# Quantitative Methods

## Human Sciences, 2020–21

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Lecture 5: 12 November 2020

# Today: some revision

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- Probability and conditional probability

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- Random variables
- Probability distributions

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  2. $\mathbb{P}(\varnothing) = 0$ and $\mathbb{P}(S) = 1$.
  3. If $A_1, \ldots, A_n$ are disjoint events, then

$$\mathbb{P}(A_1 \cup \cdots \cup A_n) = \mathbb{P}(A_1) + \cdots + \mathbb{P}(A_n).$$

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- Frequentist view: a long-run frequency over many repetitions of an experiment.
- Bayesian view: a degree of belief about the event in question.
- It is possible to assign probabilities to hypotheses like "Joe Biden will win the election" or "the defendant is guilty" even though we cannot repeat the same election or the same crime over and over again.

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(*Vandermonde's identity*.)

# More counting

Books

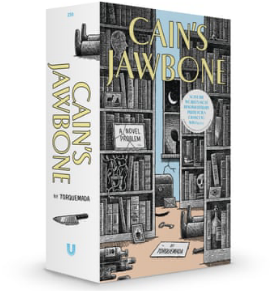
# Literary puzzle solved for just third time in almost 100 years

**British comedy writer John Finnemore has solved Cain's Jawbone, a murder mystery that has 32m possible combinations**

Alison Flood

Tue 10 Nov 2020 12.11 GMT

430




▲ Cain's Jawbone by Edward Powys Mathers. Photograph: Unbound

One of the world's most fiendish literary puzzles - a murder mystery in which all the pages are out of order - has been solved for just the third time in almost a century.

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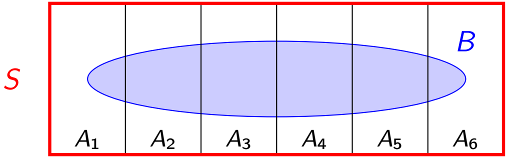
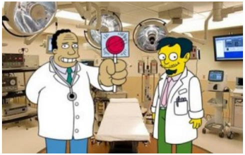
- Bayes' Rule:
$$\mathbb{P}(A \mid B) = \frac{\mathbb{P}(B \mid A)\mathbb{P}(A)}{\mathbb{P}(B)}.$$
- Law of Total Probability:
$$\mathbb{P}(B) = \sum_{i=1}^{n} \mathbb{P}(B \mid A_i)\mathbb{P}(A_i).$$

# Visualising the Law of Total Probability



# Simpson's paradox



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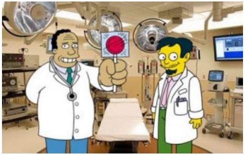

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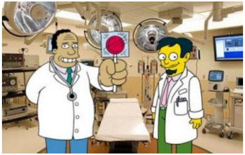

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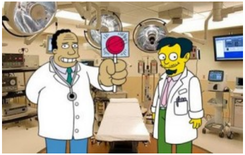

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| Dr. H. | Heart | Plaster |
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| Success | 70 | 10 |
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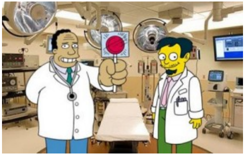

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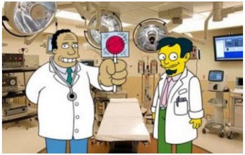

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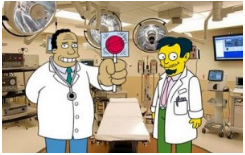

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- The association between surgeon and outcome is *confounded* by surgery type: we need to condition on whether or not $C$ occurred.

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- $F(x) = \mathbb{P}(X \leq x)$: sum of all densities up to $x$.
- Famous distributions: Bernoulli, Binomial, Poisson, Uniform, Exponential, Normal, etc.

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- Notation: $\mu$, $\sigma^2$, $\sigma_{XY}$.

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- To standardise, subtract mean and divide by standard deviation:

$$Z = \frac{\overline{X} - \mu}{\sigma}.$$

- Standard Normal distribution: $\mu = 0$, $\sigma^2 = 1$.
- Formal notation: $Z \sim \mathcal{N}(0, 1)$.

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- How to use (simulate from) the distribution.
- E.g. `rnorm()`, `dnorm()`, `pnorm()`.

# Simulating in R

```r
rnorm(n = 1, mean = 0, sd = 1)

## [1] 1.473838
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```r
rnorm(1, -5, 7)
```

```
## [1] -20.85814
```

# Simulating in R (cont.)

```
x <- seq(-1, 1, 0.5)
dnorm(x)

## [1] 0.2419707 0.3520653 0.3989423 0.3520653 0.2419707
```

# Simulating in **R** (cont.)

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```
dnorm(20)

## [1] 5.520948e-88
```

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x <- seq(-1, 1, 0.5)
dnorm(x)

## [1] 0.2419707 0.3520653 0.3989423 0.3520653 0.2419707
```

```
dnorm(20)

## [1] 5.520948e-88
```

```
dnorm(50)

## [1] 0
```

# Simulating in **R** (cont.)

```
pnorm(0)

## [1] 0.5
```

# Simulating in R (cont.)

```
pnorm(0)

## [1] 0.5
```

```
pnorm(1) - pnorm(-1)

## [1] 0.6826895
```

# Simulating in **R** (cont.)

```r
pnorm(0)

## [1] 0.5
```

```r
pnorm(1) - pnorm(-1)

## [1] 0.6826895
```

```r
pnorm(2) - pnorm(-2)

## [1] 0.9544997
```

# Simulating in **R** (cont.)

```r
pnorm(0)
```

```
## [1] 0.5
```

```r
pnorm(1) - pnorm(-1)
```

```
## [1] 0.6826895
```

```r
pnorm(2) - pnorm(-2)
```

```
## [1] 0.9544997
```

```r
pnorm(3) - pnorm(-3)
```

```
## [1] 0.9973002
```

# Simulating in R

```
rnorm(n = 1, mean = 0, sd = 1)

## [1] 0.8698619
```

# Simulating in R

```r
rnorm(n = 1, mean = 0, sd = 1)

## [1] 0.8698619
```

```r
rnorm(n = 1, mean = 50, sd = 14)

## [1] 34.32478
```

# Simulating in R

```r
rnorm(n = 1, mean = 0, sd = 1)

## [1] 0.8698619
```

```r
rnorm(n = 1, mean = 50, sd = 14)

## [1] 34.32478
```

```r
rnorm(1, -5, 7)

## [1] 3.651719
```

# Simulating in **R** (cont.)

```
x <- seq(-1, 1, 0.5)
dnorm(x)

## [1] 0.2419707 0.3520653 0.3989423 0.3520653 0.2419707
```

# Simulating in **R** (cont.)

```
x <- seq(-1, 1, 0.5)
dnorm(x)

## [1] 0.2419707 0.3520653 0.3989423 0.3520653 0.2419707
```

```
dnorm(20)

## [1] 5.520948e-88
```

# Simulating in R (cont.)

```r
x <- seq(-1, 1, 0.5)
dnorm(x)

## [1] 0.2419707 0.3520653 0.3989423 0.3520653 0.2419707
```

```r
dnorm(20)

## [1] 5.520948e-88
```

```r
dnorm(50)

## [1] 0
```

# Simulating in **R** (cont.)

```
pnorm(0)

## [1] 0.5
```

# Simulating in **R** (cont.)

```
pnorm(0)

## [1] 0.5
```

```
pnorm(1) - pnorm(-1)

## [1] 0.6826895
```

# Simulating in R (cont.)

```r
pnorm(0)

## [1] 0.5
```

```r
pnorm(1) - pnorm(-1)

## [1] 0.6826895
```

```r
pnorm(2) - pnorm(-2)

## [1] 0.9544997
```

# Simulating in R (cont.)

```r
pnorm(0)
```

```
## [1] 0.5
```

```r
pnorm(1) - pnorm(-1)
```

```
## [1] 0.6826895
```

```r
pnorm(2) - pnorm(-2)
```

```
## [1] 0.9544997
```

```r
pnorm(3) - pnorm(-3)
```

```
## [1] 0.9973002
```

# The 68-95-99% rule