Time Series Analysis & Recurrent Neural Networks

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Exercise 2

To be uploaded before the exercise group on May 5th, 2021

Task 1. AR(p) models
Consider the univariate time series contained in file 'Tut2_file1.mat' (Tut2_file1.txt):

1. Estimate the coefficients $\{a_0, a_1\}$ of an AR(1) process from this series using linear regression. Why do you obtain this specific value for a_1, and how does it relate to the properties of the time series?

2. Detrend the time series by performing linear regression on time (see the last exercise sheet) and repeat the analysis above. How does the coefficient (and the AR process) change?

3. Generate predictions from your estimated AR(1) model up to five time steps ahead (Hint: to achieve this, take the datapoint x_t at time t and run the estimated AR(1) model with x_t as initial state forward for 5 steps. Then compare this with the true datapoint at time x_{t+5}, and compute the error by squaring the residual. Do this for the entire time series and compute the average). How well do they agree with the true signal? What is the error for one-step ahead prediction as compared to five step ahead predictions?

Task 2. Estimating an AR(1) model
Consider the univariate time series contained in file 'Tut2_file2.mat' (Tut2_file2.txt'). Estimate the coefficients $\{a_0, a_1\}$ of an AR(1) process from this series as above, using linear regression. Is a linear time series model suitable for this type of signal? Plot the signal in various ways (refer back to the last tutorial, e.g. return plots), and examine the autocorrelations in the residuals.

Task 3. Autocorrelation and partial autocorrelation.
The file 'Tut2_file3.mat' (Tut2_file3.txt') contains two time-series (termed x_AR and x_MA) generated by an AR and a MA process, respectively. Determine the order of both processes by plotting the autocorrelation and the partial autocorrelation function of x_AR and x_MA [Try to implement the functions yourself. The autocorrelation function can be conceptually a little tricky, but understanding how it works is very illustrative for several of the key concepts we are currently discussing].