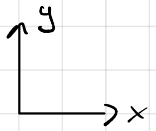
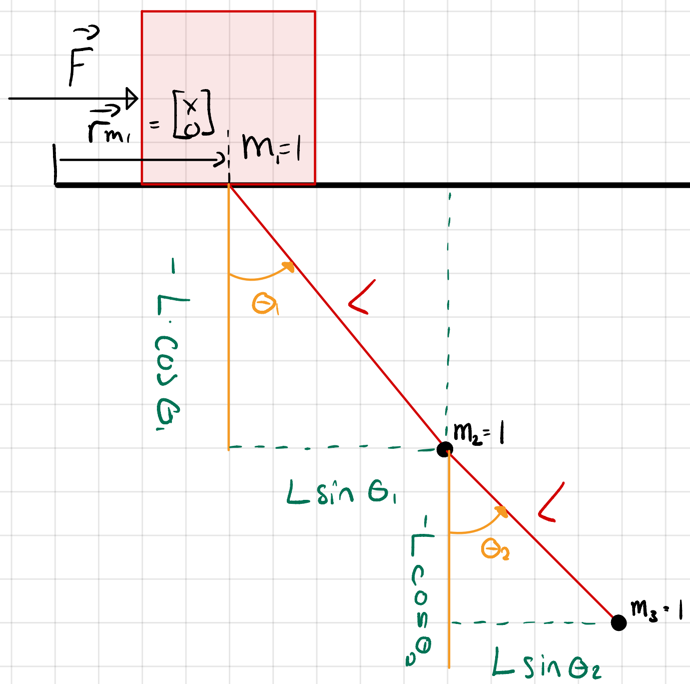
Assignment 5 TDT4130 Elias Olsen Almenningen

Problem 1:

a)





$$q = \begin{bmatrix} x \\ \theta_1 \\ \theta_2 \end{bmatrix}$$

$$\vec{p}^{\,0}_{m_1} = \begin{bmatrix} x \\ 0 \end{bmatrix} + \begin{bmatrix} L\sin\theta_1 \\ -L\cos\theta_1 \end{bmatrix} = \begin{bmatrix} x + L\sin(\theta_1) \\ -L\cos\theta_2 \end{bmatrix}$$

$$\vec{p}^{\,m_1}_{m_2} = \begin{bmatrix} L\sin\theta_2 \\ -L\cos\theta_2 \end{bmatrix}$$

$$\vec{p}^{\,0}_{m_2} = \vec{p}^{\,0}_{m_1} + \vec{p}^{\,m_1}_{m_2} = \begin{bmatrix} x + L\sin(\theta_1) + L\sin\theta_2 \\ -L\cos(\theta_1) - L\cos(\theta_2) \end{bmatrix}$$

4) Kinetic energy:

$$T = \frac{1}{2}\dot{q}^T W(q)\,\dot{q}$$

$$W_1 = J_1^T J_1 m_1 = \frac{\partial p_1}{\partial q}^T \frac{\partial p_1}{\partial q} m_1 = \underset{1\times 3}{\begin{bmatrix} \frac{\partial p_1}{\partial x} \\ \frac{\partial p_1}{\partial \theta_1} \\ \frac{\partial p_1}{\partial \theta_2} \end{bmatrix}^T} \underset{3\times 1}{\begin{bmatrix} \frac{\partial p_1}{\partial x} \\ \frac{\partial p_1}{\partial \theta_1} \\ \frac{\partial p_1}{\partial \theta_2} \end{bmatrix}} m_1$$

$$= \begin{bmatrix} 0 & L\cos(\theta_1)\dot{\theta}_1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ L\cos(\theta_1)\dot{\theta}_1 \\ 0 \end{bmatrix} m_1 = L^2\cos^2(\theta_1)\dot{\theta}_1^2 m_1$$

Cart: $\vec{p}_{m_1} = \begin{bmatrix} x \\ 0 \end{bmatrix}$

$$W_1 = \frac{\partial P_{m_1}}{\partial q}^T \frac{\partial P_{m_1}}{\partial q} m_1 = \underset{3\times 2}{\begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}} \cdot \underset{2\times 3}{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}} m_1 = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

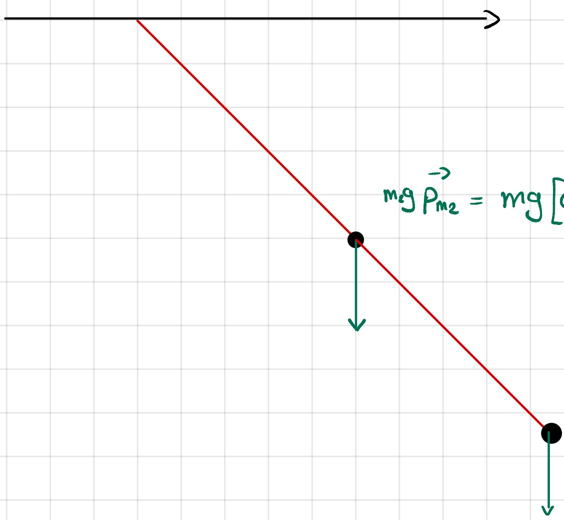
$$\begin{bmatrix} x \\ 0 \end{bmatrix} \quad \frac{\partial P_{m_1}}{\partial q} = \begin{bmatrix} \frac{\partial x}{\partial x} & \frac{\partial x}{\partial \theta_1} & \frac{\partial x}{\partial \theta_2} \\ \frac{\partial 0}{\partial x} & \frac{\partial 0}{\partial \theta_1} & \frac{\partial 0}{\partial \theta_2} \end{bmatrix}$$

$$T = \frac{1}{2} q^T W_1 q = \frac{1}{2} m x^2$$

mass 2 $\quad W_{m_2} = \left(J_2^T J_2\right) m_2$

$T_2 = \frac{1}{2} \dot{q}^T$

Potential energy:



$$m g \vec{P}_{m_2} = m g \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} L \sin\theta_1 \\ -L \cos\theta_1 \end{bmatrix} = -m g L \cos\theta_1$$

$$m_3 g \vec{P}_{m_3} = m_3 g \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} L \sin(\theta_2) \\ -L \cos\theta_2 \end{bmatrix} = -m_3 g L \cos(\theta_2)$$

b)

$$\dot{\boldsymbol{x}} = \begin{bmatrix} \dot{\boldsymbol{q}} \\ \overbrace{\frac{\partial^2 \mathcal{L}}{\partial \dot{q}^2}^{-1}}^{W} \left(\boldsymbol{Q} + \frac{\partial \mathcal{L}}{\partial q} - \frac{\partial^2 \mathcal{L}}{\partial \dot{q} \partial q} \dot{\boldsymbol{q}} \right) \end{bmatrix} \qquad F = -10x - \dot{x}$$

$$X = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$$

PendulumDynamics(parameters, state, t) skal lages

PendulumODEMatrices$(\underbrace{\text{in1}}_{\text{states}}, F, \underbrace{\text{in2}}_{\text{params}})$

c)

Pendulum Dynamics:

PendulumSymbolicTemplate.m × PendulumDynamics.m * × PendulumODEMatrices.m × PendulumSimulation.m × PendulumPosition.m × +

```
function   x_dot = PendulumDynamics(t, state, parameters)
    % [x; theta_1; theta_2]
    q = state(1:3);
    % [x_dot; theta_1_dot; theta_2_dot]
    qd = state(4:6);
    % F = -10*x - x_dot
    F = -10*q(1)-qd(1);

    % x_dot = q_dot; d^2(L^-1)/dq_dot^2) ( Q + dL/dq ..... )
    %                detter er W          detter er RHS
    [W,RHS] = PendulumODEMatrices(state,F,parameters);

    x_dot = [qd;W\RHS];
end
```

Pendulum simulation:

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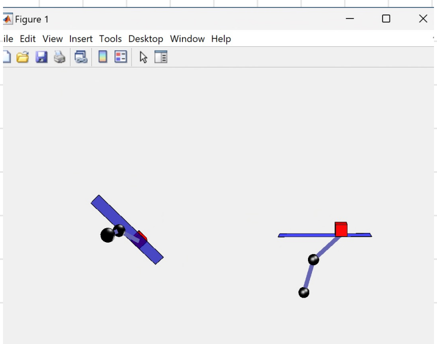
PendulumSymbolicTemplate.m × PendulumDynamics.m × PendulumODEMatrices.m × PendulumSimulation.m ×

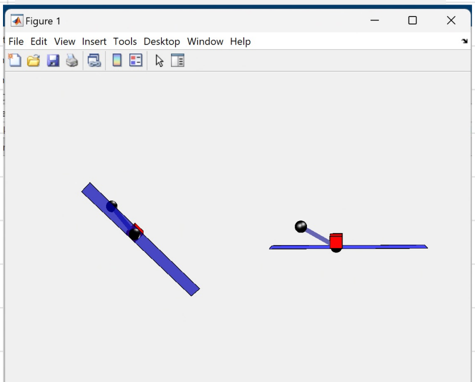
```
% Parameters and initial states
tf = 20;

x0 = 0;
g = 9.81;
L = 1;
m = 1;
M = 1;

theta1_0 = pi/4;
theta2_0 = pi/2;

q = [x0;theta1_0;theta2_0];
qd = [0;0;0];

state = [q;qd];
parameters = [m;M;L;g];

```

```
    % Unwrap state.
    [p1,p2] = PendulumPosition(x_disp,parameters); % position 1st and 2nd ball
    x = x_disp(1); % position cart
```

The animation

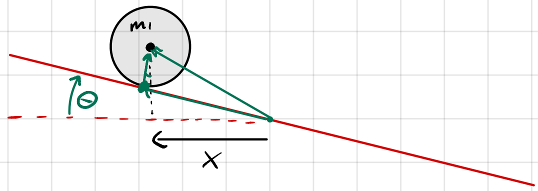


The pendulum seems to work. When the box is mooving, the pendulum is "still". and when the pendlulum mooves. the box is still.

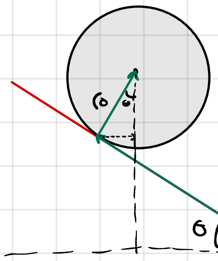


Problem 2:

$M_1 = 10\,kg$
$R = 0.25$



a) $$q = \begin{bmatrix} x \\ \theta \end{bmatrix}$$



$$\vec{r}_0 = R\begin{bmatrix} -\sin\theta \\ \cos\theta \end{bmatrix}$$

$$\vec{p}_{m_1} = \begin{bmatrix} x\cos(\theta) \\ x\sin(\theta) \end{bmatrix} + \vec{r}_0$$

$$\vec{p}_M = \begin{bmatrix} x\cos(\theta) - R\sin(\theta) \\ x\sin(\theta) + R\cos(\theta) \end{bmatrix}$$

b) The total angular momentum of the ball is the sum of the angular momentum from the ball rolling and from the ball and beam rotating.

$$\omega_{ball} = \underbrace{\frac{\dot{x}}{R}}_{\text{ball rolling (purely)}} + \underbrace{\dot{\theta}}_{\text{beam "rolling"}}$$

c) The total kinetic energy of the ball is the sum of the "linear"- and "rotational"- kinetic energy:

$$T_{ball} = \frac{1}{2}\dot{q}^T W \dot{q} + \frac{1}{2} I\omega^2 \quad , \quad W = \iint M, \; I = \frac{2}{5}MR^2$$

d) The kinetic energy of the beam is only rotational

$$T_{beam} = \frac{1}{2} J \dot{\theta}^2 \quad , \; J = 1$$

e) 5) $V = m \cdot g \cdot h = M \cdot g \cdot [0 \;\; 1] \, \vec{p}_{m_1}$

f/g)

BallAndBeamSimulation.m

```
clear all
close all
clc

% Parameters and initial states
tf = 15;

J = 1;
M = 10;
R = 0.25;
g = 9.81;

x0 = 1;
theta0 = 0;
q = [x0;theta0];
dq = [0;0];

state = [q;dq];
parameters = [J; M; R; g];
```

```
        % Unwrap state.
        pos = BallPosition(x_disp,parameters);
        theta = x_disp(2); % beam angle
        pos = [pos(1);0;pos(2)]; % ball position
```

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BallAndBeamSimulation.m | BallAndBeamDynamics.m *

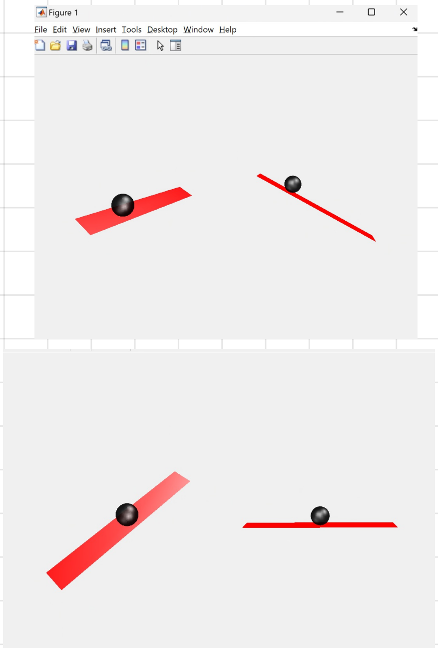
```
function x_dot = BallAndBeamDynamics(t, state, parameters)

    q = state(1:2);
    dq = state(3:4);

    T = 200*(q(1)-q(2)) + 70*(dq(1)-dq(2));

    [W, RHS] = BallAndBeamODEMatrices(state,T,parameters);

    x_dot = [dq;W\RHS];
end
```



The simulation behaves as expected, the beam counteracts the ball, making it stop.