Text exam for the lecture

Fundamentals of Simulation Methods

WS 2019/20

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30.01.2020

1	2	3	4	Total

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1) Short questions [15pt]

Please answer concisely, but always motivate/support your answer.

a) Representation of integer numbers:
The product of the two numbers $a = 13$ and $b = 10$ is $c = a \cdot b = 130$. If, however, we define a, b and c as 8-bit integers, we obtain $c = -126$.

1) Usually, the two's complement is used in this case to represent negative integers. Explain what this means. [2pt]
2) What range can a 8-bit integer represent, if we want to account for negative numbers, too? [2pt]
3) Why do we get a wrong result in the above calculation? [1pt]
4) Why is the result we get negative? [1pt]
5) Show why c is returned as -126 in the above implementation. [2pt]

b) N-body codes with gravity

1) Why are direct N-body gravity solvers unpractical for large systems? [1pt]
2) Why do we usually need a softening length? [2pt]
3) Describe qualitatively the idea behind tree methods to solve the problem, aud explain why they can help to overcomes the central problem of direct N-body gravity solvers. [2pt]
4) What is tbe idea behind particle-mesh codes, and how do they help to solve the gravitational N-body problem? [2pt]

Force on one particle is compu...

2) Numbers on a computer [6pt]

In the interion of a star energy is produced through nuclear fusion. The Triple-α-Process is extremely sensitive to the temperature of the gas. The energy production rate is

$$\epsilon = 8.24 \times 10^{-31} \rho X_{CNO} \left(\frac{T}{10^6 \mathrm{K}} \right)^{19.9} ; . \tag{1}$$

1. Explain why, if you use single precision, it is important to keep the 10^6 inside the brackets, and not write the formula as [3pt]

$$\epsilon = 2.07 \times 10^{89} \rho X_{CNO} T^{19.9} . \tag{2}$$

2. If we want to take the 3.5th power of a numer x, what is wrong with the following program (in most programming languages):

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y=x^(7/2)
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where the caret ^ means to-the-power? [3pt]

3) Equations of hydrodynamics used in numerical hydro codes [13pt]

The equations of isothermal ideal gas dynamics are given as the following two coupled equations:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathrm{v}) = 0 , \tag{3}$$

$$\frac{\partial (\rho \mathrm{v})}{\partial t} + \nabla \cdot (\rho \mathrm{v}\mathrm{v} + P \mathbb{1}) = 0 , \tag{4}$$

where P is the pressure given by $P = \rho c_s^2$ with c_s^2 being the isothermal sound speed and ρ the density, and where v is the velocity. In this notation vv is the matrix $(v_i v_j)$ and $\mathbb{1} = \delta_{ij}$. We use cartesian coordinates (x, y, z). This is the form of the equations in the *laboratory frame*. We can, however, also recast these equations in the *comoving frame* (the so-called Lagrange form of the equations):

$$\frac{D\rho}{Dt} = -\rho \nabla \cdot \mathrm{v} \tag{5}$$

$$\frac{D\mathrm{v}}{Dt} = -\frac{1}{\rho} \nabla P , \tag{6}$$

where D/Dt is the *comoving derivative* defined as $D/Dt = \partial/\partial t + \mathrm{v} \cdot \nabla$.

1. Which of the two sets of equations is used by grid-based codes (such as Riemann solvers)? Explain your answer. [2pt]
2. Which of the two sets of equations is used in smooth particle hydrodynamics (SPH)? Explain. [2pt]
3. The first set of equations are *conservation equations*. Which quantities are conserved? [2pt]

4. What are the mass and momentum fluxes in x-direction? [2pt]

5. Derive the comoving-frame equations from the laboratory frame equations. [3pt]

6. Give in words the meaning of the right-hand-side terms in the comoving-frame equations. [2pt]

4) Multigrid approach to the Poisson equation [24pt]

Consider the following equation in 1-D:

$$\frac{d^2y(x)}{dx^2} = f(x) \tag{7}$$

The grid in x is equal-spaced and has 9 grid points between $x = -L$ and $x = +L$, i.e. $x[0] = -L$ and $x[8] = +L$. The boundary conditions are $y = 0$ at both ends.

(a) Write down the discrete form of the equation for the non-boundary grid points (i.e. for grid points 1 to 7).

(b) We now make successively coarser grids. The first level of coarsening (level 1) has 5 grid points, the next level (level 2) has 3 grid points. Give the expression for the restriction operators for the restriction from level 0 to 1, and from level 1 to 2. These should be given as a 5×9 matrix and a 3×5 matrix respectively.

(c) Give the prolongation matrices, and explain how they relate to the restriction matrices.

(d) Use the information of the previous parts to compute the discrete form of the equation at level 1.