Assignment 1

TTK4215

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Contents

Problem 1

Read Chapter 1 in the textbook [1] and answer the following questions:

1a)

One of the primary motivations for research on adaptive control systems where the design of autopilots for high-performance aircraft. Aircraft operate over a wide range of speeds and altitudes, and their dynamics are nonlinear and conceptually time-varying. The dynamics of an aircraft can be approximated by a linear system, but as the speeds and altitudes varies, so does the system matrices A, B, C and D. Therefore one can argue that, in principle, since the measured outputs carry information about the state and parameters, a sophisticated feedback controller could learn the parameter changes, by processing the outputs, and use the appropriate adjustments to accommodate them. This argument led to a feedback control structure on which adaptive control is based [1].

1b)

Robust control designs a controller to handle uncertainties without adaptation, while adaptive control actively adapts the controller based on real-time parameter estimation and system identification.

1c)

One of the disadvantages of gain scheduling is that the adjustment mechanism of the controller gains is precomputed offline and, therefore, provides no feedback to compensate for incorrect schedules [1].

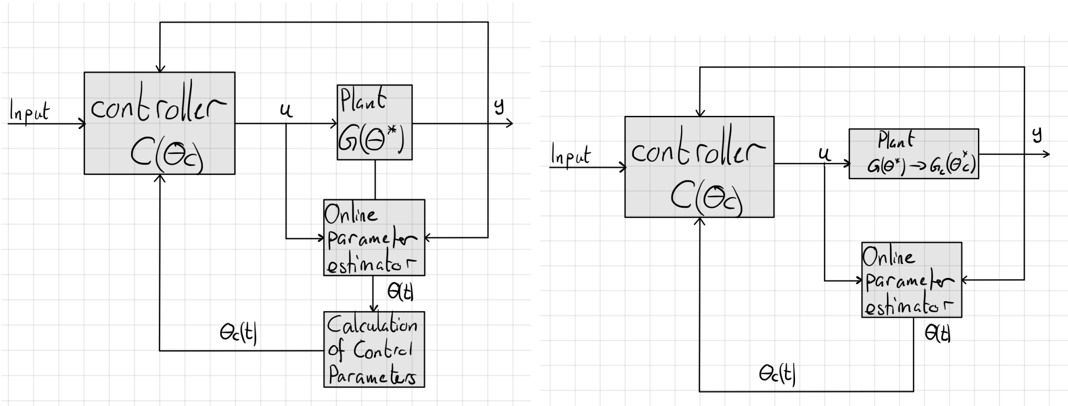
1d)

Indirect adaptive control first estimates the plant parameters in a vector $\theta(t)$, which is used to calculate the controller parameter vector $\theta_c(t)$, see Figure 1a.

The direct adaptive control structure parameterizes the plant model, $G(\theta^*)$, in terms of the unknown controller parameter θ_c^*, for which $G_c(\theta_c^*)$ exactly matches the I/O characteristics of $G(\theta^*)$. $G_c(\theta_c^*)$ is then used to provide the direct online estimate $\theta_c(t)$, see Figure 1b

1e)

For a LTI-system to be minimum-phase all their zeroes are located in the left half plane.



(a) Indirect adaptive control structure

(b) Direct adaptive control structure

Figure 1: Direct- and indirect adaptive control structure

1f

In both cases, direct- and indirect adaptive control, the estimated parameters are treated as the true parameters for control design purposes. This design approach is called *certainty equivalence* [1].

1g)

Mention some basic methods used to design adaptive laws (on-line parameter estimators).

Problem 2

Implement the peak-seeking algorithm in MATLAB/Simulink and apply it to the static function $f : R \to R$

$$f(\theta) = -\theta^2 + 2\theta + 5 \tag{1}$$

Enclose a plot of $\hat{\theta}$ as a function of time for at least 3 different initial conditions $\hat{\theta}(0)$ in your report.

The implemented peak-seeking algorithm can be seen in figure 2.

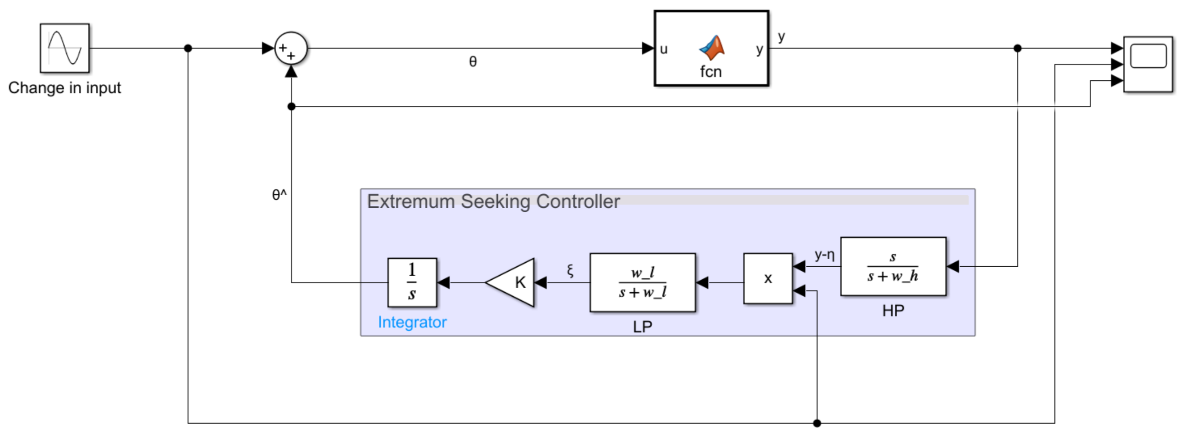


Figure 2: Peak-seeking algorithm implemented in MatLab and Simulink

Since equation 1 doesn't have any dynamics, we can somewhat freely choose $a, \omega, K, \theta_0, \omega_l$ and ω_h. One still has to ensure that these parameters produce a feasible trajectory. The parameters used can be found in the script below

```
%% SIM param
dt = 0.01;   % Sample time

%% Parameters
theta_0 = 2;% Initial condition -1 0 2

w_h = 1;     % High Pass cutoff frequenzy
w_l = 10;    % Low Pass cutoff frequenzy
K = 25;      % Gain

a = 0.1;     % Sine wave amplitude
w = 5;       % Sine wave frequenzy

%% Run simulink
sim("Problem_2_Simulink.slx");
```

The plots of three different initial conditions can be seen in figures 3, 4 and 5.

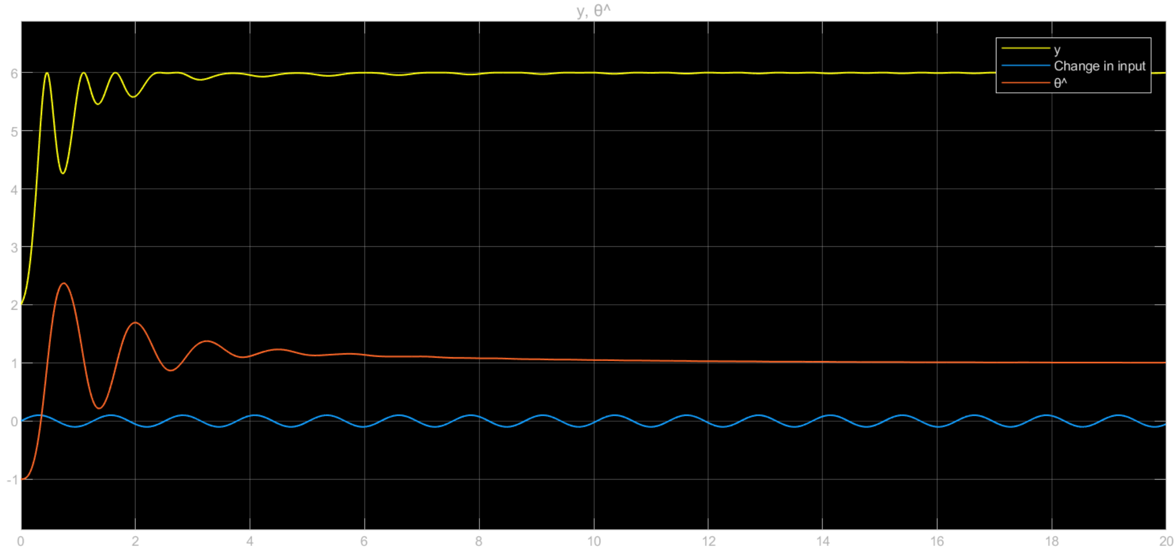


Figure 3: $\theta_0 = -1$

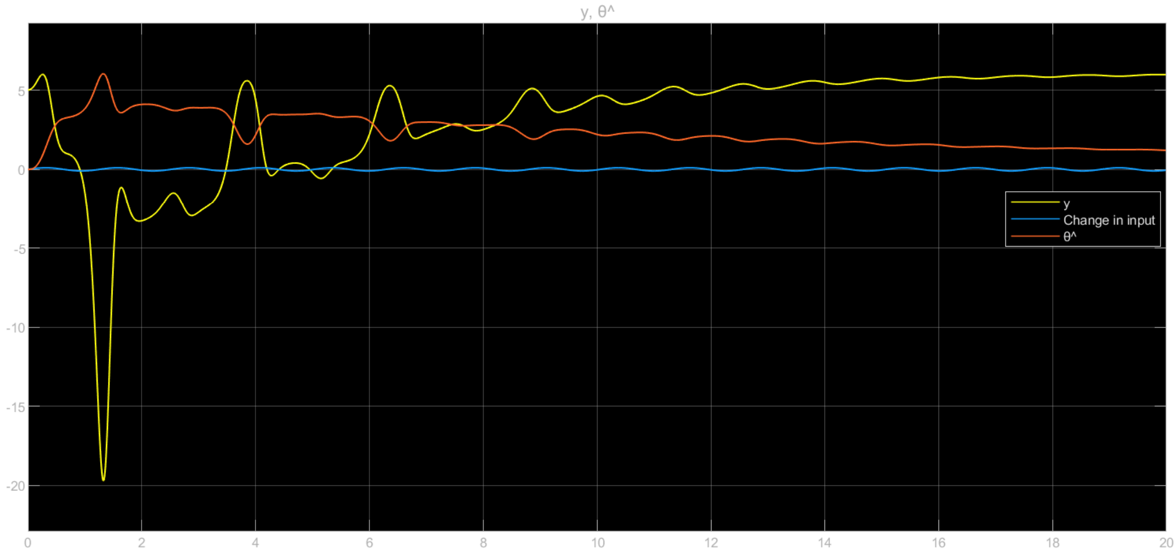


Figure 4: $\theta_0 = 0$

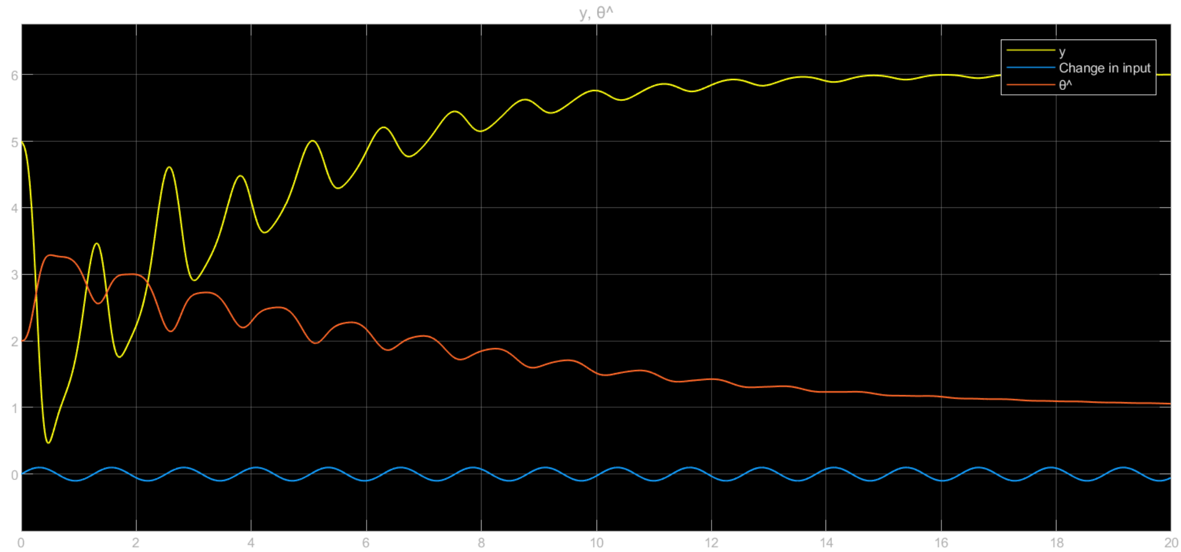


Figure 5: $\theta_0 = 2$

Problem 3

Consider the plant

$$y_s = \frac{k_c \omega_s^2}{s^2 + 2\xi_s \omega_s s + \omega_s^2} r_s \tag{2}$$

where $k_s = 2$ is the steady-state gain, $\omega_s = 10$ is the natural frequency, $\xi_s = 0.1$ is the damping ratio, and r_s, y_s are the reference input and plant output, respectively. The plant performance is a function of time due to the presence of a periodic disturbance, and the plant output, and is given by

$$y = f(t, y_s) = -(0.5 - 0.1 sin(0.01t) - y_s)^2 \tag{3}$$

It is desirable to maximize the plant performance (y) using extremum seeking. Suggest reasonable cut-off frequencies for the low-pass filter and high-pass filter of the extremum seeking algorithm, along with an appropriate frequency for the perturbation signal. Justify your choices. Implement the plant and extremum seeking algorithm in MATLAB and simulate. Include appropriate figures in your report and discuss them.

[2] says that:

> As it will become apparent later, a also needs to be small.
>
> From (8) and (9) we see that the cut-off frequencies of the filters need to be lower than the frequency of the perturbation signal. In addition, the adaptation gain K needs to be small. Thus, the overall feedback system has three time scales:
>
> - fastest : the plant with the stabilizing controller
> - medium : the periodic perturbation

- slow : the filters in the peak seeking scheme

Thus a, ω, K, ω_l and ω_h depends on ω_s. Since the fastest time scale is $\omega_s = 10$, the periodic perturbation (ω) has to be lower than 10, and ω_l and ω_h has to be lower than ω.

The parameters in Table 1 where the ones I ended up with through trial and error.

Table 1: Parameters

Parameter	Value
ω	3
a	0.1
ω_l	1
ω_h	0.1
K	10

The trajectory with these parameters can be found in figure 6.

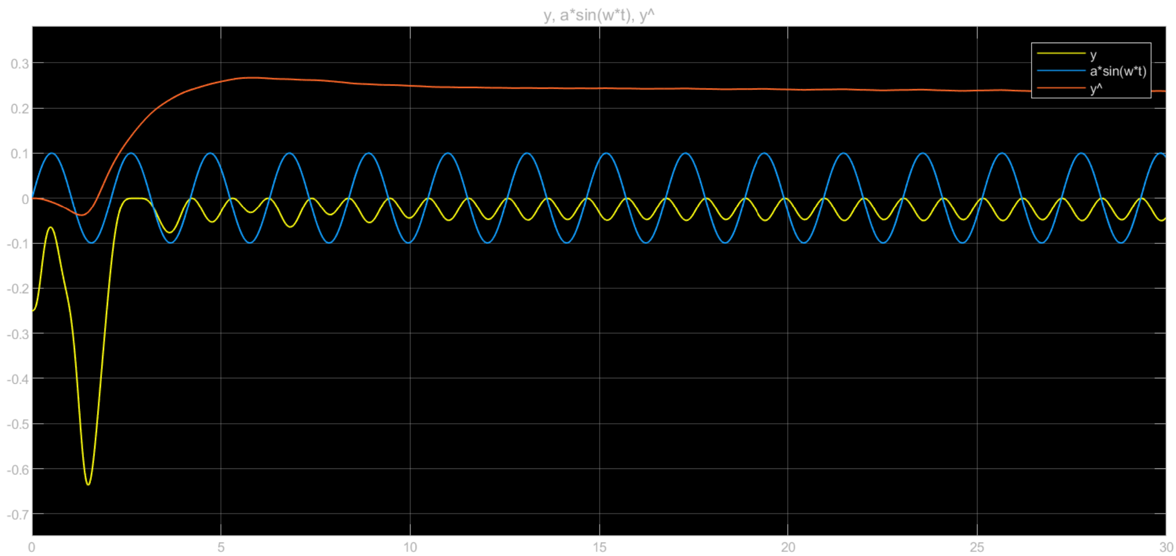


Figure 6: Trajectories when following [2]

In the lecture notes it's however stated that $\omega_h \ll \omega \ll \omega_l$. So changing the filter frequencies as shown in Table 2 resulted in the trajectory in Figure 7

There is a difference between the different setups, perhaps most clearly in $\hat{y}$. When following the paper's intructions, it converges much smoother towards 0.25 compared to the instructions of the lecture notes.

Table 2: Parameters

Parameter	Value
ω	3
a	0.1
ω_l	5
ω_h	1
K	10

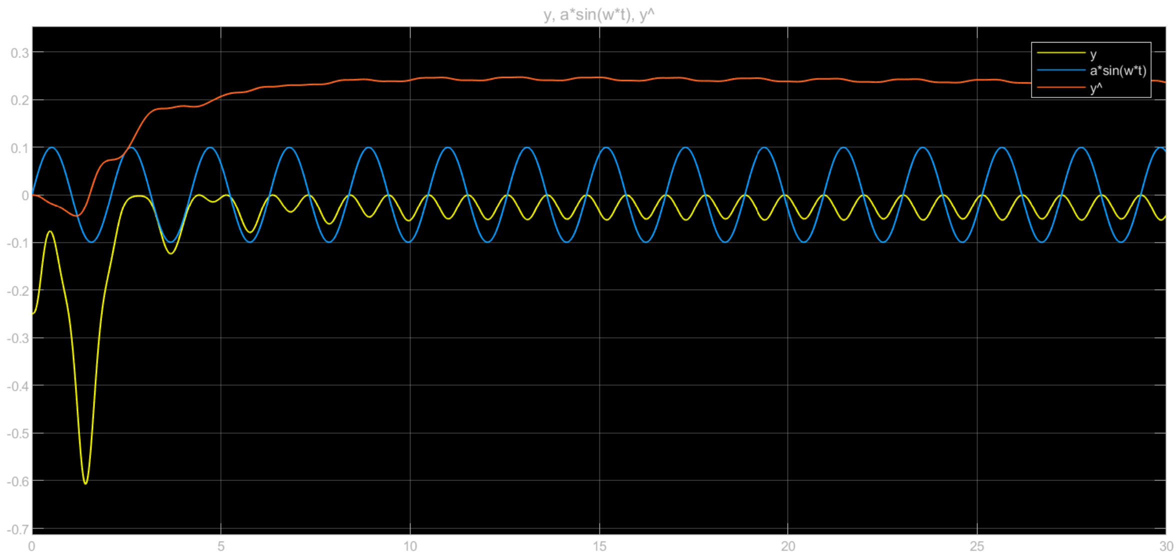


Figure 7: Trajectories when following the lecture notes

Problem 4

Suppose that $V : [0, \infty) \to R$ is non-increasing and that $V(t) \geq 0$ for all $t \geq 0$. Show that $\lim_{t\to\infty} V(t)$ exists and is finite.

Since V is non-increasing, it means that the value of V is at its maximum at $V(t = 0)$. Therefore its upper bounded by $V(t = 0)$. Its also from the definition lower bounded by zero. This means that the value of $V(t = \infty)$ has to lie somewhere between $V(t = 0)$ and 0.

This is illustrated in Figure 8

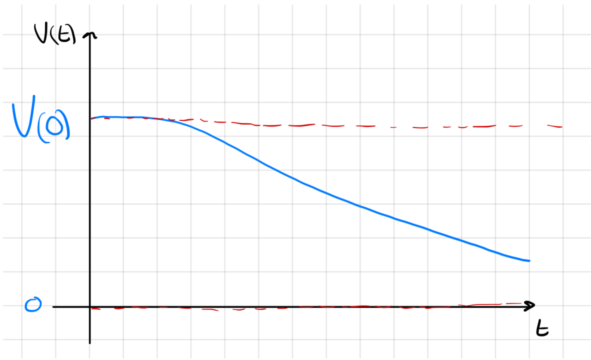


Figure 8: V(t)

References

[1] P. Ioannou and B. Fidan. *Adaptive control tutorial*. en. Advances in Design and Control. Society for Industrial and Applied Mathematics, 2006. ISBN: 99780898716153. DOI: `https://doi.org/10.1137/1.9780898718652`.

[2] Miroslav Krsti and Hsin-Hsiung Wang. “Stability of extremum seeking feedback for general nonlinear dynamic systems”. In: *Automatica* 36.4 (Apr. 2000), pp. 595–601. DOI: `10.1016/s0005-1098(99)00183-1`.