handed out: May 5, 2020
handing in: May 14, 2020
presentation/discussion: May 15, 2020

1. **Second Iterate** present ☐

   The second iterate of the logistic map (3.7) is

   $$f^2(u) = f\big(f(u)\big) = \mu^2 u[1-u]\big[1-\mu u[1-u]\big] \ .$$

   Define a new map as $u_{i+1} = f^2(u_i)$.

   (a) Without implementing and running the iteration: What sequences do you expect? How does the attracting set look like, actually: will there be a single attractor? How does this set develop into the chaotic regime?

   (b) Implement the model and reflect your expectations.

   (c) Extend to higher iterates, both conceptually and by numerical simulation.

2. **Bifurcation Diagram of Logistic Map** present ☐

   Reproduce the bifurcation diagram shown in Figure 3.3 and explore some of its details. Specifically

   (a) Implement the logistic map (3.7) for a fixed value of μ and initial value u_0. Iterate it for the spin-up phase, say for $4'000$ steps, without plotting the resulting states. Continue iteration for 400 steps, say, plotting each of the states.

   (b) Integrate the map into a stepper that starts at μ_0 and sweeps to μ_1 in N_μ steps, say with $N_\mu = 1'000$. Reproduce Figure 3.3.

   (c) Choose some interesting intervals for μ and plot the corresponding part of the bifurcation diagram.

   (d) What limits the detail you can zoom into?

   Comments: (i) The suggested numbers are from experience. Think about their impact on the final figure. (ii) The plot can be produced by just putting a mark, say a small filled circle at the location of each state to be plotted. This leads to huge file sizes that can be easily compressed by converting to the .png-format. (iii) For those more into programming, an economic approach is to setup an $n \times m$-array of integers that discretizes $[\mu_0, \mu_1] \times [u_0, u_1]$ and allows to count the number of hits in each of the cells. This may then be drawn as a color bitmap. Figure 3.9 in the lecture notes was produced in this way using a $5'000 \times 5'000$ array with $N_{\text{spinup}} = 4'000$ and $N_{\text{mark}} = 80'000$. The latter number is so large because it has to provide the color-resolution.

3. **Transient Phase of Logistic Map** present ☐

   Repeat the previous problem but drop the spin-up phase. The resulting graph is no more a bifurcation diagram because it does not just show the pure attractor but also the approach to it.

   (a) Explain the overall appearance for $\mu \in [0, 4]$.

   (b) Select some interesting parts, e.g., a window within the chaotic regime, explore and explain.

4. **The Last Question** want to discuss ☐

(a) What are the key messages you took home from the lecture?

(b) What are still open questions?

(c) What associated issues did you miss?