Assignment 10

TTK4215

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Contents

Problem 9 Chapter 7

Consider the second-plant

$$y_p = \frac{b_1 z + b_0}{z^2 + a_1 z + a_0} u_p \tag{1}$$

where a_0, a_1, b_0, b_1 are unknown constants. The reference model is given by

$$y_m = \frac{1}{z + 0.1} r. \tag{2}$$

The control objective is to guarantee close-loop stability and force y_p to track y_m.

a)

What assumptions will you make in order to achieve the control objective?

In order to meet the MRC objective, the following assumptions on the plant and reference model must be satisfied:

Plant assumtions:

- P1. $Z_p(z)$ is a monic Hurwitz polynomial.
- P2. An upper bound n of the degree n_p of $R_p(z)$ is known.
- P3. The relative degree $n^* = n_p - m_p$ of $G_p(z)$ is known, where m_p is the degree of $Z_p(z)$.
- P4. The sign of the high-frequency gain k_p is known.

Reference model assumtions:

- M1. $Z_m(z)$, $R_m(z)$ are monic Hurwitz polynomials of degree q_m, p_m, respectively, where $p_m \leq n$.
- M2. The relative degree $n_m^* = p_m - q_m$ of $W_m(z)$ is the same as that of $G_p(z)$, that is, $n_m^* = n^*$.

b)

Assume that a_0, a_1, b_0, b_1 are known. Design an MRC law to achieve the control objective.

Using the same control law as last time

$$u_p = \theta_1^{*T} \frac{\alpha(z)}{\Lambda(z)} u_p + \theta_2^{*T} \frac{\alpha(z)}{\Lambda(z)} y_p + \theta_3^* y_p + c_0^* r \tag{3}$$

where

$$\alpha(z) = \begin{cases} [z^{n-2}, z^{n-3}, \ldots, z, 1]^T & \text{if } n \geq 2 \\ 0 & \text{if } n = 1 \end{cases}$$
$$\Lambda(z) = \Lambda_0(z) Z_m(z)$$

where $\Lambda_0(z)$ is a monic Hurwitz polynomial of degree $n_0 \triangleq n - 1 - q_m$ and c_0^*, $\theta_3^* \in \mathbb{R}$; θ_1^*, $\theta_2^* \in \mathbb{R}^{n-1}$ are controller parameters to be designed.

Equation 1 can be written as

$$y_p = b_1 \frac{(z + \frac{b_0}{b_1})}{z^2 + a_1 s + a_0} u_p, \tag{4}$$

then

- $k_p = b_1$
- $Z_p = z + \frac{b_0}{b_1}$
- $deg(Z_p) = m_p = 1$
- $R_p = z^2 + a_1 z + a_0$
- $deg(R_p) = n_p = 2$
- $n^* = n_p - m_p = 1$

Equation 2 can be written as

$$y_m = 1 \frac{1}{z + 0.1} r, \tag{5}$$

then

- $k_m = 1$
- $Z_m = 1$
- $deg(Z_m) = q_m = 0$
- $R_m = z + 0.1$
- $deg(R_m) = p_m = 1$
- $n_m^* = p_m - q_m = 1$

Then $\Lambda(z) = \Lambda_0(z)Z_m(z) = \Lambda_0 = z + \lambda_0$ and $\alpha(z) = 1$, since $n = 2$. c_0^* becomes $\frac{1}{b_1}$ which makes the matching equation

$$\left(\Lambda - \theta_1^{*T}\alpha\right) R_p - k_p Z_p \left(\theta_2^{*T}\alpha + \theta_3^*\Lambda\right) = Z_p\Lambda_0 R_m$$
$$\downarrow$$
$$\theta_1^{*T}\alpha R_p + k_p \left(\theta_2^{*T}\alpha + \theta_3^*\Lambda\right) Z_p = \Lambda R_p - Z_p\Lambda_0 R_m$$
$$\downarrow$$
$$\theta_1^{*T}\alpha R_p + k_p\theta_2^{*T}\alpha Z_p + k_p\theta_3^*\Lambda Z_p = \Lambda R_p - Z_p\Lambda_0 R_m$$
$$\downarrow$$
$$S\bar{\theta}^* = p$$

Inserting our values gives:

$$\theta_1^{*T}(z^2+a_1 s+a_0)+b_1\theta_2^{*T}(z+\frac{b_0}{b_1})+b_1\theta_3^*(z+\lambda_0)(z+\frac{b_0}{b_1}) = (z+\lambda_0)(z^2+a_1 s+a_0)-(z+\frac{b_0}{b_1})(z+\lambda_0)(z+0.1)$$

Setting the coefficients in front of z^2, z^1, z^0 equal on each side of the equation and rearranging with the θ^*'z on the left hand side, we obtain

$$\boldsymbol{z^2} : \theta_1^{*T} + \theta_3^{*T} b_1 = a_1 - 0.1 - \frac{b_0}{b_1}$$
$$\boldsymbol{z^1} : \theta_1^* a_1 + \theta_2^* b_1 + \theta_3^* b_1\lambda_0 + \theta_3^* b_0 = a_0 + a_1\lambda_0 - 0.1\lambda_0 - 0.1\frac{b_0}{b_1} - \frac{b_0}{b_1}\lambda_0$$
$$\boldsymbol{z^0} : \theta_1^* a_0 + \theta_2^* b_0 + \theta_3^*\lambda_0 b_0 = \lambda_0 a_0 - 0.1\frac{b_0}{b_1}\lambda_0$$

which in matrix form becomes

$$z = \begin{bmatrix} 1 & 0 & b_1 & 0 \\ a_1 & b_1 & b_1\lambda_0 + b_0 & 0 \\ a_0 & b_0 & \lambda_0 b_0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \bar{\theta}^* = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ c_0^* \end{bmatrix} \tag{6}$$

$$p = \begin{bmatrix} a_1 - 0.1 - \frac{b_0}{b_1} \\ a_0 + a_1\lambda_0 - 0.1\lambda_0 - 0.1\frac{b_0}{b_1} - \frac{b_0}{b_1}\lambda_0 \\ \lambda_0 a_0 - 0.1\frac{b_0}{b_1}\lambda_0 \\ \frac{1}{b_1} \end{bmatrix} \tag{7}$$

We can set up our control law as

$$u_p = \theta^{*T}\omega$$

, where

$$\omega = \begin{bmatrix} \frac{\alpha(z)}{\Lambda(z)}[u_p] \\ \frac{\alpha(z)}{\Lambda(z)}[y_p] \\ y_p \\ r \end{bmatrix} = \begin{bmatrix} \omega_1 \\ \omega_2 \\ y_p \\ r \end{bmatrix}$$

c)

Design a direct MRAC law when a_0, a_1, b_0, b_1 are unknown.

By filtering both sides of Equation 3 with $W_m(z)$ gives

$$W_m u_p = \theta_1^{*T} W_m(z) \frac{\alpha(z)}{\Lambda(z)} u_p + \theta_2^{*T} W_m(z) \frac{\alpha(z)}{\Lambda(z)} y_p + \theta_3^* W_m(z) y_p + c_0^* W_m(z) r$$

$$\downarrow$$

$$\underbrace{W_m u_p}_{z} = \underbrace{[\theta_1,\, \theta_2,\, \theta_3,\, C_0]}_{\theta} \underbrace{\left[W_m \frac{\alpha(z)}{\Lambda(z)} u_p,\, W_m \frac{\alpha(z)}{\Lambda(z)} y_p,\, W_m y_p,\, y_p\right]^\top}_{\phi^\top}$$

Using for example instantaneous cost, the adaptive law becomes

$$\varepsilon = \frac{z - \theta^{*T} \phi}{1 + \alpha \phi_p^T \phi_p}$$
$$\dot{\theta}_p = \Gamma \varepsilon \phi_p,$$

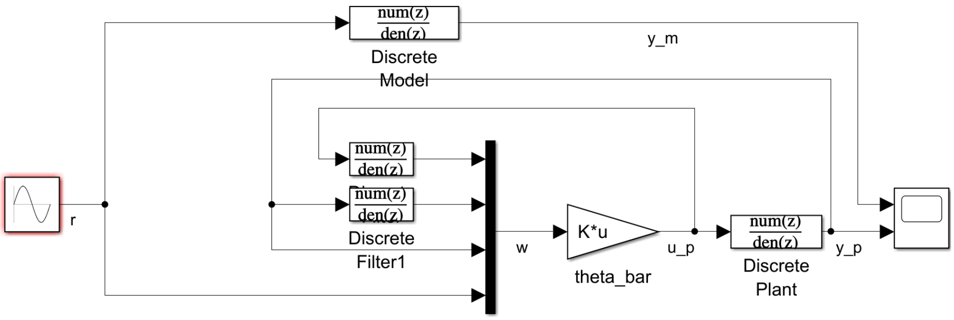
and inserting the estimate of θ into section gives us a MRAC law.

d)

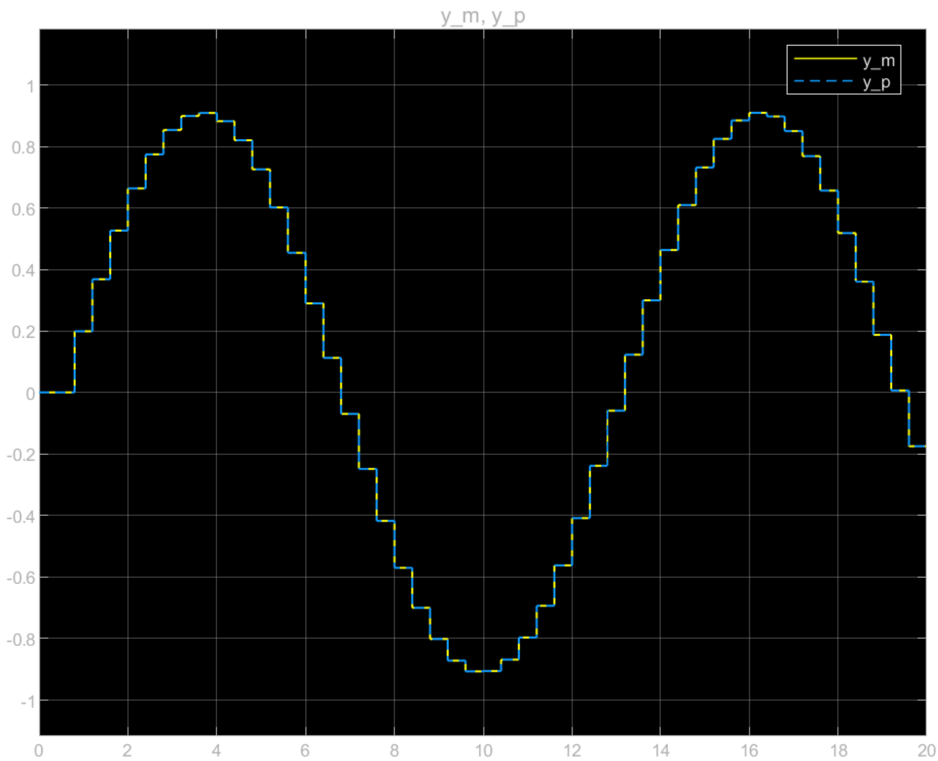
Simulate b) and c) Using simulink to simulate MRC and MRAC as shown below

MRC

The previous version was in continious time, which is incorrect. I hace since modified the simulink model, and it works.



(a) MRC simulink model

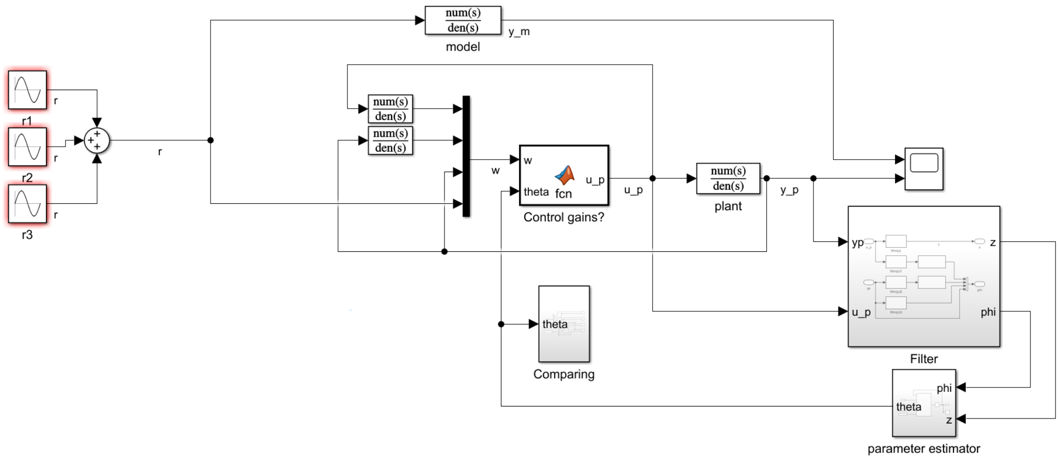


(b) y_p follows y_m

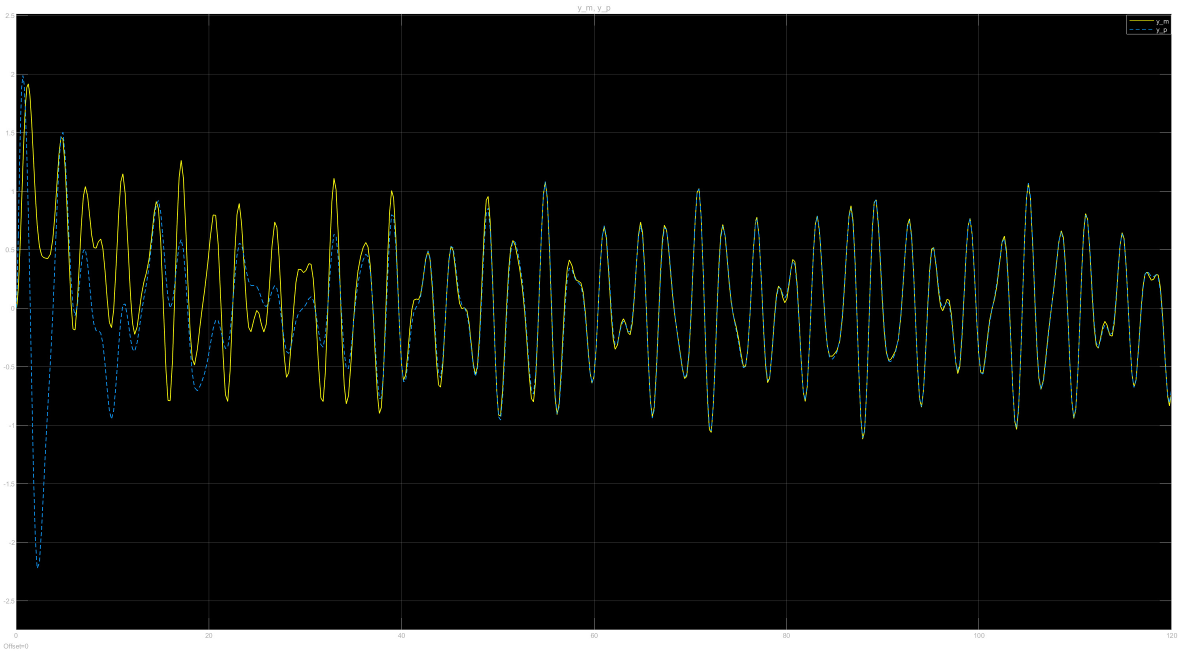
Figure 1: MRC simulation and result

MRAC

The previous version was in continious time, which is incorrect. I hace since modified the simulink model, but now nothing works.



(a) MRAC simulink model



(b) y_p follows y_m after Θ converges

Figure 2: MRAC simulation and result