# TSA Tutorial 2
# 5.5.2021

Quick recap AR+MA models

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# Quick notes regarding grades

If you need a grade (not only fail pass), you can tell us (after the lecture is over and all 12 exercise sheets are done) which 6/8 (depending if you hand in solo or together) of the 12 sheets you want to hand in as sheets to be graded

We will then go over these sheets in more detail and grade you based on the points you obtained relative to the maximum number of points obtainable.

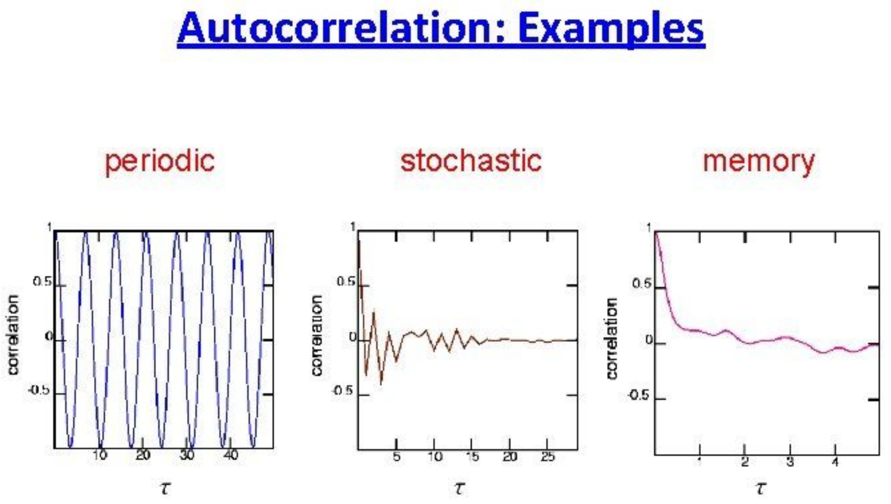
# Handing in sheets

Please hand in files as either html or pdf’s.
No zipping needed, no notebooks needed.

# Autoregressive Models

Make use of information in the present to predict the future

How long is the information in the present useful to make a meaningful prediction?



# AR models and regression

Simple linear regression for target, lagged target and error term

$$\underbrace{Y_t}_{\text{Target}} = \omega + \phi \underbrace{Y_{t-1}}_{\text{Lagged Target}} + \underbrace{e_t}_{\text{Error}}$$

# Long term vs. short term contributions

AR models: predict solely on previous values (lags). Previous values still influence current value, but can drop off exponentially if the absolute value is smaller than 1

$$Y_t = \omega^* + \phi^2 Y_{t-2} + \phi e_{t-1} + e_t$$

$$Y_{t-2} = \omega + \phi Y_{t-3} + e_{t-2}$$

$$Y_t = \omega^* + \phi^3 Y_{t-3} + \phi^2 e_{t-2} + \phi e_{t-1} + e_t$$

$$Y_t = \omega^* + \phi^3 Y_{t-3} + \phi^2 e_{t-2} + \phi e_{t-1} + e_t$$

$$Y_t = \frac{\omega}{1-\phi} + \phi^t Y_1 + \phi^{t-1} e_2 + \phi^{t-2} e_3 + \cdots + e_t$$

# Rephrased in terms of duality of AR and MA processes: AR(1) is MA(infinite)

$$
\begin{aligned}
x_t &= a_0 + a_1 x_{t-1} + \epsilon_t \\
&= a_0 + a_1(a_0 + a_1 x_{t-2} + \epsilon_{t-1}) + \epsilon_t \\
&= a_0 + a_1(a_0 + a_1(a_0 + a_1 x_{t-3} + \epsilon_{t-2}) + \epsilon_{t-1}) + \epsilon_t \\
&= a_0 \sum_{i=1}^{t-1} a_1^i + \sum_{i=0}^{t-1} a_1^i \epsilon_{t-i}
\end{aligned}
$$

# Autocorrelation functions in AR(1) models

AC drops off, but is never zero

$$\rho(h) = \frac{\gamma(h)}{\gamma(0)} = \phi^h, \quad h \geq 0,$$

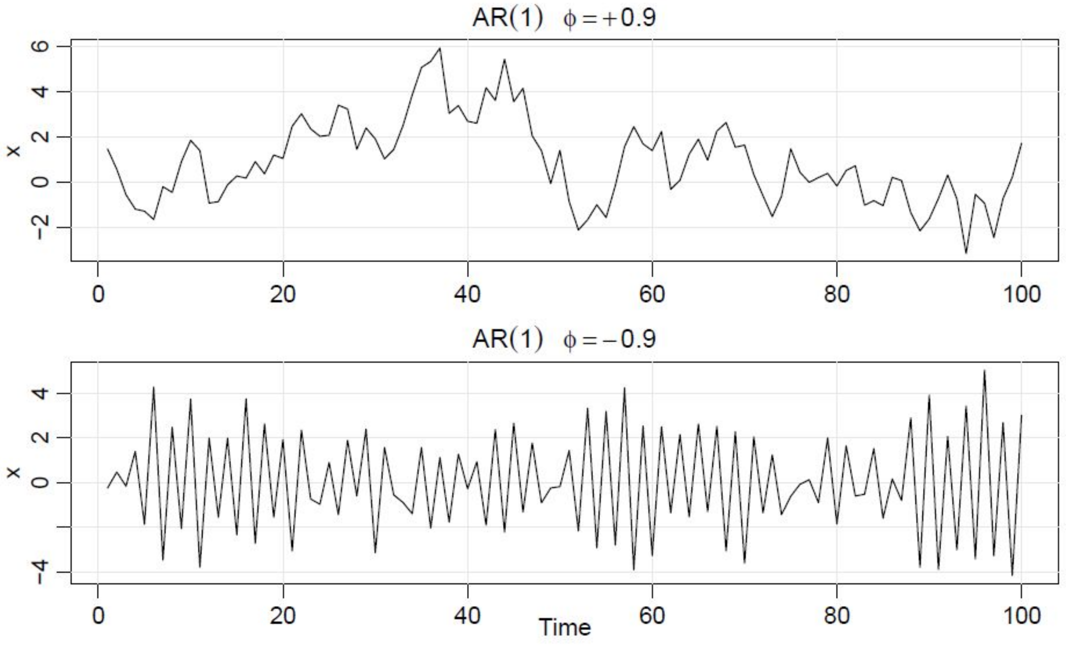
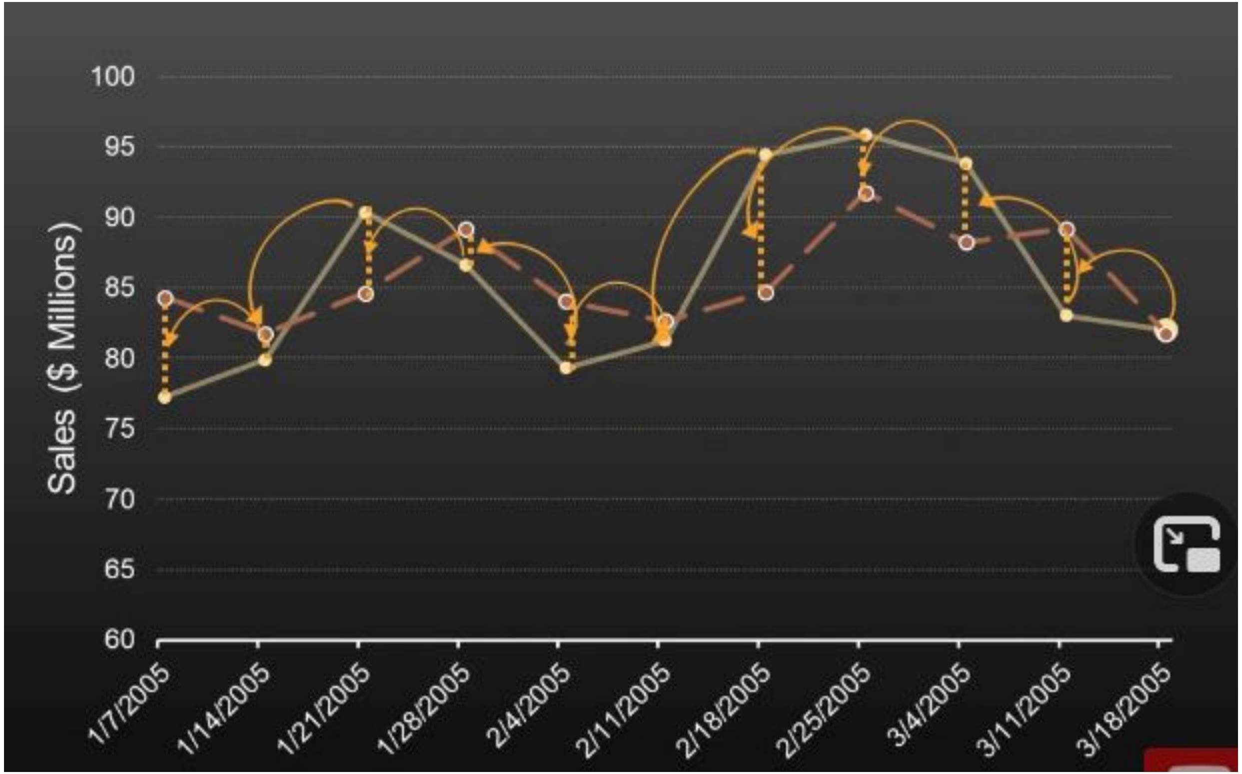



***Fig. 3.1.*** *Simulated AR(1) models: $\phi = .9$ (top); $\phi = -.9$ (bottom).*

# Moving Average Models

Say we have a real time series and an estimated model. Our model then takes into account how “wrong” we were in the previous time step



# Short term contributions

Errors don't last long into the future, and MA models are short memory models.

$$Y_{t-1} = \omega + \theta e_{t-2} + e_{t-1}$$

$$Y_t = \omega + \theta e_{t-1} + e_t$$

$$Y_{t+1} = \omega + \theta e_t + e_{t+1} \longrightarrow \text{No more } e_{t-1}\text{!!}$$

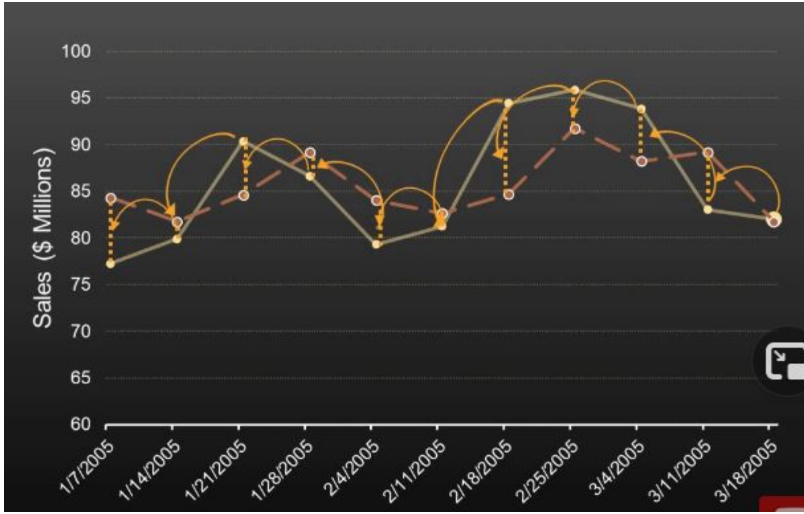
# How do we estimate MA models?

The usual starting point can just be the mean of the time series.

Overall, fitting the MA estimates is more complicated than it is in AR models because the lagged error terms are not a priori observable.

→ estimates have to be generated sequentially

We need iterative non-linear procedures
in place of linear least squares
that estimates everything simultaneously!



# Autocorrelation functions in MA(1) models

Consider the MA(1) model $x_t = w_t + \theta w_{t-1}$. Then, $E(x_t) = 0$,

and the ACF is

$$\rho(h) = \begin{cases} \dfrac{\theta}{(1+\theta^2)} & h = 1, \\ 0 & h > 1. \end{cases}$$

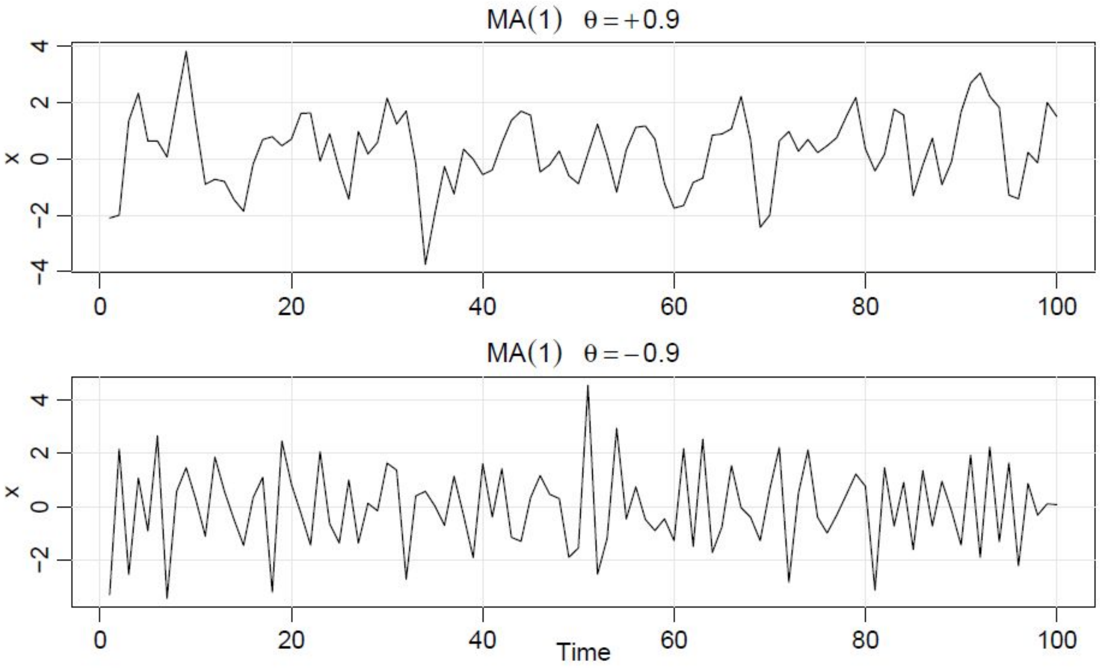



***Fig. 3.2.*** *Simulated MA(1) models: $\theta = .9$ (top); $\theta = -.9$ (bottom).*