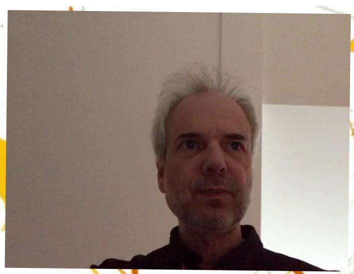


TIME SERIES ANALYSIS & RECURRENT NEURAL NETWORKS #4

- Granger causality
- AR count process models
- AR point process models

Main lecture: Daniel Durstewitz

Exercises: Leonard Bereska, Manuel Brenner, Daniel Kramer, Georgia Koppe

Heidelberg University

Granger causality

X, Y time series processes

$X \to Y$ "Granger causes"

Z : all other world knowledge

$$\Pr(Y_t \in U \mid \underbrace{Z_{t\text{-past}}}, X_{t\text{-past}}) \neq \Pr(Y_t \in U \mid Z_{t\text{-past}})$$

$$\{Z_{t-1} \dots Z_{t-\infty}\}$$

Reformulate in terms of AR models

$K \times K$

$$(1)\ y_t = a_0 + \sum_{i=1}^{p} A_i y_{t-i} + \cancel{\sum_{j=1}^{q} B_j x_{t-j}} + \sum_{l=1}^{r} C_l z_{t-l} + \varepsilon_t$$

$$\varepsilon_t \sim N(0, \Sigma)$$

$$(11)\ y_t = a_0 + \sum_{i=1}^{p} A_i y_{t-i} + \sum_{l=1}^{r} C_l z_{t-l} + \varepsilon_t$$

LLR

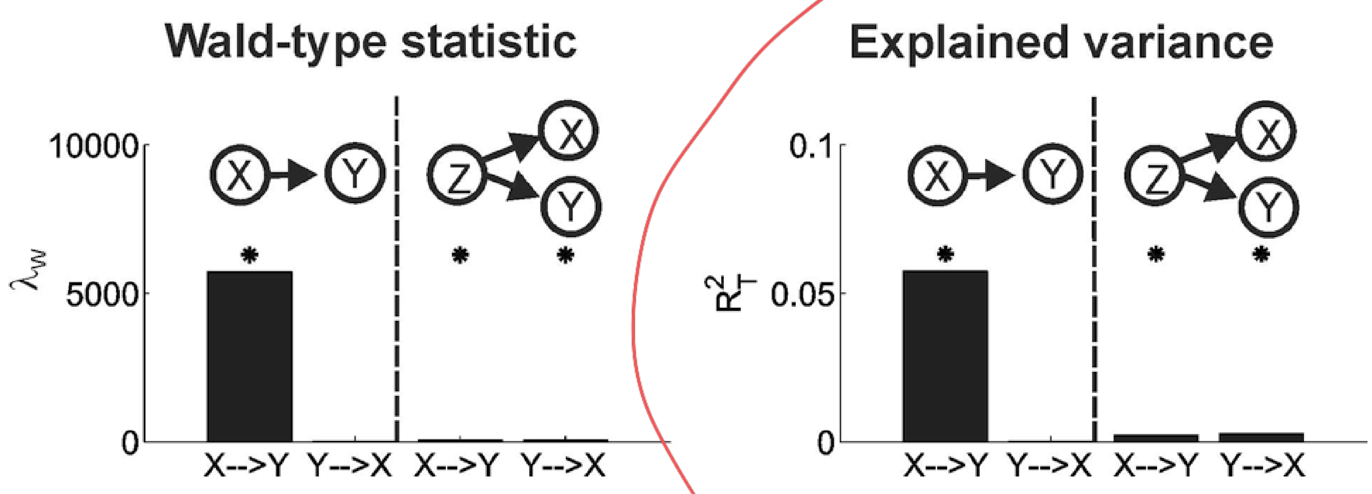
$$D \approx (T - \max(p,q,r))\left(\log|\Sigma_{(11)}| - \log|\Sigma_{(1)}|\right) \sim \chi^2_{q \cdot K^2}$$

$$\Pr(D \geq D^{obs}) \leq 0.05 \quad \alpha\text{-level}$$

$\to$ $X \to Y$ "Granger-causes"

$$\begin{tikzcd}
 & X \\
U \arrow[ur] \arrow[dr] & \\
 & Y \text{ (time-lag)}
\end{tikzcd}$$

Source: Durstewitz (2017) *Advanced Data Analysis in Neuroscience.* Springer.

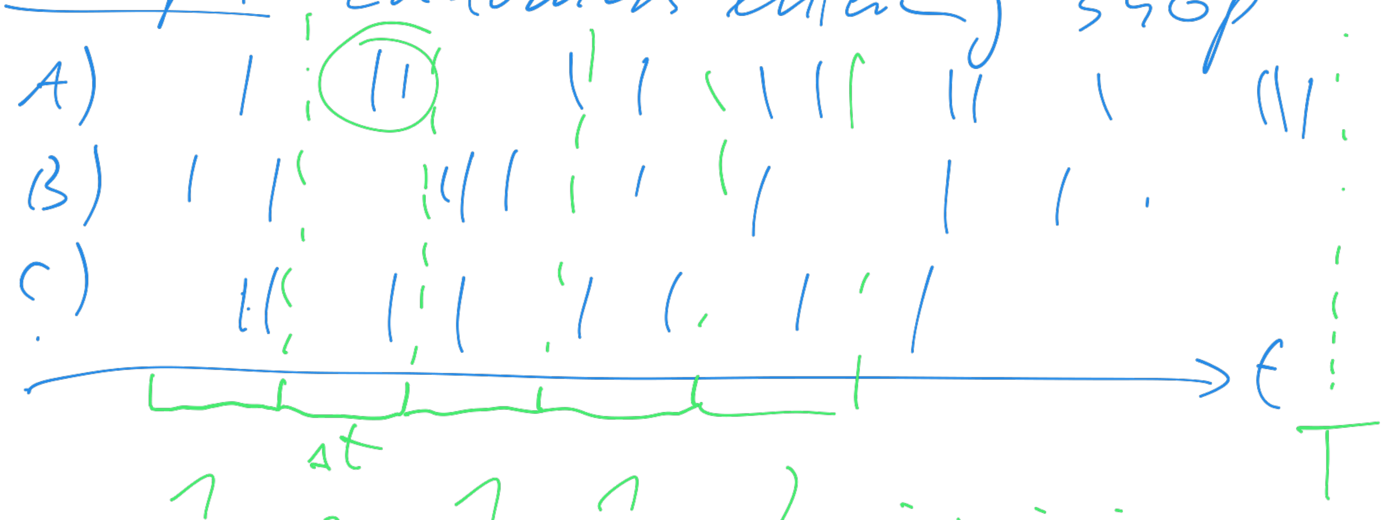


AR for count & point processes

$$x_t = a_0 + \sum^{p} a_i x_{t-i} + \varepsilon_t \,, \quad \varepsilon_t \sim N(0, \sigma^2)$$

$$x_t \mid x_{t-p} \dots x_{t-1} \sim N(\;, \sigma^2)$$

Example: customers entering shop



1 2 1 1 2 . . . . .

$$c_t: \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix}, \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}, \dots$$

binomial ⟶ Poisson

$\Delta t \to 0$

$T/\Delta t \to \infty$

$$\{c_t\}, \; t = 1 \dots T, \quad c_t \; M \times 1, \quad c_{it} \in \mathbb{N}, \; i = 1 \dots M$$

$$c_{it} \mid c_{t-p} \dots c_{t-1} \sim \text{Poisson}(\lambda_{it}) = \frac{\lambda_{it}^{c_{it}}}{c_{it}!} e^{-\lambda_{it}}$$

log

$$\ln \lambda_t = a_0 + \sum_{j=1}^{p} A_j c_{t-j}$$

functional interactions

natural link function

max. likelihood

$$p(c_t \mid c_{t-p} \dots c_{t-1})$$

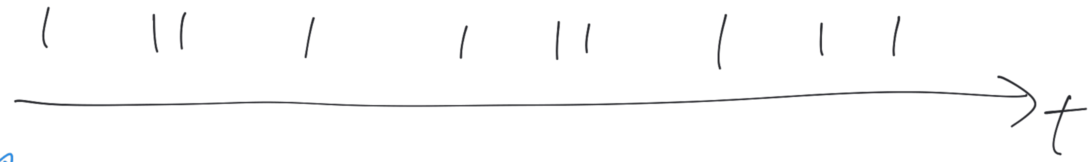
$$\log L_{\{c_t\}}(A_j, a_0) = \log \left[\prod_{t=p+1} \prod_{i=1}^{M} \frac{\lambda_{it}^{c_{it}}}{c_{it}!} e^{-\lambda_{it}} \right]$$

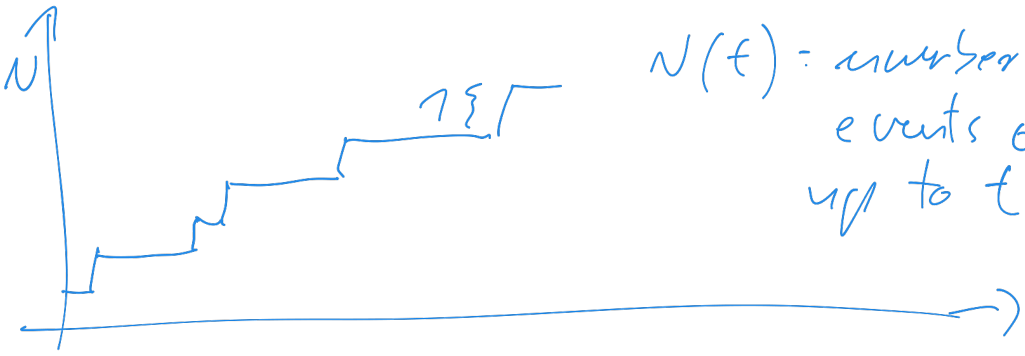
$$= \sum_{t=p+1}^{T} \sum_{i=1}^{M} c_{it}$$

$\Delta t \to 0$

Point process

N: cumulative event count

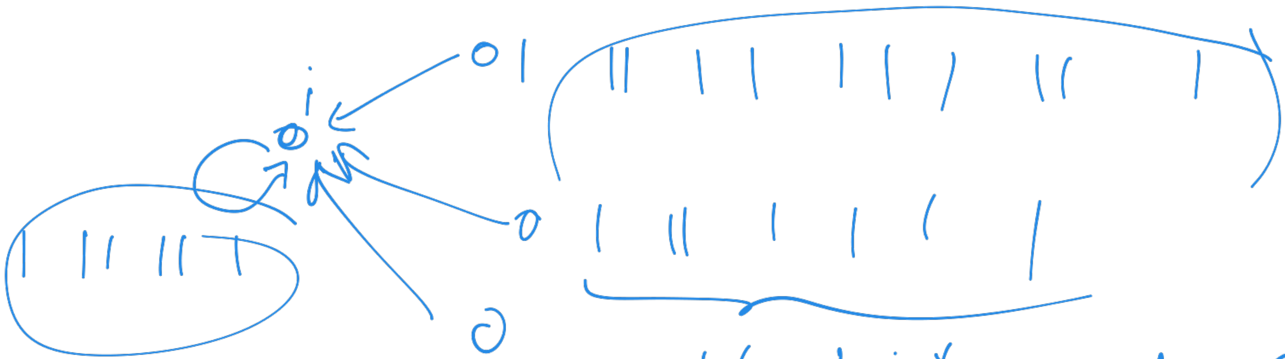




$N(t)$ = number of events occ. up to t

Def. condit. intensity func.:

$$\lambda_i(t \mid H_i(t)) \equiv \lambda_t^{(i)} := \lim_{\Delta t \to 0} \frac{Pr[N_i(t+\Delta t) - N_i(t) = 1 \mid H_i(t)]}{\Delta t}$$



H: history of all events in all t. s.

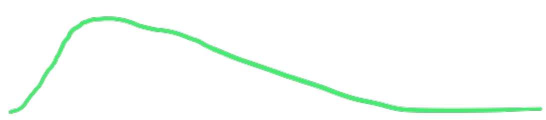
time series → M

$$\log \lambda_t^{(i)} = a_{i0} + \sum_{j=1}^{M} \sum_{\{t_n^{(j)} < t\}} a_{ij} h_{ij}(t - t_n^{(j)})$$

temp. interaction kernel

$$h_{ij}(t - t_n^{(j)}) = e^{-(t - t_n^{(j)})/\tau_{ij}}$$

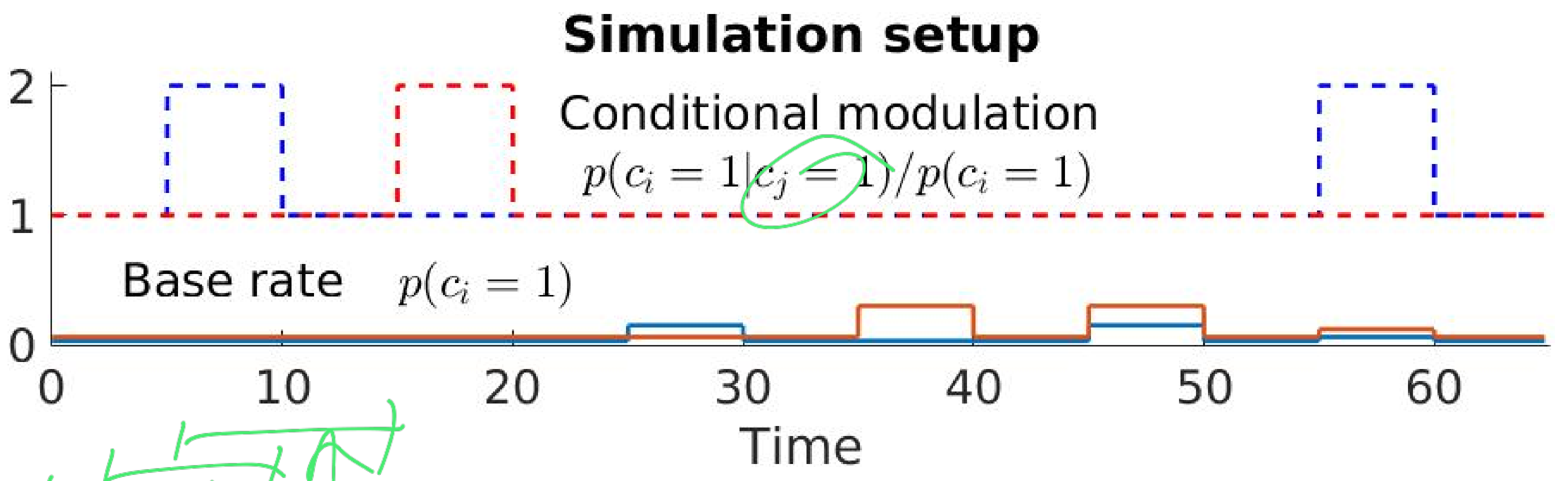
$$h_{ij}(t - t_n^{(j)}) = \frac{(t - t_n^{(j)})}{\tau_{ij}} e^{-(t - t_n^{(j)})/\tau_{ij}}$$



max. $\log L$ w.r.t. $\theta = \{a_{i0}, a_{ij}, \tau_{ij}\}$

$$\log L = \sum_{t=p+1}^{T} \sum_{i=1}^{M} \left[c_{it} \log \lambda_{it} - \log \lambda_{it} - \text{const.} \right]$$

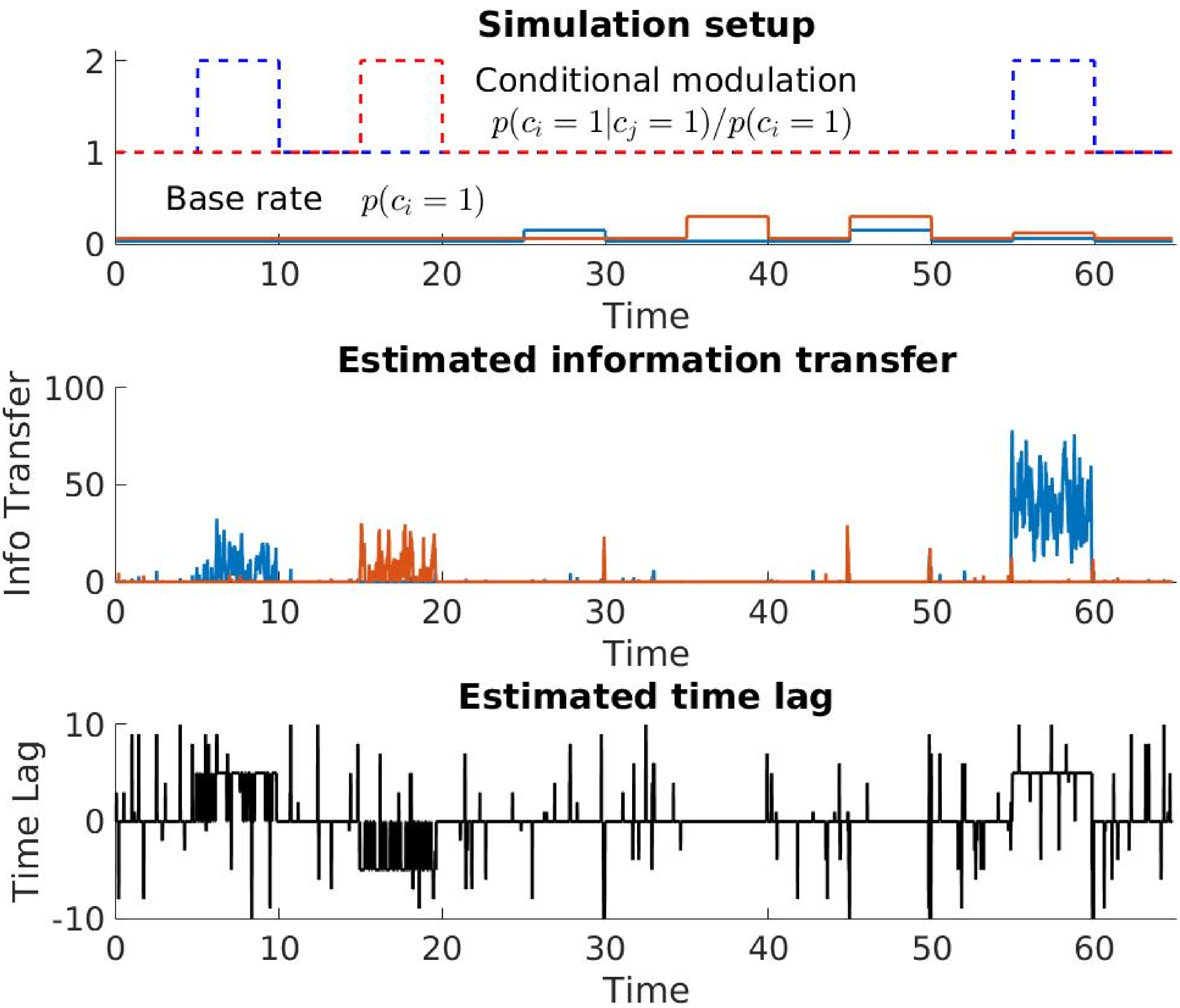
$P($"



<u>Source:</u> Durstewitz (2017) *Advanced Data Analysis in Neuroscience.* Springer.

(I) full

(II) reduced



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Latent variable / state space models

generative models

conditional independence

$$\{x_t\},\ t = 1 \ldots T$$

ocean temp. across locations

occurrence of species

$p_{\theta_{obs}}(x_t \mid g(z_t))$

observation model

decoder

climate model

$$z_t = f(z_{t-1}) + \varepsilon_t\ ,\ z_t \sim p_{\theta_{lat}}[f(z_{t-1})]$$

$$\dot{z} = f(z) + \eta(t)\ ,\ p(z(t))$$

Langevin SDE $\longrightarrow$ Fokker-Planck

DS

biol. ecosystem

latent model process DS

- scientific mechanisms
- best predictability

1 Markov property

2 $p(x_t, x_{t'} \mid z_t, z_{t'})$

$$= p(x_t \mid z_t, z_{t'})\, p(x_{t'} \mid z_t, z_{t'})$$

$$= p(x_t \mid z_t)\, p(x_{t'} \mid z_{t'})$$

$$\not\Rightarrow p(x_t, x_{t'}) = p(x_t)\, p(x_{t'})$$

Euler rule $\quad z_t = z_{t-\Delta t} + \Delta t\, f(z_{t-\Delta t})$