = \begin{pmatrix} 0.2 & -.1 \\ 0.1 & 0.1 \end{pmatrix}$, $A_2 = \begin{pmatrix} 0.1 & -.1 \\ 0.1 & 0.1 \end{pmatrix}$, and $\mu_0 = \begin{pmatrix} .5 & .5 \end{pmatrix}^\intercal$. Assume no base rate (i.e. $a_0 = \begin{pmatrix} 0 & 0 \end{pmatrix}^\intercal$).
2. Given the data generated in (a), vary the parameters $A_1(1, 1)$ and $A_2(2, 1)$ between 0 and 0.4 with 0.01 increments. For each parameter value pair, compute the log-likelihood of the data (keeping all other parameters fixed!). Plot the log-likelihood landscape surface as a function of these two parameters. Does the real parameter pair value correspond (or is close) to an extreme point in the approximate log-likelihood landscape? What kind of an extreme point is it?