The structure of social space

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We seek to provide a formal definition of what constitutes a social *space* and to specify those properties that endow it with a *structure*. Intuitively speaking, a space is a set where some notion of proximity and distance between its elements is established. The structure of a space delineates the ways in which the elements of the set relate to and interact with one another. A social space, then, can be viewed as a system of positions structured by social relations between individuals and collectives. Previous scholarship has tended to frame these relations as dual in nature — both material and symbolic — and relate them to their (long-run) historical conditions of possibility. Moreover, they have typically been attributed a certain moral valence derived from the distribution of social power, where power can be defined as the capacity to appropriate socially useful energy. In more prolix terms, then, social spaces are historically situated constellations of social positions, relations, and actions that are animated by the distribution of material and symbolic goods. Such distributions, in turn, are governed by durably institutionalised networks of social power.

Both theoretical and empirical accounts of social spaces rely, implicitly or explicitly, on a choice of 'focal variable' with respect to which relational comparisons are made. Common choices include income and wealth, years of education, or access to political power. From this perspective, a social space may be construed as a metric space — a pair (S, d) composed of a set S and a distance function $d : S \times S \to \mathbb{R}_{\geq 0}$ (satisfying a set of basic axioms) — whereby relative positions within the space become defined with respect to a chosen metric. Hence two social spaces are considered 'equivalent' or 'similar' if they have similar distributions of a socially effective asset or resource. For instance, Scandinavian societies are often viewed as 'similar' insofar as they have similar income distributions. More generally, two metric spaces are considered 'the same' if there is a distance-preserving bijection between them (an isometric isomorphism).

Such an approach has the advantage of simplifying the comparative analysis of societies, yet it espouses a relatively rigid understanding of social spaces and the set of possible mappings that relate them to one another. Indeed, there are isomorphic spaces that possess very different metrics but which nonetheless have important properties in common. Many such properties are preserved by homeomorphism — i.e., isomorphic mappings between topological spaces. A topological space can be viewed as a set of points coupled with a set of neighbourhoods for each point that satisfy a collection of basic axioms relating points and neighbourhoods. The category of topological spaces (loosely speaking, the collection of topological spaces that are linked by continuous maps) thus seems to offer a more flexible yet comprehensive framework through which the notion of a social space — and its dual, a social structure — can be mathematically formalised than does that of metric spaces.

A few points to consider:

- ▲ A rigorous theory hinges on rendering explicit the duality between a space and a structure.
- ▲ What constitutes a basis of the space — i.e., a minimal amount of information or data with which we may recover the whole space and its endowed structure?
- ▲ Is a general model-theoretic or algebraic understanding of a structure — a set along with a collection of well-defined finitary relations — a sufficiently refined account of what constitutes a social structure?
- ▲ Should the space be metrisable — i.e., should it be homeomorphic to at least one metric space?
- ▲ What measure-theoretic concepts are required to assign a meaningful 'measure' to positions in a social space?
- ▲ How are quotient spaces and non-commutative geometries/algebras to the conceptualised?