Design of a Two Stage Reduction Gearbox

Group D

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Introduction

The design of a gearbox is a process requiring attention to multiple moving parts that act dependently on other components within the design. Because of this, it is vital for mechanical engineers to be familiar with and capable of recognizing how the different parts of a transmission gearbox interact with each other. This project focused on the design and analysis of a two stage reduction gearbox consisting of four gears, an input shaft, an output shaft, and a countershaft. For this design, certain requirements had to be met for the gearbox to be deemed acceptable. These requirements were:

- Input speed, ω_{in} = 1500 rpm
- Output speed, ω_{out} = 68-78 rev/min
- Center distance, Y = 6-9"
- Countershaft length, $L_{CS} \leq 15"$
- Power, P = 14hp
- Pressure angle, $\theta = 20°$

With these requirements, a two stage reduction gearbox was designed to meet these requirements from which schematics could be made in 3D using a CAD software. Additionally, the design was scaled down with a 1:3 ratio for 3D printing and the input/output power for this new design was calculated.

Gear Design

For the design of the gears, the reduction in speed was calculated using the given specifications for the design. The output speed of the gearbox was chosen to be 75rpm for this purpose and the total reduction in speed was calculated using the following equation:

$$\frac{\omega_{in}}{\omega_{out}} = \omega_{eff} \qquad (1)$$

From Equation 1, the total reduction was found to be 20rpm and the square root of this reduction was then taken to find the gear ratio of both gear sets within the gearbox. The gear ratio (GR) was found to be approximately 4.5:1. By using this gear ratio, the size of the pinion and gear in the first stage of the gearbox would be equal to the other pinion and gear for the second stage. To begin the calculations for the diameter of the pinion and gear, the relationship between the diameter and speed of a pinion and gear in the same stage was used to set the diameter of the pinion and gear equal to each other,

$$\frac{d_g}{d_p} = \frac{\omega_p}{\omega_g} = GR$$

$$d_g = GR \cdot d_p \tag{2}$$

To find the diameter of both pinions and gears, a center distance of 8.25” was chosen from the range specified by the design requirements. With both the gear ratio and center distance, Equation 2 can be plugged into the equation to find the center distance to find the diameter of the pinion,

$$Y = \frac{d_p + d_g}{2}$$

$$Y = \frac{GR \cdot d_p + d_p}{2}$$

$$2Y = d_p(GR + 1)$$

$$\frac{2Y}{GR+1} = d_p \tag{3}$$

The pinion and gear diameter could be found using Equation 3 and Equation 2 respectively to give a pinion diameter of 3” and a gear diameter of 13.5”. To finish the design of the pinion and gear, the diametral pitch (P) for both pinion and gear was assumed to be 6. From this, the equation for diametral pitch could be used to find the number of teeth (N) for both the pinion and gear.

$$P = \frac{N}{d}$$

$$P \cdot d = N \tag{4}$$

From Equation 4, the number of teeth for the pinion and gear for both stages of the gearbox were found to be 18 and 81 respectively.

The face width of the gears (b) also had to be calculated for the gears. This was done using an equation that involved the stress on the gear (σ), the safety factor (n), and endurance limit of the gear (S_n).

$$b = \frac{\sigma \cdot n}{S_n} \tag{5}$$

To calculate the face width, the endurance limit of the gears were found first. The material of the gears and pinion had been selected to be AISI 4140 normalized steel with a machine surface finish. The ultimate strength (S_{ut}) and yield strength (S_y) for this material was found to be 148 ksi and 95 ksi. For the endurance limit to be found, certain factors had to be defined,

Table 1. Factors for Gear Endurance Limit

C_L	C_G	C_S	C_T	C_R
1.0	1.0	0.7	1.0	0.814

With these values defined, Moore's endurance limit (S'_n) had to be calculated using the ultimate strength.

$$S'_n = 0.5 \cdot S_{ut} \tag{6}$$

From equation 6, Moore's endurance limit was found to be 74 ksi and the endurance limit of the gears could be calculated.

$$S_n = C_L \cdot C_G \cdot C_S \cdot C_T \cdot C_R \cdot S'_n \tag{7}$$

From equation 7, the endurance limit of the gears was found to be 59.3 ksi. With the endurance limit found, the stress on the gear was calculated next using the formula,

$$\sigma = \frac{F_t \cdot P}{b \cdot J} \cdot K_V \cdot K_O \cdot K_M \tag{8}$$

For this, the tangential force of the gears was calculated as shown in the static analysis section below. The factors for equation 8 were also defined as,

Table 2. Factors for Gear Stress

J	K_O	K_M
0.24	1.25	1.6

The factor K_V varied for each gear stage and was 1.69 for the first stage gears and 1.32 for the second stage gears. With these variables found, Equation 7 and Equation 8 were combined to solve for the face width for each gear. From this calculation, the minimum face width for the gears in the first stage was 0.56” and the minimum face width for the gears in the second stage was 2.06”. At this point, the face width for the gears in the first stage was increased to 1.5” as standards set by the design required a minimum face width of 1.5”.

Once the face width had been calculated for each gear, it would be used to check that the stress of the gear did not exceed the endurance limit of the gear. This was done using a safety factor of 1.0 and the endurance limit,

$$\sigma = \frac{S_n}{n} \tag{9}$$

Static Analysis

With the design of the gear complete, work on designing the countershaft could begin. The length of the countershaft had been decided to be 15” from the requirements of the design. Work on the countershaft began by first calculating the forces acting on the shaft. For this, the torque (T) acting on the input and output shaft were calculated first so the tangential force on each gear stage could be found.

$$T = \frac{5252 \cdot P}{\omega} \tag{10}$$

To find the torque acting on the countershaft using Equation 10, the speed of the countershaft had to be calculated. This was done using a formula relating the speed of the input shaft and the speed of the countershaft with the gear ratio. For this, ω_{cs} was used for the speed of the countershaft.

$$\frac{\omega_{in}}{\omega_{cs}} = GR$$

$$\frac{\omega_{in}}{GR} = \omega_{cs} \tag{11}$$

From equation 11, the speed of the countershaft was calculated to be about 334rpm. Plugging this into Equation 10, the torque on the countershaft was found to be 220 ft*lb and the torque on the input shaft was found to be 49 ft*lb. With these torques, the tangential force acting on the countershaft at the gear and pinion could be calculated.

$$F_t = \frac{T}{r_p} \tag{12}$$

From Equation 12, the tangential force at the first stage was calculated to be 392 lbf and the tangential force at the second stage was 1760 lbf. With the tangential force of each gear found, the radial force could be calculated as well by using the pressure force.

$$F_r = F_t \cdot tan(\theta) \tag{13}$$

From Equation 13, the radial forces for the first stage and second stage gears were found to be 143 lbf and 642 lbf respectively. From this point on, the first stage gear and second stage pinion mounted on the countershaft were referred to as gear 2 (G_2) and gear 3 (G_3) respectively.

With the tangential and radial forces of the gear found, the force of the bearings on the countershaft could be calculated. This was done by taking the countershaft to be in static equilibrium with forces acting on it only from the two gears and two bearings mounted onto the shaft. From here, the moment about one bearing could be taken to find the forces of the other bearing before taking the sum of the forces in a direction to find the forces of the first bearing. From the forces found, the freebody diagram of the countershaft could be drawn as shown in Figure 1 with the values for the diagram in the table below.

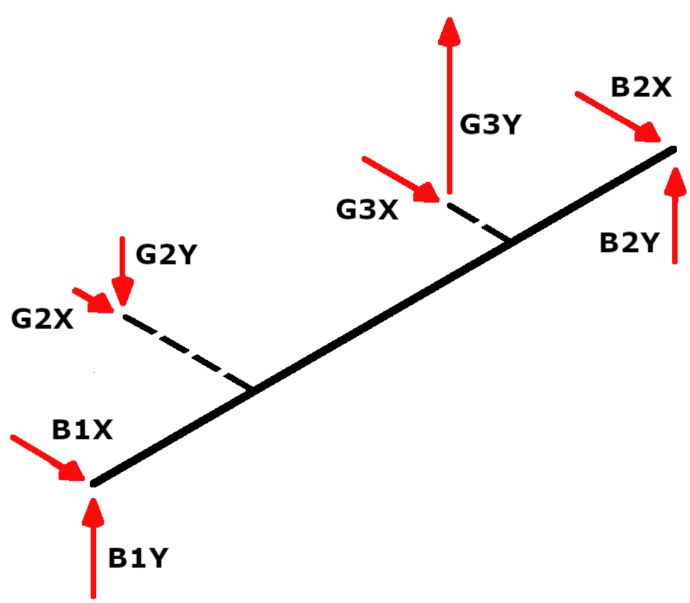


Figure 1. Countershaft Freebody Diagram

Table 3. Forces on Countershaft

B1y (lbf)	**G2y (lbf)**	**G3y (lbf)**	**B2y (lbf)**
111.111	392.158	-1764.710	1261.441
B1x (lbf)	**G2x (lbf)**	**G3x (lbf)**	**B2x (lbf)**
-259.300	142.734	642.302	-525.736

Countershaft Design

With the forces on the countershaft known, the design of the shaft could begin. For this design, AISI 1050 steel with a machined surface finish was chosen as the material for both the input and output shaft as well as the countershaft. The ultimate strength (S_{ut}) of the material was found to be 108.5 ksi [1] and was used to calculate the Moore endurance limit (S'_n).

$$S'_n = 0.5 \cdot S_{ut} \tag{14}$$

From Equation 14, the Moore endurance limit was calculated to be 54.25 ksi. This limit could then be used to estimate the modified endurance limit of the countershaft. For this, multiple factors had to be found. These factors are shown in the table below.

Table 4. Factors for Countershaft Endurance Limit

C_L	C_G	C_S	C_T	C_R
1.0	0.8	0.75	1.0	0.897

With these factors and the Moore endurance limit, the modified endurance limit could be calculated using equation 7. From this equation, the modified endurance limit was calculated to be 29.2 ksi. With the modified endurance limit and ultimate strength of the countershaft found, the moment of gear 2 and gear 3 about the countershaft were calculated to be 987 lb*in and 4783 lb*in.

Before the diameter of the shaft at both gears could be calculated, certain ratios for the countershaft had to be declared. These ratios were the shaft diameter to shoulder diameter (D/d) and the fillet radius to shoulder diameter (r/d) which were 1.25 and 0.03. Additionally, the fillet radius was taken to be 2mm. With these values, the stress concentration factors of the countershaft at the shoulder and keyways could be found.

Table 5. Stress Concentration Factors

	Keyways	Shoulders
K_f^{bend}	2.0	1.891
K_f^{tors}	1.6	1.765

With all of these values, the diameter of the countershaft at gear 2 and gear 3 could be calculated for both infinite life and for static failure. From these two diameters, the larger diameter would be chosen for the design of the countershaft. The minimum diameter of the countershaft at the gears for infinite life was calculated using,

$$d = \sqrt[3]{\frac{16*n}{\pi}\left[\frac{\sqrt{4(K_f^{bend}*M_m)^2+3(K_f^{tors}*\tau_m)^2}}{S_{ut}} + \frac{\sqrt{4(K_f^{bend}*M_a)^2+3(K_f^{tors}*\tau_a)^2}}{S_n}\right]} \quad (15)$$

and the diameter of countershaft at the gears for static failure was calculated using,

$$d = \sqrt[3]{\frac{16*n}{\pi*S_y}\sqrt{4\left(K_f^{bend} * M_{max}\right)^2 + 3\left(K_f^{tors} * \tau_{max}\right)^2}} \quad (16)$$

From Equation 15 and Equation 16, the diameter of the countershaft at both gears for infinite life was calculated to be 1.37” for gear 2 and 2.1” for gear 3. The diameter of the shaft for static failure was calculated to be 0.82” for gear 2 and 1.10” for gear 3. From these values, the diameter of the shaft was chosen to be the largest required diameter for both of the gears which was 2.1”. A static failure check using the final diameter in and stress concentration factors for keyways in Equation 16 produced a safety factor of 13.5 for gear 2 and 5.5 for gear 3.

A different process was used to find the diameter of the shaft at the bearings considering torque, bending moment, and axial load were all zero at the shaft ends. The diameter was determined by considering the diameter of other sections, shoulder ratios, final bearing selection, and ease of scaling for 3D printing.

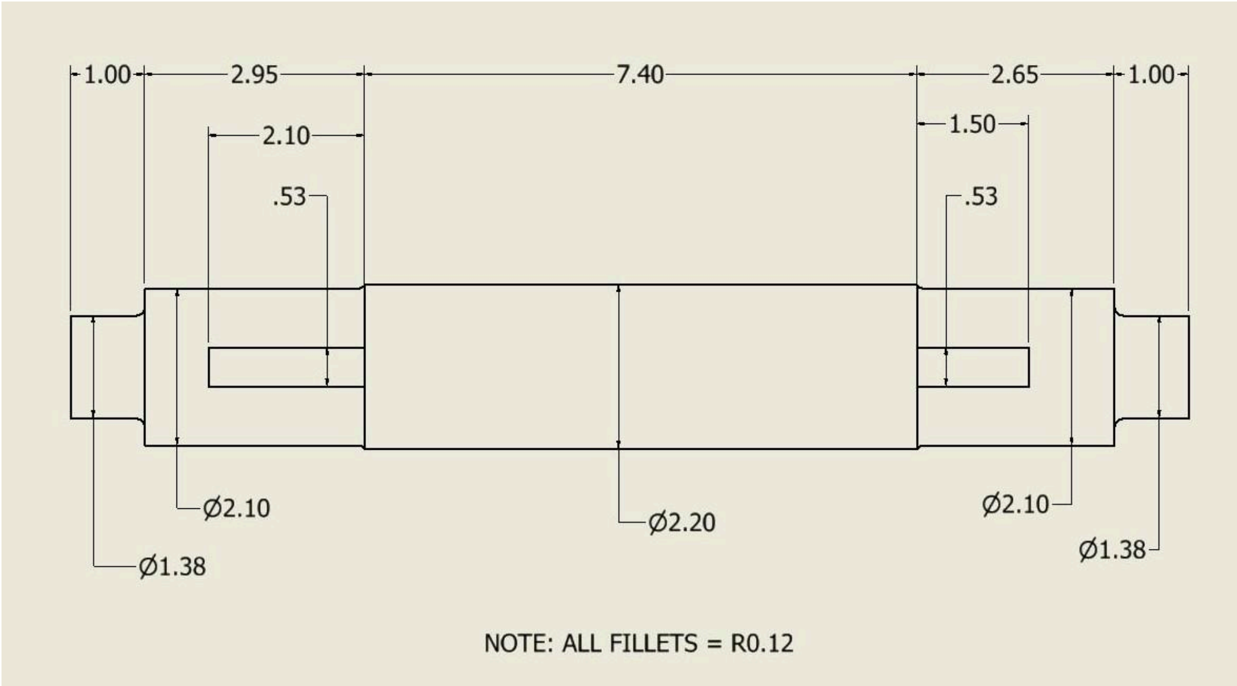


Figure 2. Countershaft With Dimensions

Bearing Selection

With the design of the countershaft complete, the bearings for the gearbox could be chosen. While the countershaft bearings were calculated from reaction forces at B1 and B2, for simplification the input and output shaft would use the same bearings. The calculations to choose the bearing type for the countershaft had some factors that were decided before the process began. They were,

- Bearing life for radial load, L = 4000 hr
- Bearing life for rated capacity, $L_r = 90 \cdot 10^6$
- Reliability factor, K_r = 1.0 (90% reliability)
- Overlord correction factor, K_a = 1.2

Additionally, the magnitude of force at bearings 1 and 2 were calculated from Table 3 to be 282.1 lb and 1366 lb. With these values, the required value for application for the bearing (C_{req}) could be calculated,

$$C_{req} = K_a \cdot F_r \cdot \left(\frac{L}{K_r \cdot L_R}\right)^{3/10} \tag{17}$$

From equation 17, the required values for bearings 1 and 2 were calculated to be 1.4 kN and 6.8 kN. As all bearings would be identical across the entire gearbox, the required value for bearing 2 was chosen to be used for all bearings as it would be able to fulfill the requirements for the other bearings. From these requirements, the type of bearing suitable for the countershaft was chosen to be the type 207 bearing from the 200 series [1]. With the bearing type selected, the diameter of the shaft at the bearing had to be readjusted to fit the bearing with a diameter of 1.378" being the new dimensions for the shaft.

Keyway Design

With the design of the countershaft complete and the bearings selected, the keyway to hold the gears on the countershaft in place had to be designed. To do this, the maximum torque acting on the countershaft at the gears had to be calculated.

$$T = \frac{\pi \cdot d^3}{16}\left(0.58 \cdot S_y\right) \tag{18}$$

The material of the keyway was chosen to be AISI 1020 steel with a yield strength (S_y) of 48 ksi and the diameter of the countershaft at the gear was used. With these values, the maximum torque calculated from Equation 16 was 4218 ft*lb which when compared to the actual torque of the countershaft at the gear proved that the forces were within the limits of the design.

With the torque acting on the countershaft due to the gear found to be within the maximum torque of the shaft, the length of the keyway (L) could be calculated using the torque from the compressive force and from the key shear. For this, a safety factor (n) of 1.6 was used.

$$L = \frac{n \cdot T \cdot 16}{S_y \cdot d^2} \tag{19}$$

$$L = \frac{8 \cdot T \cdot n}{0.5 \cdot S_y \cdot d^2} \tag{20}$$

From both Equations 19 and 20, the length of the keyway was calculated to be 0.31" at the minimum. The height and width of the keyway were both calculated using the same equation.

$$H = W = 0.25 \cdot d \tag{21}$$

From Equation 21, the height and width of the keyway was calculated to be 0.525".

Finite Element Analysis

The loading conditions from the static analysis section were applied at annular regions where components interact with the rest of the assembly. At gear and pinion locations, loads were assigned as calculated in the static analysis. Additionally, bearing locations were also constrained by using a fixed constraint on one side and frictionless constraint on the opposite side.

Fig 3. Loading Conditions

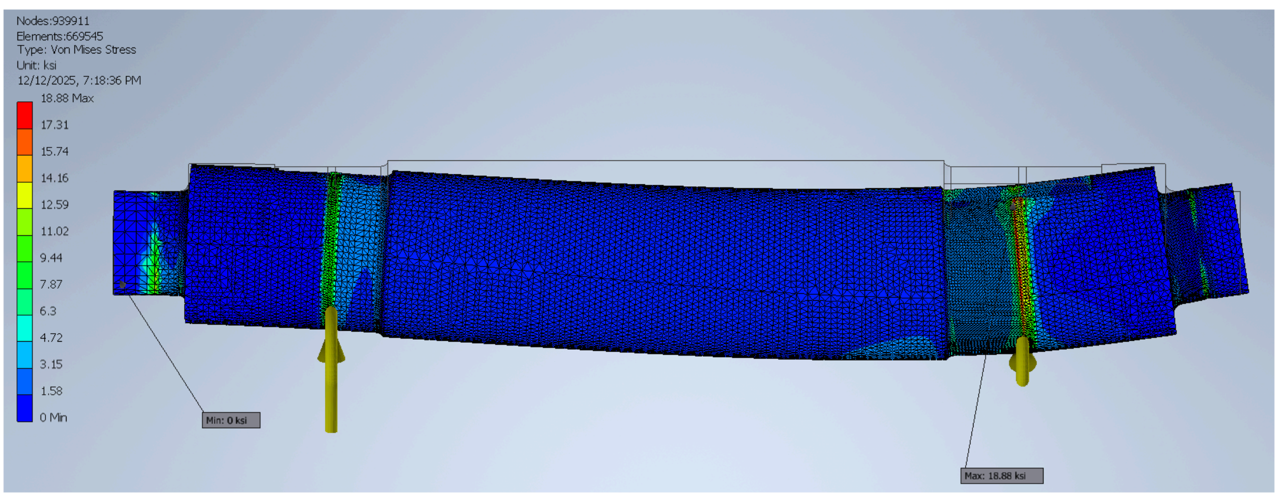
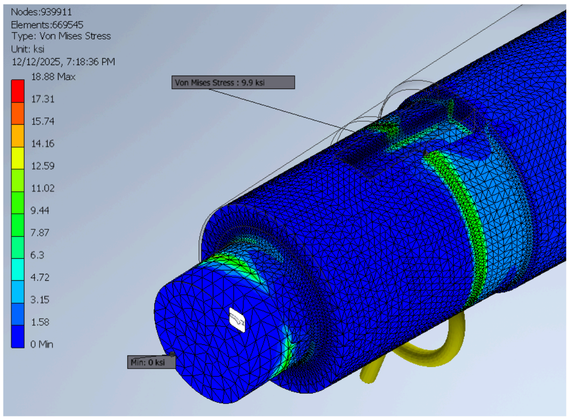


Figure 4. FEA results, x1 deflection adjustment.



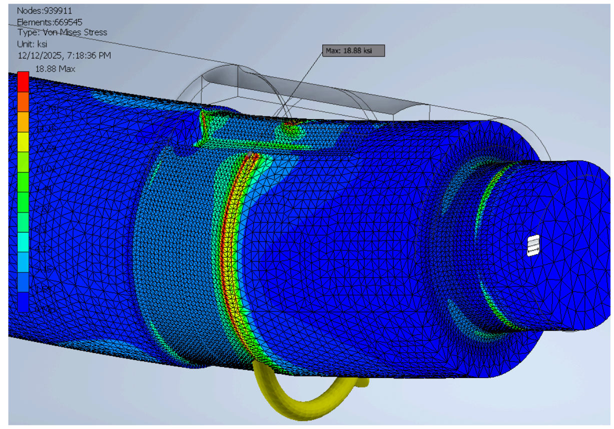


Figure 5. G2 location

Figure 6. G3 location (most critical)

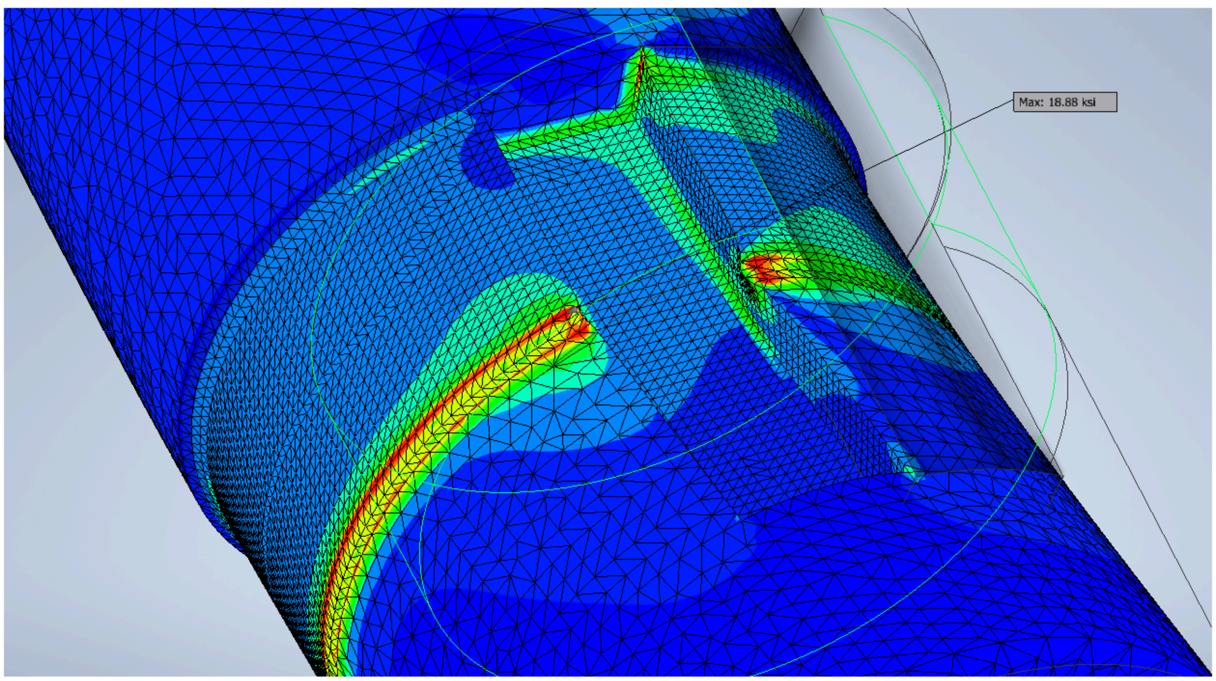


Figure 7. G3 close-up

FEA results showed a maximum local stress of 18.88 ksi at G3 and near the walls of the keyway. This correlates with our static analysis of the countershaft as G3 was expected to be the location under the most stress. This is due to the combined loading on the counter shaft as well as the smaller size of the pinion at that location which would intensify the effect of the forces on the shaft.

Final Analysis

The aim of this project was to design a two stage reduction gearbox to meet certain requirements. This involved the designing of the individual gears, an analysis of the forces acting on the countershaft, the designing of the countershaft, the selection of bearings fit for the design, and the designing of the keyways. The gearbox had to be able to deliver 14 hp with an input speed of 1500 rpm, an output speed of 75 rpm, a center distance of 8.25”, a countershaft length of 15” or less, and a pressure angle of 20 degrees on the gear teeth. To meet these requirements, a gear ratio of 4.5:1 was used for the first and second stage of the gearbox. From this, the diameter of the gears were calculated to be 3” for the pinions and 13.5” for the gears. A diametrical pitch of 6 was assumed for the gears and the number of teeth for the pinion and gears were calculated to be 18 and 81. The face width for the gears in the first stage and second stage were also calculated, the number of teeth being 1.5” for the first stage and 2.06” for the second stage. AISI 1050 steel with a machined finish was selected as the material for the countershaft and the Moore’s endurance limit was calculated for this material. From this calculation, the diameter of the countershaft was found to be 1.378” at the bearings and 2.1” at the gears with a safety factor of 13.5 for gear 2 and 5.5 for gear 3. From this, the bearings for the countershaft were selected to be the type 207 bearings. At each bearing and gear, the countershaft would be stepped such that the shoulders would hold each component in place with keyways modeled to prevent the gears from rotating on the countershaft.

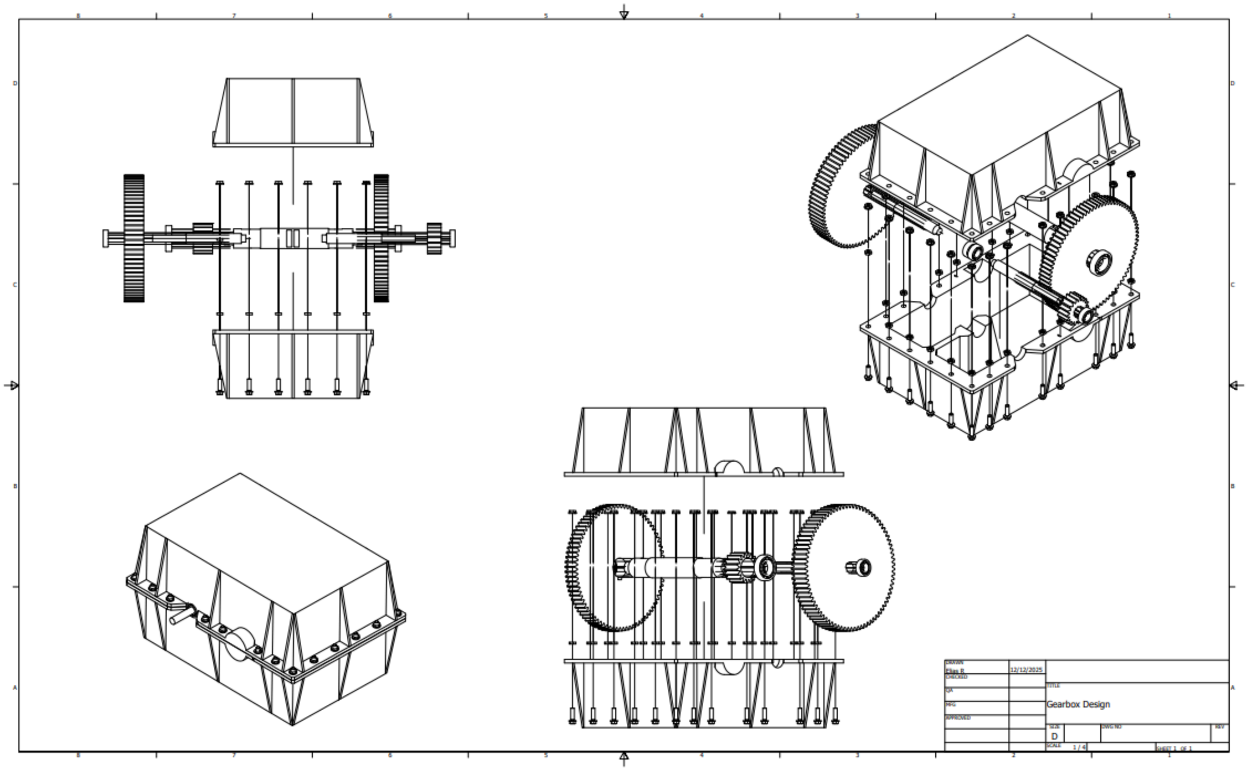
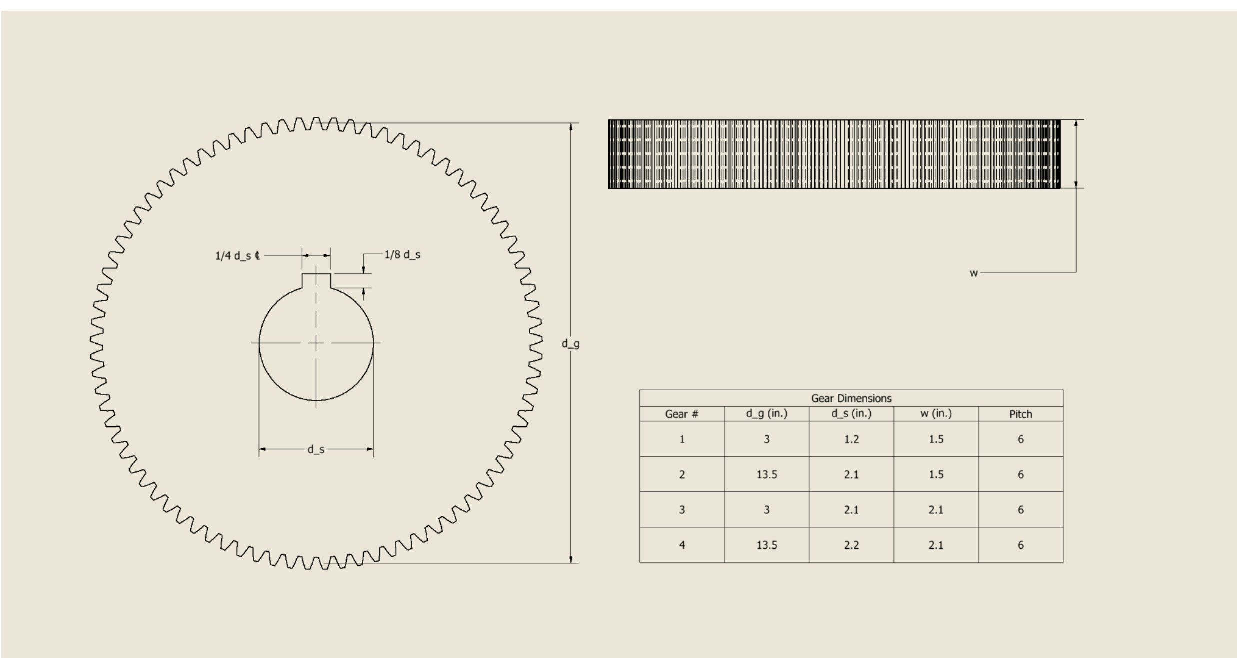


Figure 8. Final assembly with exploded views



Gear #	d_g (in.)	d_s (in.)	w (in.)	Pitch
1	3	1.2	1.5	6
2	13.5	2.1	1.5	6
3	3	2.1	2.1	6
4	13.5	2.2	2.1	6

Figure 9. G2, countershaft gear

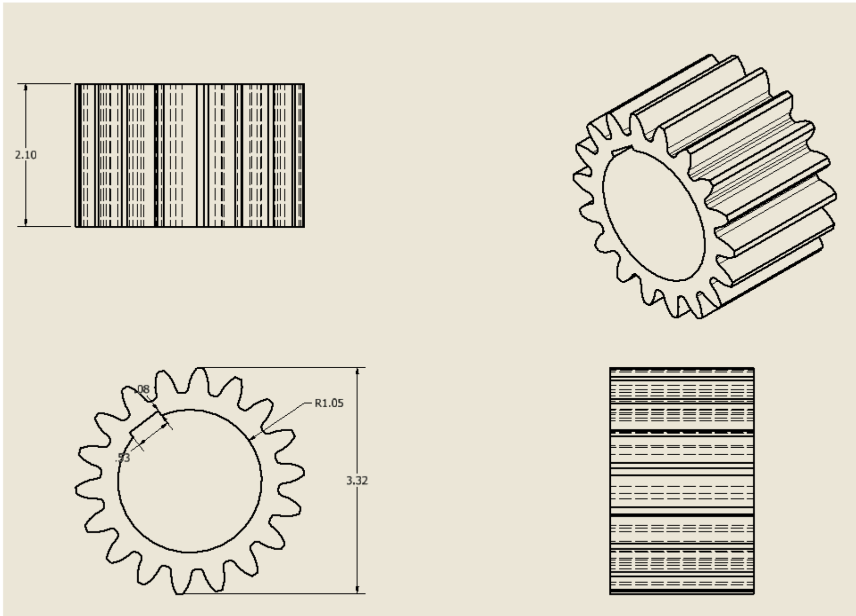


Figure 10. G3, countershaft pinion

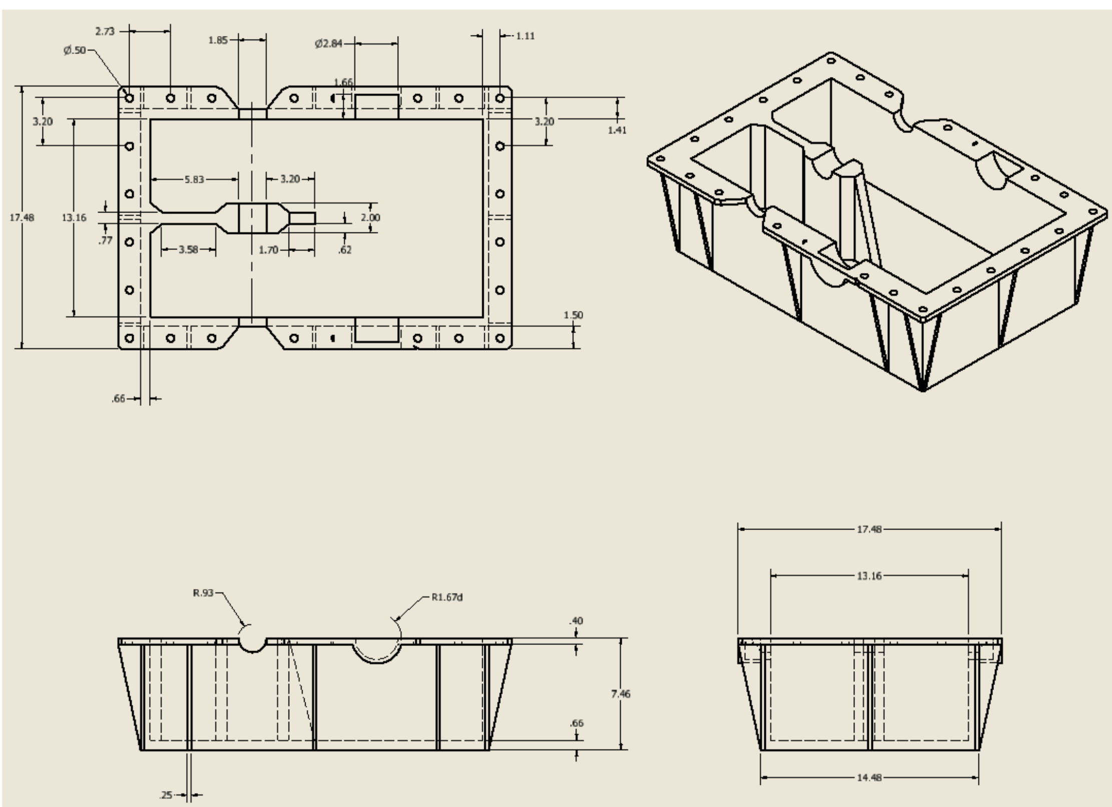


Figure 11. Gearbox body

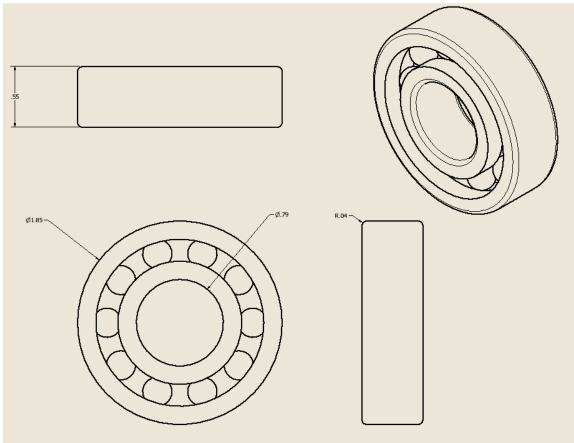


Figure 12. N204 input shaft bearing [2]

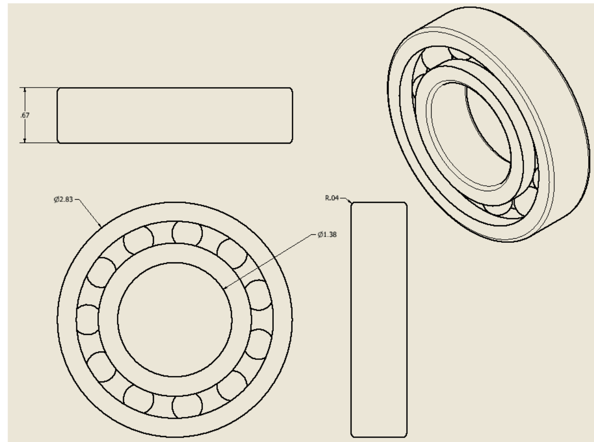


Figure 13. N207 countershaft bearing [3]

3D Printing Calculations:

In preparation for 3D printing the shaft, it is needed to know the maximum power the gearbox can handle with a 1/3rd reduction and change of the material to Polycarbonate. The ultimate tensile strength and yield strength for polycarbonate filament were found to be 69.7 MPa and 59.7 MPa respectively [4]. Converted to Ksi, these values are 9.848 and 8.398. S_n was found using equation 7 with the same modifications as the original shaft and was calculated to be 2.260 Ksi.

Using the modified Goodman at our most critical point as a base with a safety factor of 2.5, we can write o_m and o_a as the following in terms of Torque and Moment.

$$\frac{o_m}{S_{ut}} + \frac{o_a}{S_n} = \frac{1}{2.5}$$

$$o_m = \sqrt{3} \cdot 1.6 \cdot \left(\frac{16}{\pi \cdot 0.17^3}\right) \cdot T_{G3} \tag{22}$$

$$o_a = 2 \cdot \left(\frac{32}{\pi \cdot 0.7^3}\right) \cdot M \tag{23}$$

The moment at the shaft's critical point is related to the force on bearing 2 by the following equation.

$$M = F_{B2} \cdot \frac{3.5}{3} \tag{24}$$

Using the geometry of the shaft, the relationship between the magnitude of the bearing force and the tangential force to Gear 3 is as follows.

$$F_{B2} = 0.774 \cdot F_{G3T} \tag{25}$$

Lastly, torque can be found from tangential force by multiplying by one half times the diameter.

$$\frac{1}{2} \cdot F_{G3T} = T_{G3} \tag{26}$$

Putting everything together, o_a can be rewritten in terms of torque on gear 3 as.

$$o_a = 2 \cdot (\frac{32}{\pi * .7^3}) \cdot \frac{3.5}{3} \cdot 0.774 \cdot T_{G3} \tag{27}$$

For sake of simplification, these will be rewritten into the goodman equation as c_1 and c_2. Together the following equations are created.

$$c_1 = \sqrt{3} \cdot \left(\frac{16}{\pi \cdot 0.7^3}\right) \cdot 1.6$$

$$c_2 = 2 \cdot \left(\frac{32}{\pi \cdot 0.7^3}\right) \cdot \frac{3.5}{3} \cdot 0.774 \cdot 2$$

$$\frac{c_1}{8.398} \cdot T_{G3} + \frac{c_2}{2.65} \cdot T_{G3} = \frac{1}{2.5}$$

$$T_{G3} = \frac{\frac{1}{2.5}}{(\frac{c_1}{9848} + \frac{c_2}{2260})} \tag{28}$$

From Equation 28, given the geometry, torque on gear 3 is found to be 7.746 lb-in.

Using the gear ratio, torque on gear 3 is related to torque on the input shaft and found to be 1.732 lb-in.

$$T_{in} = T_{G3} \cdot \frac{1}{\sqrt{20}}$$

From the input RPM, and input torque, using the following equation, the maximum horsepower is found to be 0.0412 hp for the one-third scale 3D printed gearbox.

$$P = \frac{1500 \cdot T_{in}}{63000}$$

References

[1] Juvinall, R. C., & Marshek, K. M. (2012). *Fundamentals of Machine Component Design* (5th ed.). John Wiley & Sons.

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