Assignment 2

TTK4215

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Contents

Problem 1

We have the following definitions:

$$
\begin{aligned}
y(t) &= \theta^* \phi(t) \\
\tilde{\theta}(t) &= \theta(t) - \theta^* \\
\varepsilon(t) &= y(t) - \theta(t)\phi(t) = -\tilde{\theta}(t)\phi(t) \\
\dot{\theta}(t) &= \gamma \varepsilon(t)\phi(t)
\end{aligned}
$$

where $\theta(t)$ is an estimate of θ^*, making $\tilde{\theta}(t)$ the parameter estimation error, and $\varepsilon(t)$ the estimation error. $\dot{\theta}(t)$ is an adaption law.

The Lyapunov function is

$$V = \frac{1}{2\gamma}\tilde{\theta}^2(t) \tag{1}$$

Differentiating (1) gives:

$$
\begin{aligned}
\dot{V} &= \frac{1}{\gamma}\tilde{\theta}(t)\dot{\tilde{\theta}}(t) \\
&= \frac{1}{\gamma}\tilde{\theta}(t)\left(\gamma\varepsilon(t)\phi(t)\right) \\
&= \tilde{\theta}(t)\varepsilon(t)\phi(t) \\
&= -\tilde{\theta}(t)\tilde{\theta}(t)\phi(t)\phi(t) \\
&= -\tilde{\theta}^2(t)\phi^2(t) \\
&= -\varepsilon^2(t)
\end{aligned}
$$

which is negative semi-definite, since $\dot{V}$ can be ≤ 0 even though $\tilde{\theta}$ is not.

1. $\theta \in \mathcal{L}_\infty$
   Since (1) is positive definite, its lower bounded by zero, and negative semi-definite, V is non-increasing. Therefore (1) is bounded and in $\mathcal{L}_\infty$. If $V \in \mathcal{L}_\infty$, then $\tilde{\theta} \in \mathcal{L}_\infty \implies \theta \in \mathcal{L}_\infty$

2. $\varepsilon \in \mathcal{L}_2$
   Since the Lyapunov function is negative semi-definite, Lemma A.4.5 states that y(t) converges to a limit as $t \to \infty$. Hence, $\varepsilon \in \mathcal{L}_2$

3. $\varepsilon \in \mathcal{L}_\infty$, provided $\phi \in \mathcal{L}_\infty$
   Since $\varepsilon = -\tilde{\theta}(t)\phi(t)$, and both $\tilde{\theta}(t), \phi(t) \in \mathcal{L}_\infty$, ε also has to be $\mathcal{L}_\infty$.

4. $\varepsilon \to 0$, provided $\phi, \dot{\phi} \in \mathcal{L}_\infty$
   Lemma A.4.7 states that if $f, \dot{f} \in \mathcal{L}_\infty$ and $f \in \mathcal{L}_p, p \in [1, \infty)$, then $f(t) \to 0$ as $t \to \infty$.

   To prove this is the case for ε, we have to check if $\dot{\varepsilon} \in \mathcal{L}_\infty$.

   $\dot{\varepsilon} = -\dot{\tilde{\theta}}(t)\phi(t) - \tilde{\theta}\dot{\phi}(t)$. $\phi, \dot{\phi} \in \mathcal{L}_\infty$, so we need to check if $-\dot{\tilde{\theta}}(t) \in \mathcal{L}_\infty$. $\dot{\theta}(t) = \gamma\varepsilon(t)\phi(t)$, and both $\varepsilon(t), \phi(t) \in \mathcal{L}_\infty$. Therefore $-\dot{\tilde{\theta}}(t) \in \mathcal{L}_\infty \implies \dot{\varepsilon} \in \mathcal{L}_\infty$.

   Then we have that $\varepsilon, \dot{\varepsilon} \in \mathcal{L}_2 \cap \mathcal{L}_\infty \implies \varepsilon \to 0$ as $t \to \infty$.

Argue why you cannot conclude that $\theta(t) \to \theta^*$. This can be proven by a counterexample:

If $\phi(t) = 0$ and our guess for $\theta(t = 0)$ is bad/wrong, then $\theta(t)$ not $\to \theta^*$, even though $\varepsilon \to 0$.