# TTK16: Project Assignments

Prof. Eduardo Camponogara

The task on "production optimization" is mandatory, whereas the one on "complexity-bounded system identification" is optional.

## 1 Production Optimization of Gas-Lifted Oil Wells

*Floating Production Storage and Offloading* (FPSO) is a large vessel which is deployed to produce oil and gas from deep-water offshore reservoirs. Such vessels have devices and controls systems that enable production from subsea wells, which can be connected directly to the platform (satellite wells) or to a manifold that gathers the production from several wells. In either case a satellite well or manifold is connected to the FPSO by a special tube called *riser*, which is somewhat flexible to cope with waves and deviation from the resting position of the FPSO. Figure 1 illustrates a production system.

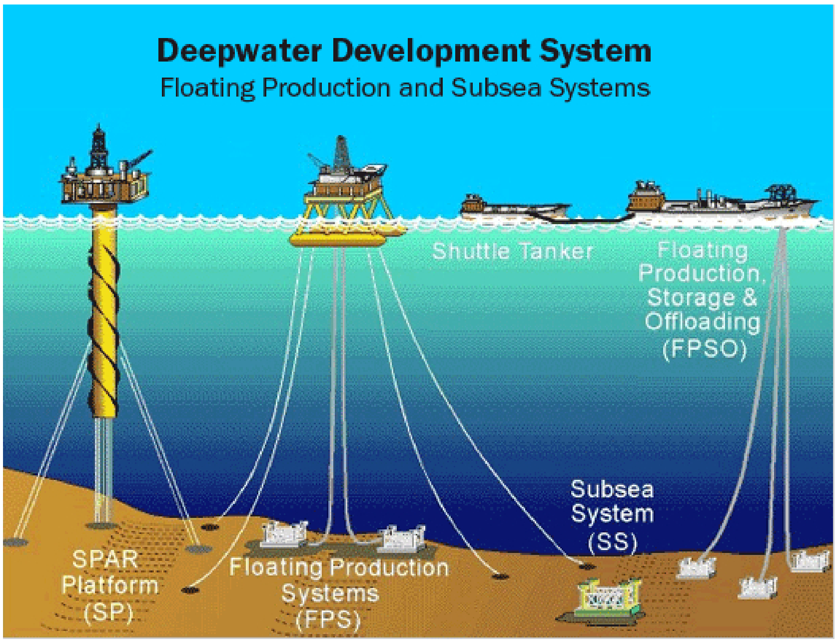


Figura 1: A typical offshore production systems .

When the oil reservoir becomes mature, the internal pressure is not sufficient to lift the fluids up to the surface and artificial lifting techniques are brought into action. The actual technique which is applied depends on a number of economic and technological issues. Among these techniques, gas-lift is a widely applied method for its relatively low installation and operating costs, robustness, and wide range of operating conditions. Gas-lift works by injecting high pressure gas at the bottom of the well, thereby generating a pressure gradient from the well bottom to the surface that promotes oil and gas production. The capacity for producing high pressure gas is limited, mostly by the compression station in the platform, giving rise to the problem of lift-gas allocation which aims to maximize production (or an economic function) while accounting for process constraints.

Once the fluids are received at the platform, they undergo a series of processes. One such process is the separation of the fluids in the three main components, oil, gas, and water. Such processes typically have more than one stage and recirculation mechanisms. The oil is stored in the vessel for later transfer to onshore facilities by special vessels called shuttle tankers. The water is treated before discarded in the sea, or reinjected in the reservoir as a mechanism for pressure support.

The actual problem can vary significantly from one system to another. In this assignment we will consider two baseline problems, one in which the wells operate at a fixed top-side pressure and another in which the pressures are also decision variables.

## 1.1 Optimizing Gas-lift Rate

Here, we consider a set $\mathcal{N} = \{1, \dots, N\}$ of $N$ gas-lifted wells. The oil production of well $n$ is a function $q_{\text{o}}^n(q_{\text{inj}}^n)$ of the lift-gas rate $q_{\text{inj}}^n$ injected into the well. Often this function has complex behavior being implemented by a simulator. The behavior of a well may be one of the following:

- *Surgent:* in this case, $q_{\text{o}}^n(0) > 0$, meaning that the well produces naturally and so the injection of lift-gas is used to boost production.
- *Kickoff-rate:* $q_{\text{o}}^n(q_{\text{inj}}^n) = 0$ for $q_{\text{inj}}^n \in [0, q_{\text{inj}}^{\text{min},n}]$. The well begins to produce only after the lift-gas rate exceeds the lower bound $q_{\text{inj}}^{\text{min},n}$.
- *Immediately flowing:* $q_{\text{o}}^n(q_{\text{inj}}^n) \geq 0$ for $q_{\text{inj}}^n > 0$.

For well $n$, the gas and water phases are obtained from the oil production. Let $\gamma_{\text{w}}^n$ be the Water Cut and $\gamma_g^n$ be the Gas-Oil Ratio (GOR) for well $n$. Then the gas and water phases are obtained as follows:

$$q_{\text{g}}^n = \gamma_{\text{g}}^n \cdot q_{\text{o}}^n(q_{\text{inj}}^n) \tag{1a}$$

$$q_{\text{w}}^n = \frac{\gamma_{\text{w}}^n}{1 - \gamma_{\text{w}}^n} q_{\text{o}}^n(q_{\text{inj}}^n) \tag{1b}$$

A number of process constraints are handled by the optimization. Typically, there is a limit $q_{\text{inj}}^{\max}$ on lift-gas injection, a limit $q_{\text{g}}^{\max}$ on gas processing, and a limit $q_{\text{liq}}^{\max}$ on fluid handling. The decision on whether or not to produce from a well $n$ is a binary variable $y^n$, which assumes the value 1 if well $n$ produces, and otherwise assumes value 0.

As the production function $q_{\text{o}}^n(q_{\text{inj}}^n)$ is complex, and the production phases appear in the objective and constraints, the resulting problem falls in the class of MINLP problems.

The conceptual formulation is given by:

$$P_1: \max \sum_{n\in\mathcal{N}} \widehat{q}_{\mathrm{o}}^n \tag{2a}$$

$$\text{s.t.} \sum_{n\in\mathcal{N}} q_{\mathrm{inj}}^n \leq q_{\mathrm{inj}}^{\max} \tag{2b}$$

$$\sum_{n\in\mathcal{N}} (q_{\mathrm{w}}^n + \widehat{q}_{\mathrm{o}}^n) \leq q_{\mathrm{liq}}^{\max} \tag{2c}$$

$$\sum_{n\in\mathcal{N}} \left(q_{\mathrm{inj}}^n + q_{\mathrm{g}}^n\right) \leq q_{\mathrm{g}}^{\max} \tag{2d}$$

$$\left\{\begin{array}{lcl} q_{\mathrm{g}}^n & = & \gamma_{\mathrm{g}}^n \cdot \widehat{q}_{\mathrm{o}}^n \\ q_{\mathrm{w}}^n & = & \frac{\gamma_{\mathrm{w}}^n}{1-\gamma_{\mathrm{w}}^n} \widehat{q}_{\mathrm{o}}^n \end{array}\right. , n \in \mathcal{N} \tag{2e}$$

$$\left\{\begin{array}{lcl} \widehat{q}_{\mathrm{o}}^n & = & q_{\mathrm{o}}^n(q_{\mathrm{inj}}^n) \\ \widehat{q}_{\mathrm{o}}^n & \leq & q_{\mathrm{o}}^{\max,n} y^n \end{array}\right. , n \in \mathcal{N} \tag{2f}$$

$$q_{\mathrm{inj}}^{\min,n} \cdot y^n \leq q_{\mathrm{inj}}^n \leq q_{\mathrm{inj}}^{\max,n} \cdot y^n,\ n \in \mathcal{N} \tag{2g}$$

$$y^n \in \{0,1\},\ n \in \mathcal{N} \tag{2h}$$

If not mentioned otherwise, all variables are nonnegative.

### 1.2 Tasks

Propose MILP reformulations for the lift-gas optimization problem $P_1$ using the following one-dimensional piecewise-linear models:

- CC
- SOS2

The data will be provided to put together a problem instance, run experiments, test and validation. Your report should provide:

- The piecewise-linear reformulations.
- Results of gas-lift optimization for a varying range of lift-gas availability ($q_{\mathrm{inj}}^{\max}$). You should provide the objective, the lift-gas rates and the operating conditions of the wells.
- Graphs describing the total oil production as a function of lift-gas capacity ($q_{\mathrm{inj}}^{\max}$) should also be provided.

## 2 Linear Least-Squares (Optional)

Now we consider the a system of linear equations $Ax = b$ without a solution, a condition that happens when $rank(A) < rank([A\, b])$. Such systems typically arise in the identification of linear system then the system has infinitely many solutions and a typical problem is the least-squares problem:

$$P_{\mathrm{ls}}: \min_x \ \|Ax - b\|_2^2 \tag{3}$$

for which the analytic solution is $x_{\text{ls}} = (A'A)^{-1}A'b$. In systems identification, vector $x$ will correspond to past control inputs and state measurements that serve as feedback for a moving average model. The least-squares solution $x_{\text{ls}}$ will tend to use all entries and unduly add complexity that can render control more challenging, particularly when the resulting model is used in Model Predictive Control (MPC). Like in the previous problem, we introduce a 0-norm bound on the least-squares solution, leading to the following problem:

$$P_{\text{ls}}(r) : \min_{x} \ \ \|Ax - b\|_2^2 \tag{4a}$$

$$\text{s.t.} : \|x\|_0 \leq r \tag{4b}$$

with $r$ being the maximum number of nonzero entries in $x$.

The complexity-bounded least-squares problem will be illustrated in the identification of dynamic system illustrated in Figure 2. A set of control signals $u(t)$ is input to the system which will result in series of outputs $y(t)$ over the time horizon $t = 0, \ldots, N$. Figure 3 shows a piecewise-constant control signal defined by $u(t)$ and the resulting output signal $y(t)$, $t = 0, 1, \ldots, N$.

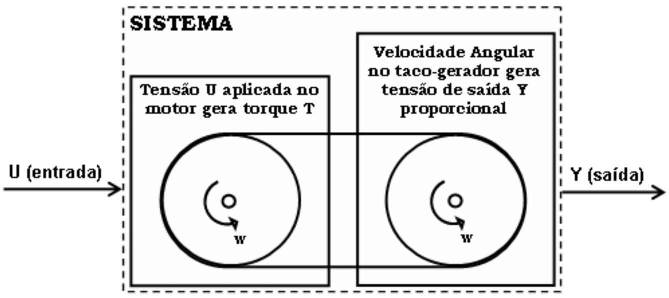


Figura 2: Dynamic system.

A simple and yet effective system model is the *moving average* with $n$ delays, given by:

$$\begin{aligned}\widehat{y}(t) = h_0u(t) + h_1u(t-1) + h_2u(t-2) + \cdots + h_nu(t-n) + \\ + w_1y(t-1) + w_2y(t-2) \cdots + w_ny(t-n)\end{aligned} \tag{5}$$

The prediction $\widehat{y}(t)$ for the output $y(t)$ produced by moving-average model is a function of current input, $u(t)$, the past $n$ inputs, $u(t-1)$, $u(t-2)$, $\ldots$, $u(t-n)$, and the past $n$ outputs, $u(t-1)$, $u(t-2)$, $\ldots$, $u(t-n)$. Thus, the prediction is a linear combination of past inputs and outputs, with $h_i$ and $w_i$ being the parameters that define the linear combination. Ideally, the prediction would be identical to the corresponding output, a condition that is expressed in mathematical notation:

$$y(t) = \widehat{y}(t) = \sum_{i=t}^{t-n} h_iu(i) + \sum_{i=t-1}^{t-n} w_iy(i), \ t = n, n+1, \ldots, N \tag{6}$$

This system of linear equations is conveniently represented as

$$Ax = b \tag{7}$$

in which $x = (h_0, \ldots, h_n, w_1, \ldots, w_n)$, $b = (y(t) : t = n, \ldots, N)$, and $A$ is a suitable matrix.

For the purpose of analysis, a set of $N = 3381$ inputs and outputs were sampled from a physical system of type illustrated in Figure 2, extending for a period of 300 s. The input signals are produced according with a PBRS distribution. The input and output signals are shown in Figure 3.

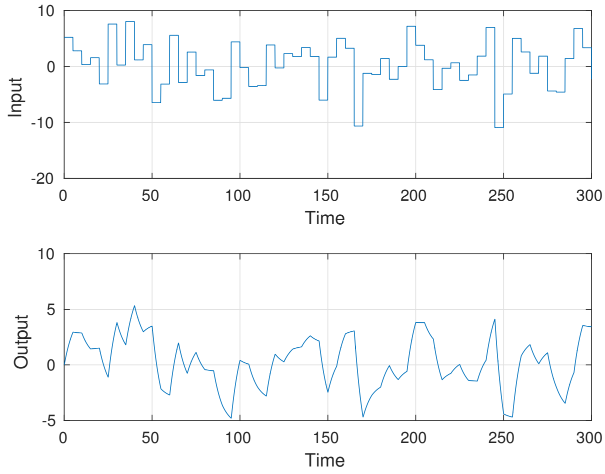


Figura 3: Input and output signals from the dynamic system.

## 2.1 Tasks

1. To consider the trade-off between prediction error and model complexity, consider models with $n = 10$ delayed signals such that the dimension of $x$ is 21.

2. Obtain the matrix $A$ and vector $b$ in order to induce the predictions given by equation (5).

3. Solve a series of problems $P_{\text{ls}}(r)$ for varying $r$. You have to formulate the problem as a Mixed-Integer Quadratic Program (MIQP). Plot the error $\|Ax - b\|_2^2$ as a function of the complexity bound $r$.