

Quantitative Methods

Human Sciences, 2020–21

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Lecture 8: 3 December 2020

Today

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- ▶ A third theory of inference.

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- ▶ Causal graphs.

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- ▶ Causal graphs.
- ▶ Three forms of systematic bias.

Neyman-Pearson hypothesis testing

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- ▶ Note: all tests are done “under” (i.e., assuming) H_0 .

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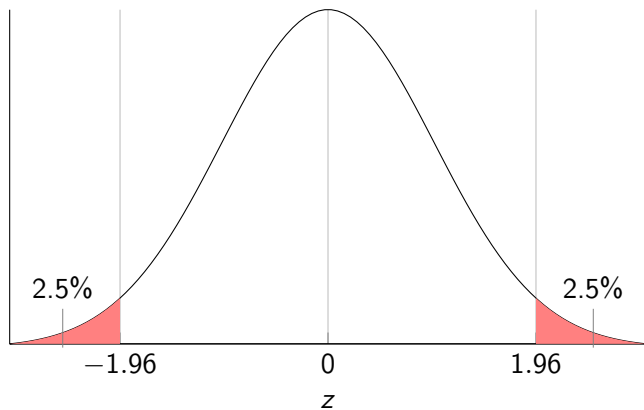
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- ▶ p -value: the probability of obtaining something at least as extreme as the observed z -statistic, assuming H_0 is true.
- ▶ Common mistake: the p -value is *not* the probability that H_1 is false. (Why not?)

Neyman-Pearson hypothesis testing (cont.)



Neyman-Pearson hypothesis testing: example

```
##
## Call:
## lm(formula = y ~ x, data = data)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -14.8208  -3.1487   0.0409   3.1336  13.1631
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  0.23379    0.50486   0.463   0.644
## x            5.11564    0.08244  62.052 <2e-16 ***
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.755 on 498 degrees of freedom
## Multiple R-squared:  0.8855, Adjusted R-squared:  0.8852
## F-statistic: 3851 on 1 and 498 DF, p-value: < 2.2e-16
```

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- ▶ A standard causal graph does not express the magnitude or sign of a causal effect.
- ▶ Conditional on its direct causes, any variable in a causal graph is independent of any other variable for which it is not a cause.
- ▶ Implication: the common causes of any pair of variables in the graph must also be in the graph.

A randomised experiment

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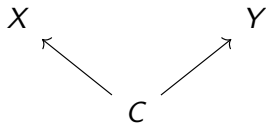
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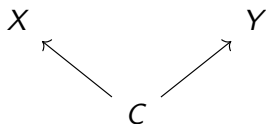
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- ▶ The causal graph depicts this knowledge for an experiment in which cigarettes are randomly assigned to a treatment group.

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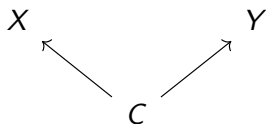
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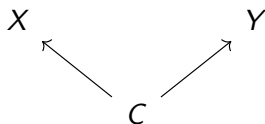


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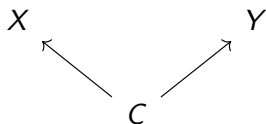


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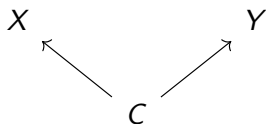


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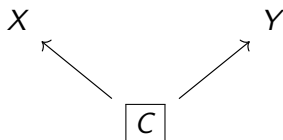


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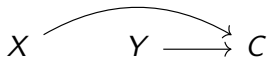


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- ▶ If we do condition on C , we block the association between X and Y (via conditional independence).

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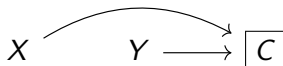


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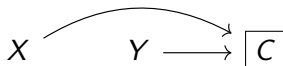


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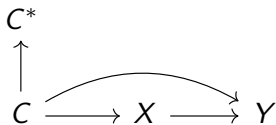


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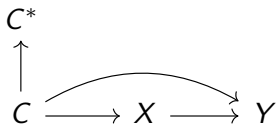
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- ▶ If we do condition on C , we induce an association between X and Y .
- ▶ This is known as *selection bias* (via conditional dependence).

Measurement bias

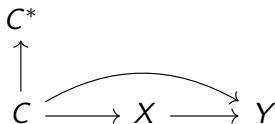


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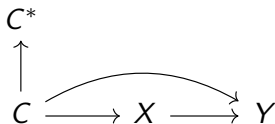
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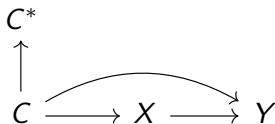
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- ▶ For instance, mismeasurement of confounders induce bias even if both treatment and outcome are perfectly measured.
- ▶ In general, the path $X \leftarrow C \rightarrow Y$ cannot be blocked by conditioning on mismeasured confounder C^* .

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3. They share a common effect and the analysis is restricted to a certain level of that common effect.
4. Random variability (disappears as $n \rightarrow \infty$).

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 1. Confounding: not conditioning on common causes.
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- ▶ These biases may arise in observational studies *and* in randomised experiments (how?).