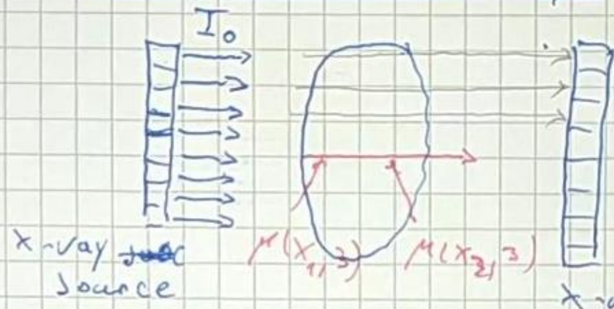
Application (homework): **Computer Tomography**

consider X-ray investigation



I_i : attenuated intensity, depending on density of body tissue in between

attenuation : $I_i = I_0 \cdot e^{-\int \mu(x,i)\, dx}$

$\mu(x,i)$: tissue density ("absorption ability") at position x along ray i

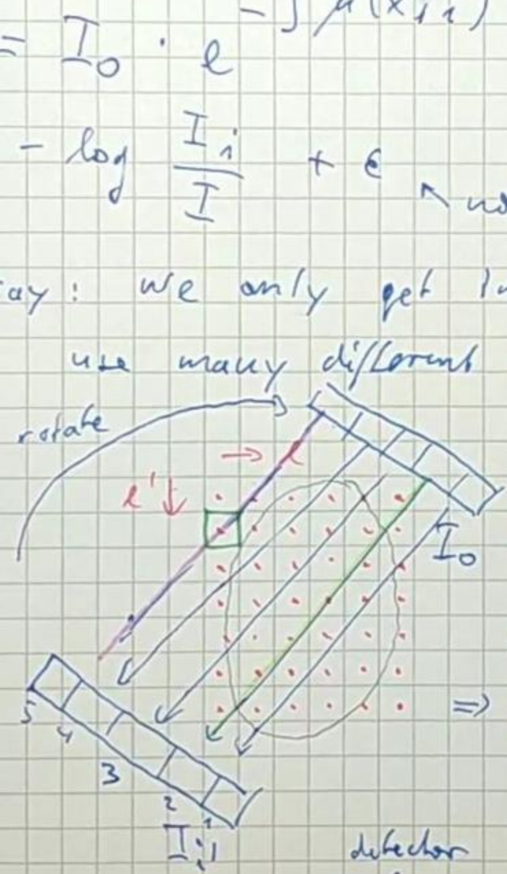
$\int \mu(x,i)\, dx = -\log \frac{I_i}{I} + \epsilon$ ← noise

problem of normal X-ray: we only get integral over the entire body

=> idea of tomography: use many different ray directions to resolve $\mu(x,i)$ within the body

- discretize body

□: $(l,l') = (2,1) \Rightarrow k = 7$



grid index : points (l,l')

flatten this into $k = \cancel{l' \cdot \text{width} + l}$

1-based indexing $(l'-1)\,\text{width}) + (l-1)$

for each rotation, we get measurement i

=> flatten into one long vector

$m = r \cdot R + i$; 1-based: $(r-1)\cdot R + (i-1)$

i ← pixel in detector; R ← size of detector; r ← current rotation index

ray 34 fully hits pixel $k=7$

$\Rightarrow X_{km} = 1$ $k=7,\ i=5$, $m=34$

$X_{km'} = 0$ $k'=7,\ i=2$, $m'=31$

- determine how much each measurement (= pixel i in rotation r) is influenced by body pixel $(l,l') = k$

detector r

⇒ linear system: $X \cdot \beta = Y + \epsilon$

↑ flattened measurements $Y_m = -\log \frac{I_m}{I_0}$

X_{mk}: how much is detector pixel m (= ray m) influenced by ~~ray~~ body pixel k

β: flattened array of absorption strength in body pixel k

X follows from the hardware design (which rotations and detectors are used), can be derived from the geometry of the setup

Y are the measurements of a particular patient

⇒ calculate β = absorption strength of the present patient

rearrange β into a 2-dimensional image → CT result