Assignment 3

TTK4215	
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Problem 1

Consider the third-order plant

$$y = G(s)u \tag{1}$$

where

$$G(s) = \frac{b_2 s^2 + b_1 s + b0}{s^3 + a_2 s^2 + a_1 s + a_0} \tag{2}$$

a)

Obtain parametric models for the plant in the form of SPM and DPM when $\theta^* = [b_2, b_1, b_0, a_2, a_1, a_0]^\top$

SPM

$$z = \theta^{*\top} \phi$$

Writing the unknown variables on the right hand side and known on the left hand side yields:

$$ys^3 = \left(b_2 s^2 + b_1 s + b_0\right) u - \left(a_2 s^2 + a_1 s + a_0\right) y$$

and dividing by the filter

$$\Lambda(s) = s^3 + \lambda_2 s^2 + \lambda_1 s + \lambda_0 \tag{3}$$

yields:

$$z = \frac{s^3}{\Lambda(s)} [y] \tag{4}$$

$$\phi = \left[\frac{s^2}{\Lambda(s)} [u], \frac{s}{\Lambda(s)} [u], \frac{1}{\Lambda(s)} [u], -\frac{s^2}{\Lambda(s)} [y], -\frac{s}{\Lambda(s)} [y], -\frac{1}{\Lambda(s)} [y] \right]^\top \tag{5}$$

DPM

The form of the DPM is

$$z = W(s) \theta^{*T} \phi \tag{6}$$

where $z = \frac{s^3}{\Lambda(s)} [y], W(s) = \frac{1}{\Lambda(s)}, \phi = \left[s^2 [u], s [u], 1 [u], -s^2 [y], -s [y], - [y]\right]^\top$

b)

$a_2 = 3, a_1 = 1, a_0 = 2$, so $\theta^* = [b_2, b_1, b_0]^\top$ so

$$z = \frac{s^3}{\Lambda(s)}[y] + \frac{3s^2}{\Lambda(s)}[y] + \frac{s}{\Lambda(s)}[y] + \frac{2}{\Lambda(s)}[y] \tag{7}$$

$$\phi = \left[\frac{s^2}{\Lambda(s)}[u], \frac{s}{\Lambda(s)}[u], \frac{1}{\Lambda(s)}[u]\right]^\top \tag{8}$$

c)

$b_2 = b_1 = 0, b_0 = 2$, so $\theta^* = [a_2, a_1, a_0]^\top$ so

$$z = \frac{s^3}{\Lambda(s)}[y] + \frac{2}{\Lambda(s)}[u] \tag{9}$$

$$\phi = \left[-\frac{s^2}{\Lambda(s)}[y], -\frac{s}{\Lambda(s)}[y], -\frac{1}{\Lambda(s)}[y]\right]^\top \tag{10}$$

Problem 2

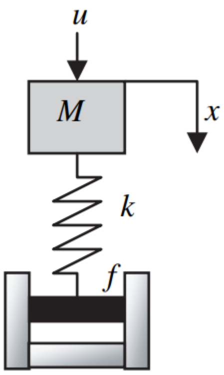

Figure 1: Mass–spring–dashplot

$$M\ddot{x} = u - kx - f\dot{x} \tag{11}$$

Consider the mass–spring–dashplot system of Figure 1 described by (11) with x, u as the only signals available for measurement. Let us assume that $M = 100$ kg and f , k are the unknown constant parameters that we want to estimate online. Develop a parametric model for estimating the unknown parameters f, k. Specify any arbitrary parameters or filters used.

Taking the Laplace transform of (11) and arranging the known variables on the left hand side and unknown on the right hand side, we get

$$Ms^2x - u = -fsx - kx \tag{12}$$

and by choosing the filter

$$\Lambda(s) = s^2 + \lambda_1 s + \lambda_0 \tag{13}$$

and SPM model can be formulated on the following form:

$$z = M\frac{s^2}{\Lambda(s)}[x] - \frac{1}{\Lambda(s)}[u] \tag{14}$$

$$\theta^* = [f, k]^\top \tag{15}$$

$$\phi = \left[-\frac{s}{\Lambda(s)}[x], -\frac{1}{\Lambda(s)}[x]\right]^\top \tag{16}$$

Problem 3

The second-order ARMA model

$$y(k) = -1.3y(k-1) - a_2 y(k-2) + b_1 u(k-1) + u(k-2) \tag{17}$$

where a_2, b_1 are unknown can be written as an linear parametric model as:

$$\theta^* = [a_2, b_1]^\top \tag{18}$$

$$z = y(k) \tag{19}$$

$$\phi = [-1.3y(k-1), -y(k-2), u(k-1), u(k-2)]^\top \tag{20}$$

Problem 4

The fourth-order ARMA model

$$y(k+4) = a_1 y(k+3) - a_2 y(k) + b_1 u(k) + u(k+2) \tag{21}$$

where a_1, a_2, b_1 are unknown. Only the current and the past four values of the signals u and y, i.e, $u(k), \dots, u(k-4), y(k), \dots, y(k-4)$, are available for measurement.

We then have to shift each side by four in the time axis, in order to write (21) as an SPM, which is equivalent to filtering each side with the fourth-order stable filter $\frac{1}{z^4}$:

$$y(k) = a_1 y(k-1) - a_2 y(k-4) + b_1 u(k-4) + u(k-2) \tag{22}$$

The SPM of (21) is then:

$$\theta^* = [a_1, a_2, b_1]^\top \tag{23}$$

$$z = y(k) - u(k-2) \tag{24}$$

$$\phi = [y(k-1), -y(k-4), u(k-4)]^\top \tag{25}$$

Problem 5

Consider the nonlinear system

$$\ddot{x} + 2\dot{x} + x = a_1 f_1(x) + a_2 f_2(x) + b_1 g_1(x)u + b_2 g_2(x)u \tag{26}$$

where a_1, a_2, b_1, b_2 are unknown constants and $x, f_1(x), f_2(x), g_1(x), g_2(x), u$ are available for measurement. Express the unknown parameters in the form of

a) the linear SPM

Taking the Laplace transform of (26) we get the system as:

$$s^2 x + 2sx + x = a_1 f_1(x) + a_2 f_2(x) + b_1 g_1(x)u + b_2 g_2(x)u \tag{27}$$

Filtering both sides with the filter $\Lambda(s) = s^2 + \lambda_1 s + \lambda_0$ we can express the system in the form of the SPM:

$$\theta^* = [a_1, a_2, b_1, b_2]^\top \tag{28}$$

$$z = \frac{s^2}{\Lambda(s)}[x] + \frac{s}{\Lambda(s)}[x] + \frac{1}{\Lambda(s)}[x] \tag{29}$$

$$\phi = \left[\frac{1}{\Lambda(s)}[f_1(x)], \frac{1}{\Lambda(s)}[f_2(x)], \frac{1}{\Lambda(s)}[g_1(x)], \frac{1}{\Lambda(s)}[g_2(x)], \right]^\top \tag{30}$$

b) the linear DPM

The form of the DPM is

$$z = W(s)\theta^{*T}\phi \tag{31}$$

where

$$\theta^* = [a_1, a_2, b_1, b_2]^\top \tag{32}$$

$$W(s) = \frac{1}{\Lambda(s)} \tag{33}$$

$$z = \frac{s^2}{\Lambda(s)}[x] + \frac{s}{\Lambda(s)}[x] + \frac{1}{\Lambda(s)}[x] \tag{34}$$

$$\phi = [[f_1(x)], [f_2(x)], [g_1(x)], [g_2(x)]]^\top \tag{35}$$

Problem 6

The system described in the I/O form

$$y = K_p \frac{s+b}{s^2 + as + c} u \tag{36}$$

where b, a, c, Kp are unknown constants. In addition, we know that $Kp > 0$ and only u and y are available for measurement. Express the unknown parameters in the form of the

a) B-SPM

B-SPM has the form:

$$z = \rho^* \left(\theta^* \phi + z_1\right) \tag{37}$$

where $z \in \Re, \phi \in \Re^n, z_1 \in \Re$ are signals available for measurement at each time t, and $\rho^* \in \Re R^n, \theta^* \in \Re^n$ are the unknown parameters.

Multiplying out (36) and rearranging gives:

$$s^2 y = K_p su + K_p bu - yas - yc \tag{38}$$

Since $K_p > 0$, we can redefine the unknown parameters to $\bar{a} = \frac{a}{K_p}, \bar{c} = \frac{c}{K_p}$, and by using the filter $\Lambda(s) = s^2 + \lambda_1 s + \lambda_0$ the system can be written as (37):

$$z = \frac{s^2}{\Lambda(s)} [y] \tag{39}$$

$$\rho^* = K_p \tag{40}$$

$$\theta^* = [b, \bar{a}, \bar{c}]^\top \tag{41}$$

$$\phi = \left[\frac{1}{\Lambda(s)} [u], -\frac{s}{\Lambda(s)} [y], -\frac{1}{\Lambda(s)} [y]\right]^\top \tag{42}$$

$$z_1 = \frac{s}{\Lambda(s)} [u] \tag{43}$$

b) B-DPM

B-DPM has the form:

$$z = W(q)\rho^* \left(\theta^* \phi + z_1\right) \tag{44}$$

where $z \in \Re, \phi \in \Re^n, z_1 \in \Re$ are signals available for measurement at each time t, and $\rho^* \in \Re^n, \theta^* \in \Re^n$ are the unknown parameters. The transfer function $W(q)$ is a known stable transfer function.

$$z = \frac{s^2}{\Lambda(s)} [y] \tag{45}$$

$$W(q) = \frac{1}{\Lambda(s)} \tag{46}$$

$$\rho^* = K_p \tag{47}$$

$$\theta^* = [b, \bar{a}, \bar{c}]^\top \tag{48}$$

$$\phi = [[u], -s[y], -[y]]^\top \tag{49}$$

$$z_1 = s[u] \tag{50}$$

c) linear SPM

$$z = \frac{s^2}{\Lambda(s)} [y] \tag{51}$$

$$\theta^* = [b, a, c, K_p]^\top \tag{52}$$

$$\phi = \left[\frac{1}{\Lambda(s)} [u], -\frac{s}{\Lambda(s)} [y], -\frac{1}{\Lambda(s)} [y], \frac{s}{\Lambda(s)} [u]\right]^\top \tag{53}$$

$$\tag{54}$$

d) linear DPM

$$z = \frac{s^2}{\Lambda(s)} [y] \tag{55}$$

$$W(s) = \frac{1}{\Lambda(s)} \tag{56}$$

$$\theta^* = [b, a, c, K_p]^\top \tag{57}$$

$$\phi = [[u], -s[y], -[y], s[u]]^\top \tag{58}$$

$$\tag{59}$$

Problem 7

Consider the nonlinear system

$$\dot{x} = f(x) + g(x)u \tag{60}$$

where the state x and the input u are available for measurement and $f(x), g(x)$ are smooth but unknown functions of x. In addition, it is known that $g(x) > 0 \forall x$. We want to estimate the unknown functions f, g online using neural network approximation techniques. It is known that there exist constant parameters $W^*_{f_i}, W^*_{g_i}$, referred to as weights, such that

$$f(x) \approx \sum_{i=1}^{m} W^*_{f_i} \varphi_{f_i}(x) \tag{61}$$

$$g(x) \approx \sum_{i=1}^{n} W^*_{g_i} \varphi_{g_i}(x) \tag{62}$$

where $\varphi_{fi}(\cdot), \varphi_{gi}(\cdot)$ are some basis functions that are known and n, m are known integers representing the number of nodes of the neural network. Obtain a parameterization of the system in the form of SPM that can be used to identify the weights $W^*_{f_i}, W^*_{g_i}$ online.

$$\dot{x} = \sum_{i=1}^{m} W^*_{f_i} \varphi_{f_i}(x) + \sum_{i=1}^{n} W^*_{g_i} \varphi_{g_i}(x) u \tag{63}$$

taking the Laplace transform of (63) results in

$$sx = \sum_{i=1}^{m} W^*_{f_i} \varphi_{f_i}(x) + \sum_{i=1}^{n} W^*_{g_i} \varphi_{g_i}(x) u \tag{64}$$

meaning the filter can be written as $\Lambda(s) = s + \lambda_0$. Writing 60 on SPD becomes:

$$z = \frac{s}{\Lambda(s)} [x] \tag{65}$$

$$\theta^* = \left[W^*_{f_1}, \ldots, W^*_{f_m}, W^*_{g_1}, \ldots, W^*_{g_n} \right] \tag{66}$$

$$\phi = \left[\frac{\phi_{f_1}}{\Lambda(s)} [u], \ldots, \frac{\phi_{f_m}}{\Lambda(s)} [u], \frac{\phi_{g_1}}{\Lambda(s)} [u], \ldots, \frac{\phi_{g_n}}{\Lambda(s)} [u] \right] \tag{67}$$

$$\tag{68}$$