NTNU | Norwegian University of Science and Technology

Project + Guest lecture

Eksamen: 14. og 15. desember

2023-12-14 09:00	Fosmo, Pål Ivar Delphin Kværnø
2023-12-14 09:00	Foss, Peder Josef Sjøgreen
2023-12-14 09:00	Rasmussen, Vetle Skifjeld
2023-12-14 09:00	Sivarasa, Sivaranjith
2023-12-14 09:00	Sommerud, Erland Schjatvet
2023-12-14 15:00	Fjalestad, Torleiv Skovgaard
2023-12-14 15:00	Gude, Tore
2023-12-14 15:00	Langklopp, Markus
2023-12-14 15:00	Prytz, Elias
2023-12-14 15:00	Trøen, Jørgen By
2023-12-15 09:00	Haughom, Håvard Sinnes
2023-12-15 09:00	Høksnes, Håkon Skau
2023-12-15 09:00	Pedersen, Aurora Åsgård
2023-12-15 09:00	Stokkeland, Andreas
2023-12-15 09:00	Sveen, Marit Skarderud
2023-12-15 15:00	Hanisch, Halvard Hodne
2023-12-15 15:00	Løver, Sondre

Main exam topics

- Modeling of MILP problems
- Solving MILP problems: Relaxations, B&B, cutting planes (principles)
- PWL modeling (from project)

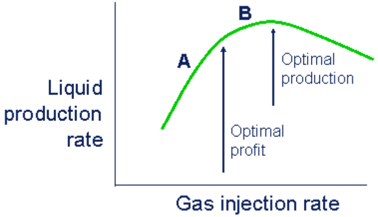
Project work

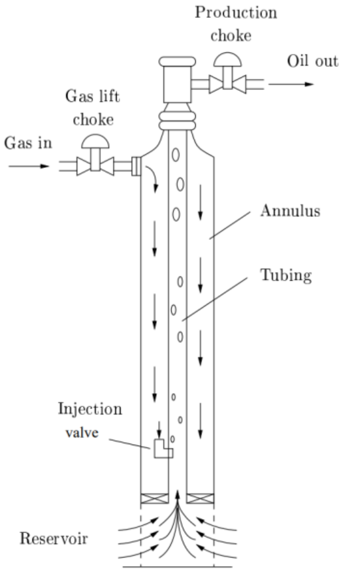
- Groups or individual
- Cooperation OK
- “Deliverable”: Code + Illustration of results

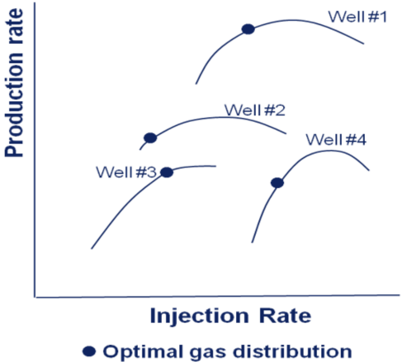
- Deadline: November 24?
  - I will set up meeting to discuss results

Gas lift optimization

- Gas lift is used for increasing production from oil wells with insufficient reservoir pressure
- A single well has typically a 'gas lift performance curve'
- An offshore oil platform has typically many wells, but not enough 'lift gas' for optimal production from each well
  - Constrained optimization problem!
- Challenge: We do not know the gas lift performance curve…
  - But we can try to measure it using test separator





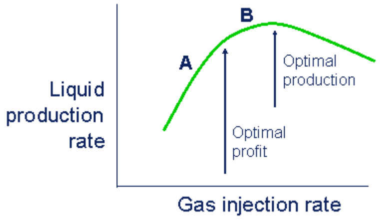


Figures from https://www.emersonautomationexperts.com/2016/industry/oil-gas/maximizing-production-optimized-gas-lifting/

Project assignment

- Use ‘sampled’ gas lift performance curves to solve the production optimization problem using MILP

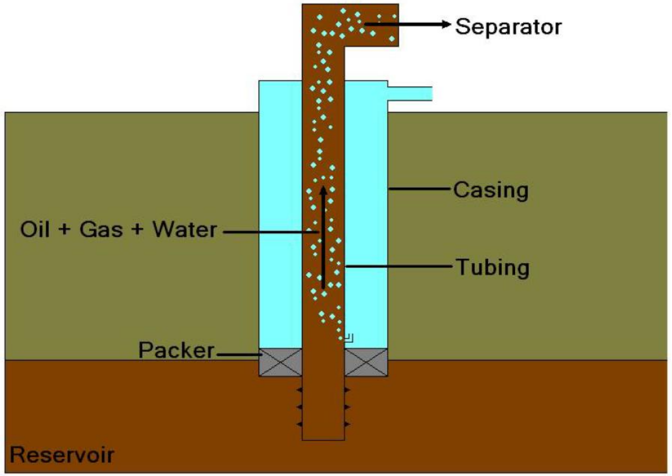
- Use piecewise linear representations to embed the gas lift performance curves in the optimization problem



$$
\begin{aligned}
P_1 : \ \max \ & \sum_{n \in \mathcal{N}} \widehat{q}_{\mathrm{o}}^{n} \\
\text{s.t.} \ & \sum_{n \in \mathcal{N}} q_{\mathrm{inj}}^{n} \leq q_{\mathrm{inj}}^{\max} \\
& \sum_{n \in \mathcal{N}} (q_{\mathrm{w}}^{n} + \widehat{q}_{\mathrm{o}}^{n}) \leq q_{\mathrm{liq}}^{\max} \\
& \sum_{n \in \mathcal{N}} \left(q_{\mathrm{inj}}^{n} + q_{\mathrm{g}}^{n}\right) \leq q_{\mathrm{g}}^{\max} \\
& \left\{ \begin{array}{lcl} q_{\mathrm{g}}^{n} & = & \gamma_{\mathrm{g}}^{n} \cdot \widehat{q}_{\mathrm{o}}^{n} \\ q_{\mathrm{w}}^{n} & = & \frac{\gamma_{\mathrm{w}}^{n}}{1-\gamma_{\mathrm{w}}^{n}} \widehat{q}_{\mathrm{o}}^{n} \end{array} \right. , \ n \in \mathcal{N} \\
& \left\{ \begin{array}{lcl} \widehat{q}_{\mathrm{o}}^{n} & = & q_{\mathrm{o}}^{n}(q_{\mathrm{inj}}^{n}) \\ \widehat{q}_{\mathrm{o}}^{n} & \leq & q_{\mathrm{o}}^{\max,n} y^{n} \end{array} \right. , \ n \in \mathcal{N} \\
& q_{\mathrm{inj}}^{\min,n} \cdot y^{n} \leq q_{\mathrm{inj}}^{n} \leq q_{\mathrm{inj}}^{\max,n} \cdot y^{n}, \ n \in \mathcal{N} \\
& y^{n} \in \{0, 1\}, \ n \in \mathcal{N}
\end{aligned}
$$

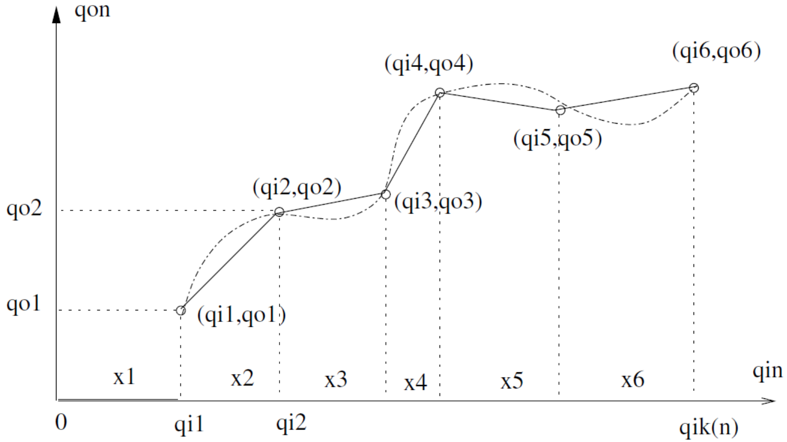
Piecewise-Linear Approximation

Models are obtained from linear (affine) combination of input-output data: $q_{oil}(q_{inj})$.



Piecewise-Linear Approximation

- Models are obtained from linear (affine) combination of input-output data: $q_{oil}(q_{inj})$.
- Data: $\{(q_{inj}^1, q_{oil}^1), (q_{inj}^2, q_{oil}^2), \ldots, (q_{inj}^n, q_{oil}^n)\}$.



Convex Combination (CC)

Givens: $\{(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)\}$.

$$x = \sum_{i=0}^{n} \lambda_i x_i \qquad y = \sum_{i=0}^{n} \lambda_i y_i$$

$$1 = \sum_{i=0}^{n} \lambda_i \qquad \lambda_i \geq 0,\ i = 0, \ldots, n$$

$$1 = \sum_{i=1}^{n} z_i \qquad z_i \in \{0, 1\},\ i = 1, \ldots, n$$

$$\lambda_0 \leq z_1 \qquad \lambda_i \leq z_i + z_{i+1},\ i = 1, \ldots, n-1$$

$$\lambda_n \leq z_n$$

Remark: $z_i = 1$ if $x \in [x_{i-1}, x_i]$.

Piecewise-Linear Model Based on SOS2

Givens: $\{(x_0, y_0), (x_1, y_1), \ldots, (x_n, y_n)\}$.

$$x = \sum_{i=0}^{n} \lambda_i x_i \qquad y = \sum_{i=0}^{n} \lambda_i y_i$$

$$1 = \sum_{i=0}^{n} \lambda_i \qquad \lambda_i \geq 0,\ i = 0, \ldots, n$$

$\{\lambda_i\}_{i=0}^{n}$ is SOS2