

Naive Logical Properties and Structural Properties*

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In their “Two Flavors of Curry’s Paradox”, forthcoming in this journal, JC Beall and Julien Murzi argue against a wide variety of *non-classical* approaches to the *semantic paradoxes*, contending that, while they might block the paradoxes involving a naive property of *truth*, they do not provide the materials for solving a paradox involving a naive property of *validity*. The specific paradox they focus on employs the metarule of *contraction* (which allows the “contraction” of multiple occurrences of the same premise or conclusion into a single one), whence they conclude that a friend of non-classical approaches could solve the paradox by rejecting that metarule. In this note, I offer some paradoxes which involve two other naive logical properties but which do *not* employ contraction. The paradoxes of naive logical properties cannot be adequately solved by simply rejecting contraction. Indeed, as will eventually emerge, they cannot be adequately solved by simply changing the logic.

I. A Paradox of Naive Validity

The paradox on which Beall and Murzi focus is in effect the contrapositive of the so-called ‘*Pseudo-Scotus* paradox’ (see section III for a version of that paradox). Let us assume a standard first-order language with, for every sentence φ , a canonical name $\langle\varphi\rangle$ referring to φ . Let \vdash be a relation of logical consequence and let V be a two-place object-language predicate expressing (single-premise, single-conclusion) \vdash -*validity* (with \vdash including the logic of validity). And, since we are envisaging the possibility that contraction fails, let us finally understand

the collections of sentences flanking \vdash as being *multisets*—that is, collections like sets save for counting the number of times with which an element occurs in them. *Naively*, the following metarules would then seem correct for V :¹

$$\frac{\Gamma_0 \vdash \Delta_0, \varphi \quad \Gamma_1, \psi \vdash \Delta_1}{\Gamma_0, \Gamma_1, V(\langle \varphi \rangle, \langle \psi \rangle) \vdash \Delta_0, \Delta_1} \text{ } V\text{-L}^{\text{IN}} \quad \frac{\varphi \vdash \psi}{\emptyset \vdash V(\langle \varphi \rangle, \langle \psi \rangle)} \text{ } V\text{-R}^{\text{IN}}$$

Unfortunately, intuitive as they are at first glance, $V\text{-L}^{\text{IN}}$ and $V\text{-R}^{\text{IN}}$ would seem to lead to catastrophe. We assume that the language contains a sentence β identical to $V(\langle \beta \rangle, \langle \varphi \rangle)$, where φ is arbitrary. It would then seem that we could reason as follows. By reflexivity, both $\beta \vdash \beta$ and $\varphi \vdash \varphi$ hold, and hence, by $V\text{-L}^{\text{IN}}$, $\beta, V(\langle \beta \rangle, \langle \varphi \rangle) \vdash \varphi$ holds. But, by definition of ‘ β ’, that is tantamount to $\beta, \beta \vdash \varphi$ holding, and hence, by contraction, $\beta \vdash \varphi$ holds, and so, by $V\text{-R}^{\text{IN}}$, $\emptyset \vdash V(\langle \beta \rangle, \langle \varphi \rangle)$ holds. However, by definition of ‘ β ’, $\beta, V(\langle \beta \rangle, \langle \varphi \rangle) \vdash \varphi$ holding is also tantamount to $V(\langle \beta \rangle, \langle \varphi \rangle), V(\langle \beta \rangle, \langle \varphi \rangle) \vdash \varphi$ holding, and hence, by contraction, $V(\langle \beta \rangle, \langle \varphi \rangle) \vdash \varphi$ holds. Thus, both $\emptyset \vdash V(\langle \beta \rangle, \langle \varphi \rangle)$ and $V(\langle \beta \rangle, \langle \varphi \rangle) \vdash \varphi$ hold, and hence, by transitivity, $\emptyset \vdash \varphi$ holds, and so, by monotonicity, \vdash is *trivial* (in the sense of relating any sentence with any sentence). (Let us label this paradox ‘paradox (V)’.)

Like the Pseudo-Scotus paradox, paradox (V) was already well-known to scholastic logicians: E.J. Ashworth mentions discussions of the paradox by Juan de Celaya, Robert Caubraith and Fernando de Enzinas, all of which were active in

¹ \emptyset is the *empty multiset*. As occurring on the *left* of \vdash , one can think of it as the *conjunction of all logical truths*; as occurring on the *right* of \vdash , one can think of it as the *disjunction of all inconsistencies*. Consequently, $\emptyset \vdash \varphi$ means that φ is a *logical truth* and $\varphi \vdash \emptyset$ means that φ is *inconsistent*.

Paris in the early 16th century.² In the scholastic context, paradox (V) and related paradoxes were discussed mainly for their bearing on the traditional definition of validity in terms of, very roughly, *the impossibility of the premise being true and the conclusion being false*. For example, when considering paradox (V), one common position maintained that the paradox shows that it is impossible that β is true, and inferred from this that it is impossible that β is true and φ is false. But that of course spells disaster for the sufficiency direction of the traditional definition, as this licences the implication from the impossibility of β being true and φ being false to the validity of the argument from β to φ (and so to the premise of the argument too). Thus, on this first scholastic position, the traditional definition had to be revised, and this was usually achieved by adding further clauses to it (for example, broadly along the lines of Pseudo-Scotus' treatment of his own paradox, both Caubraith and Enzinas insisted on an extra clause to the effect that the argument should not “invalidate” itself). However, given that the traditional definition plays no role in the specific version of paradox (V) that I have given (and in the specific versions of other paradoxes of naive logical properties that I shall give), the position cannot offer an adequate solution to paradox (V). Another common position maintained that paradox (V) and related paradoxes, rather than manifesting any inadequacy in the traditional definition of validity, belong to the same family of *insolubilia* as the semantic paradoxes, and should thus receive the same treatment as those (for example, Albert of Saxony explicitly applied a Buridan-style solution to the Pseudo-Scotus

² See E.J. Ashworth, *Language and Logic in the Post-Medieval Period* (Dordrecht: Reidel, 1974), p. 125. As far as I know, paradox (V) is thus attested later than the Pseudo-Scotus paradox, which seems to have been formulated before or around the middle of the 14th century (see Henrik Lagerlund, *Modal Syllogistics in the Middle Ages* (Leiden: Brill, 2000), pp. 165–66). This is perhaps not surprising given that, while paradox (V) is for validity what Curry’s paradox is for truth, the Pseudo-Scotus paradox is for validity what the strengthened Liar paradox is for truth.

paradox). Interestingly, paradox (V) has also received some attention by contemporary logicians, and an early reference is explicit in adopting this second scholastic position.³ However, while the position might be tenable when the envisaged approach to the semantic paradoxes is of a type that grants classical logic and takes issues instead with some of the other assumptions at work in the paradoxes (no doubt, the type of approach favoured for instance by both Albert and Read), what we will see is that the position becomes untenable when the envisaged approach to the semantic paradoxes is of a type that grants all the other assumptions at work in the paradoxes and revises instead classical logic.

To start to see this, observe that, as Beall and Murzi correctly point out, paradox (V) does not involve any principle concerning *particular logical operations* like negation or the conditional. It follows that *traditional* non-classical approaches to the semantic paradoxes, which consist in rejecting some such *operational* principle,⁴ cannot but find fault with either $V\text{-L}^{\text{IN}}$ or $V\text{-R}^{\text{IN}}$.

It is not clear to me that Beall and Murzi also manage to make out a persuading case that this is a great cost of such approaches, especially with respect to $V\text{-L}^{\text{IN}}$. For starters, a standard derivability predicate D_{PA} for first-order Peano Arithmetic (PA) fails to validate the analogue of $V\text{-L}^{\text{IN}}$ for it ($D_{\text{PA}}\text{-L}^{\text{IN}}$). For $D_{\text{PA}}\text{-L}^{\text{IN}}$ implies as a special case the *modus-ponens*-like rule

³ Stephen Read, "Self-Reference and Validity," *Synthese*, XLII (1979): 265–74.

⁴ Somewhat roughly, the operational principle rejected is usually either the law of *excluded middle* (concerning the interaction between disjunction and negation), or the law of *non-contradiction* (concerning the interaction between conjunction and negation), or the metarule of *reasoning by cases* (concerning disjunction). For future reference (in section III), let us note that, on the first and third approach, both the Liar sentence and its negation are rejected, while, on the second approach, both the Liar sentence and its negation are accepted.

$\varphi, D_{\text{PA}}(\langle \varphi \rangle, \langle \psi \rangle) \Rightarrow_{\text{PA}} \psi$ (where $\Gamma \Rightarrow_{\text{PA}} \varphi$ means that φ is derivable from Γ in PA and $\Rightarrow_{\text{PA}} \varphi$ means that φ is provable—i.e. derivable on no assumptions—in PA).

That rule has as instance $0 = 0, D_{\text{PA}}(\langle 0 = 0 \rangle, \langle \psi \rangle) \Rightarrow_{\text{PA}} \psi$, which implies

$0 = 0 \Rightarrow_{\text{PA}} D_{\text{PA}}(\langle 0 = 0 \rangle, \langle \psi \rangle) \rightarrow \psi$, which in turn implies $\Rightarrow_{\text{PA}} D_{\text{PA}}(\langle 0 = 0 \rangle, \langle \psi \rangle) \rightarrow \psi$. The last statement contradicts in effect *Löb's Theorem* (assuming the consistency of PA), and hence $D_{\text{PA}}\text{-L}^{\text{IN}}$ has got to fail. Yet, in the usual sense, D_{PA} is an adequate derivability predicate for \Rightarrow_{PA} and few would be tempted to think that \Rightarrow_{PA} does not include an appropriate deductive system for D_{PA} (obviously, \Rightarrow_{PA} does not prove every truth about itself—that is, about D_{PA} —but we know that no interesting standard derivability relation can do that).

Moreover, given that PA is a theory in a standard first-order language, *derivability* in PA actually coincides with *validity* relative to PA. It then becomes utterly unclear why, in view of these facts, one should still expect $V\text{-L}^{\text{IN}}$ to be correct for V .⁵

Another worry about $V\text{-L}^{\text{IN}}$, which also focuses on its implying as a special case the *modus-ponens*-like rule $\varphi, V(\langle \varphi \rangle, \langle \psi \rangle) \vdash \psi$, is that, while instances of that rule in which $V(\langle \varphi \rangle, \langle \psi \rangle)$ is true are unexceptionable, on reflection instances of that rule in which $V(\langle \varphi \rangle, \langle \psi \rangle)$ is false lack a clear and compelling rationale. For, in our context, the natural supporting thought for such instances seems to be, roughly, that \vdash should *conform itself to how premises represent validity to be*.⁶

⁵ Notice that the *modus-ponens*-like metarule from $\Rightarrow_{\text{PA}} \varphi$ and $\Rightarrow_{\text{PA}} D_{\text{PA}}(\langle \varphi \rangle, \langle \psi \rangle)$ to $\Rightarrow_{\text{PA}} \psi$ is still correct for D_{PA} , and that may well be all that intuition really requires from a derivability or validity predicate by way of *modus ponens* (an impression that will be reinforced in the next paragraph).

⁶ In other contexts, the obvious supporting thought (which would not require \vdash to conform itself to how premises represent validity to be) would be that there is an operation (presumably, a conditional) expressed by \rightarrow such that $V(\langle \varphi \rangle, \langle \psi \rangle) \vdash \varphi \rightarrow \psi$ holds and such that *modus ponens* is

But that thought cannot be correct in full generality on pain of making virtually every principle *formally not valid*. For example, the thought in question would lead to regard as not valid the rule from ‘Adjunction is not valid’ and ‘Snow is white’ to ‘Adjunction is not valid and snow is white’, and so to regard adjunction as formally not valid. And the recipe obviously generalises to any rule with at least one premise (including the *modus-ponens*-like rule implied by $V\text{-L}^{\text{IN}!}$).

Moreover, the thought in question cannot be correct in full generality on pain of also making virtually every principle *defeasible*. For example, the thought in question would lead to regard as not valid the rule from ‘The law of excluded middle is not valid’ to ‘Snow is white or snow is not white’ even if the rule from no premises to ‘Snow is white or snow is not white’ is valid, and so to regard monotonicity as not valid *qua* applied to that rule (in a way that makes the rule defeasible). And the recipe obviously generalises to monotonicity *qua* applied to any rule. Thus, even if \vdash is to include the logic of validity, on pain of making virtually every principle both formally not valid and defeasible \vdash cannot always conform itself to how premises represent validity to be—if \vdash is to be something recognisable as a formal, deductive logic, it must be able, as it were, “critically to rise above” the ways in which premises represent validity to be. A more perverse thought that would support the instances of the *modus-ponens*-like rule in which $V(\langle\varphi\rangle, \langle\psi\rangle)$ is false would be that, generally, if $\varphi \vdash \psi$ does not hold, $\emptyset \vdash \neg V(\langle\varphi\rangle, \langle\psi\rangle)$ holds, which, under some attractive if controversial assumptions

valid for \rightarrow (if you catch yourself thinking something like “Suppose that φ holds and $V(\langle\varphi\rangle, \langle\psi\rangle)$ holds. Then, by the latter, surely if φ holds so does ψ , and hence, by the former, ψ holds”, it is the obvious supporting thought that, at bottom, you are thinking). But, given $V\text{-R}^{\text{IN}}$, that there is *any* operation with both these properties is exactly what traditional non-classical approaches to the semantic paradoxes are well known to deny (see for example my “Naive *Modus Ponens*,” *Journal of Philosophical Logic*, XLII (2013): 575–93). Since, in our context, that justificational route is thus blocked, it is hard to see in what way $V(\langle\varphi\rangle, \langle\psi\rangle)$ could still licence the validity of the argument from it and φ to ψ if not by *affecting* \vdash in the way envisaged in the text.

about \neg and \vdash , would imply said instances. For what is worth, I am actually sympathetic to this thought and to the accompanying assumptions, but they clearly have such radical implications (some of which I in fact explore in section III) as to make them unsuitable for an effective defence of $V\text{-L}^{\text{IN}}$ in our context.

Be that as it may, in the remainder of this note I shall mostly assume that Beall and Murzi's assessment is right, so that naive truth—the main goal of non-classical approaches—should come along with naive validity (and, as we will see, with other naive logical properties). What I want to take issue with in the remainder of this note is rather Beall and Murzi's conclusion that a friend of non-classical approaches could solve paradox (V) by rejecting some principle concerning *general structural properties* of \vdash in abstraction from any particular logical operation, in particular the *structural* metarule of contraction:

$$\frac{\Gamma, \varphi, \varphi \vdash \Delta}{\Gamma, \varphi \vdash \Delta} \text{ W-L}$$

$$\frac{\Gamma \vdash \Delta, \varphi, \varphi}{\Gamma \vdash \Delta, \varphi} \text{ W-R}$$

While contraction does undoubtedly play a crucial role in paradox (V) itself, there are other paradoxes of naive logical properties that do not employ contraction.⁷ Indeed, as will eventually emerge, there are other paradoxes of naive logical properties that do not employ any operational or structural principle at all.

⁷ Given only paradox (V), Beall and Murzi's conclusion is of course a natural one. In fact, when I myself first stumbled across paradox (V) some years ago, I took it straightforwardly to favour the non-contractive approach to the semantic paradoxes that I have since developed and defended in my "Truth without Contra(di)ction," *The Review of Symbolic Logic*, IV (2011): 498–535. But deeper reflection, some of whose results I am presenting in this note, has later convinced me that matters here are fascinatingly more complex.

II. A Paradox of Naive Inconsistency

Let I be a one-place object-language predicate expressing \vdash -*inconsistency* (with \vdash including the logic of inconsistency and $\not\vdash$ being the negation of \vdash). *Naively*, the following metarules would seem correct for I :

$$\frac{\varphi \not\vdash \emptyset}{I(\langle \varphi \rangle) \vdash \emptyset} \quad I\text{-L}^{\text{IN}} \qquad \frac{\varphi \vdash \emptyset}{\emptyset \vdash I(\langle \varphi \rangle)} \quad I\text{-R}^{\text{IN}}$$

Unfortunately, intuitive as they are at first glance, $I\text{-L}^{\text{IN}}$ and $I\text{-R}^{\text{IN}}$ would seem to lead to catastrophe. We assume that the language contains a sentence ι identical to $I(\langle \iota \rangle)$. It would then seem that we could reason as follows. Suppose for *reductio* that $\iota \vdash \emptyset$ does not hold. Then, by $I\text{-L}^{\text{IN}}$, $I(\langle \iota \rangle) \vdash \emptyset$ holds. But, by definition of ' ι ', that is tantamount to $\iota \vdash \emptyset$ holding, and hence, by *reductio* (in our metatheoretical reasoning about \vdash), $\iota \vdash \emptyset$ holds. Therefore, by $I\text{-R}^{\text{IN}}$, $\emptyset \vdash I(\langle \iota \rangle)$ holds. But, by definition of ' ι ', that is tantamount to $\emptyset \vdash \iota$ holding. Thus, both $\emptyset \vdash \iota$ and $\iota \vdash \emptyset$ hold, and hence, by transitivity, $\emptyset \vdash \emptyset$ holds, and so, by monotonicity, \vdash is trivial. (Let us label this paradox ' (I_0) '.)

Subject to a qualification to be made in the next paragraph, there do not seem to be interesting differences between paradox (V) and paradox (I_0) that should lead us to treat them differently: they both concern a *natural logical property*; they both appeal to what seems to be an *intuitively correct* pair of left-

and right-metarules connecting facts about the metalinguistic notion expressed by \vdash with facts about the object-language notion expressed by the relevant predicate; they both understand such metarules in a characteristic, paradox-breeding *type-free* fashion; they both make use of *self-referential* devices; they both do not involve any principle concerning particular logical operations, but *only* metarules concerning structural properties and the left- and right-metarules concerning the relevant naive logical property; they both proceed by *first* establishing, by those metarules and by substitutions licenced by self-reference, that the same sentence both is a logical truth and has an unacceptable consequence—either an arbitrary sentence or the empty multiset of conclusions—and *then* conclude, by transitivity and monotonicity, that \vdash is trivial. Given these crucial similarities, we can defeasibly conclude that something is an adequate solution to paradox (V) only if it is an adequate solution to paradox (I_0). Unfortunately, however, paradox (I_0) does nowhere employ contraction, and so paradox (V) (let alone paradox (I_0)) cannot be adequately solved by simply rejecting that metarule.

It should be recorded that there is an interesting difference between paradox (V) and paradox (I_0) consisting in the different character of their left-metarule. $V\text{-L}^{\text{IN}}$ is a standard metarule, in that it is an *implication* from certain rules' *holding* to another rule's *holding*—in other words, $V\text{-L}^{\text{IN}}$ says that, if certain rules are valid, another rule is valid. $I\text{-L}^{\text{IN}}$ is a non-standard metarule, in that it is an *implication* from a certain rule's *failing to hold* to another rule's *holding*—in other words, $I\text{-L}^{\text{IN}}$ says that, if a certain rule is not valid, another rule is valid. $I\text{-L}^{\text{IN}}$ is still an implication, and it is still an implication whose output is

the holding of a rule, but it is an implication whose input is not the holding of a rule, but the failure to hold of a rule.

Is such a feature of $I\text{-L}^{\text{IN}}$ objectionable? It might be if something like $I\text{-L}^{\text{IN}}$ were considered for inclusion as a basic principle in a *deductive system*, since paradigmatic deductive systems (for example, the sequent calculus) are not fit for proving results of underivability that could then be fed as input into $I\text{-L}^{\text{IN}}$ to give it interesting deductive power. But considerations of what would be appropriate to include as a basic principle in a deductive system are quite out of place in our context. For recall from section I that \vdash is a relation of *logical consequence*, and reflect that the evaluation of candidate principles for a relation of logical consequence is subject to different criteria than the evaluation of candidate principles for inclusion as basic principles in a deductive system (even if the relation of derivability generated by the deductive system should eventually coincide with the relation of logical consequence). In particular, all that matters in evaluating candidate metarules for I with respect to \vdash is simply whether they correctly characterise the relevant features of the extension of \vdash , not whether, taken as basic principles, they would naturally interact with other principles so as to yield an adequate deductive system for \vdash . Letting \neg be a negation operator, compare with the non-standard metarule from $\varphi \not\vdash \emptyset$ and $\emptyset \vdash \psi$ to $\psi \not\vdash \neg\varphi$: that metarule is not typically included as a basic principle in adequate deductive systems for, say, classical logic, but it does nevertheless correctly characterise the relevant feature of the extension of the relation of classical logical consequence.

When so viewed, $I\text{-L}^{\text{IN}}$ and $I\text{-R}^{\text{IN}}$ enjoy the same naive compellingness as $V\text{-L}^{\text{IN}}$ and $V\text{-R}^{\text{IN}}$: just like $V\text{-R}^{\text{IN}}$ says that an effect of the metalinguistic fact that the rule from φ to ψ is valid is that the object-language sentence $V(\langle\varphi\rangle, \langle\psi\rangle)$ is a logical truth (is “proved by logic”), so $I\text{-R}^{\text{IN}}$ says that an effect of the metalinguistic fact that φ is inconsistent is that the object-language sentence $I(\langle\varphi\rangle)$ is a logical truth, and, conversely, $I\text{-L}^{\text{IN}}$ says that an effect of the metalinguistic fact that φ is not inconsistent is that the object-language sentence $I(\langle\varphi\rangle)$ is inconsistent (is “refuted by logic”—such principles are all instances of the same general idea that *logic should reflect (positive or negative) logical facts by means of the logical status of object-language sentences concerning such facts* (in fact, in this respect, $I\text{-L}^{\text{IN}}$ might be thought to stand on firmer ground than $V\text{-L}^{\text{IN}}$, as it might be thought not to be subject to a worry analogous to the second worry about $V\text{-L}^{\text{IN}}$ developed in section I). The conclusion of the third last paragraph thus stands undefeated: given the crucial similarities observed, something is an adequate solution to paradox (V) only if it is an adequate solution to paradox (I_0). And since simply rejecting contraction is not an adequate solution to paradox (I_0) (which does nowhere employ that metarule), it is not an adequate solution to paradox (V) either.

III. A Paradox of Naive Compatibility

Essentially the same lesson could have been learnt from the original Pseudo-Scotus paradox, for it too—with a bit more footwork—can be seen not to employ contraction. Let C be a two-place object-language predicate expressing \vdash -

compatibility (with \vdash including the logic of compatibility). *Naively*, the following metarules would seem correct for C :

$$\frac{\varphi \vdash \psi}{C(\langle\varphi\rangle, \langle\neg\psi\rangle) \vdash \emptyset} \quad C\text{-L}^{\text{IN}} \quad \frac{\varphi \not\vdash \psi}{\emptyset \vdash C(\langle\varphi\rangle, \langle\neg\psi\rangle)} \quad C\text{-R}^{\text{IN}}$$

Unfortunately, intuitive as they are at first glance, $C\text{-L}^{\text{IN}}$ and $C\text{-R}^{\text{IN}}$ would seem to lead to catastrophe. We assume that the language contains a sentence κ identical to $C(\langle\varphi\rangle, \langle\neg\kappa\rangle)$, where φ is arbitrary. It would then seem that we could reason as follows. Suppose for *reductio* that $\varphi \vdash \kappa$ does not hold. Then, by $C\text{-R}^{\text{IN}}$, $\emptyset \vdash C(\langle\varphi\rangle, \langle\neg\kappa\rangle)$ holds. But, by definition of ' κ ', that is tantamount to $\emptyset \vdash \kappa$ holding, and hence, by monotonicity, $\varphi \vdash \kappa$ holds, and so, by *reductio* (in our metatheoretical reasoning about \vdash), $\varphi \vdash \kappa$ holds. Therefore, by $C\text{-L}^{\text{IN}}$, $C(\langle\varphi\rangle, \langle\neg\kappa\rangle) \vdash \emptyset$ holds. But, by definition of ' κ ', that is tantamount to $\kappa \vdash \emptyset$ holding. Thus, both $\varphi \vdash \kappa$ and $\kappa \vdash \emptyset$ hold, and hence, by transitivity, $\varphi \vdash \emptyset$ holds, and so, by monotonicity, \vdash is trivial. (Let us label this paradox ‘paradox (C)’.)

Virtually all the same crucial similarities between paradox (V) and paradox (I_0) noted in section II extend *mutatis mutandis* to paradox (C). I say ‘virtually’ because a notable difference between paradox (V) and paradox (I_0) on the one hand and paradox (C) on the other hand is that paradox (C) does partly concern a particular logical operation, *negation*. When the relevant sense of compatibility is properly understood, the requirements placed on negation should however be uncontroversial in our context. For notice that, if compatibility is understood as something like, roughly, *compossibility of truth of*

the first sentence and truth of the second sentence, both $C\text{-L}^{\text{IN}}$ and $C\text{-R}^{\text{IN}}$ do become objectionable at least on certain non-classical approaches to the semantic paradoxes: as for $C\text{-L}^{\text{IN}}$, on non-classical approaches that accept both a Liar sentence λ and its negation (see fn 4), $\varphi \vdash \lambda$ holds even if, so understood, $C(\langle\varphi\rangle, \langle\neg\lambda\rangle)$ is accepted for any consistent φ , and hence presumably even if, so understood, $C(\langle\varphi\rangle, \langle\neg\lambda\rangle) \vdash \emptyset$ does not hold; as for $C\text{-R}^{\text{IN}}$, on non-classical approaches that reject both λ and its negation (see fn 4), $\varphi \vdash \lambda$ does not hold for any consistent φ even if, so understood, $C(\langle\varphi\rangle, \langle\neg\lambda\rangle)$ is rejected, and hence presumably even if, so understood, $\emptyset \vdash C(\langle\varphi\rangle, \langle\neg\lambda\rangle)$ does not hold. If compatibility is however understood rather as something like, roughly, *compossibility of truth of the first sentence and lack of falsity of the second sentence* (and, on the non-classical approaches in question, it *must* be so understood if lack of compatibility is to correlate with validity in the expected ways),⁸ then $C\text{-L}^{\text{IN}}$ and $C\text{-R}^{\text{IN}}$ become unobjectionable even on those non-classical approaches (such understanding will henceforth be presupposed). In fact, I do not know of any non-classical approach having an account of negation that can offer a principled resistance to the naive compellingness of $C\text{-L}^{\text{IN}}$ and $C\text{-R}^{\text{IN}}$.⁹

Notice that, in the same way as $I\text{-L}^{\text{IN}}$, $C\text{-R}^{\text{IN}}$ is also a non-standard metarule, but, for the same reason observed in section II, such feature should not be objectionable in our context. Indeed, considerations analogous to those

⁸ I naturally use ‘lack’ so that ‘lack of a ’ means that the existence of a should be rejected.

⁹ There are of course non-classical logics having an account of negation that does offer such resistance: for example, an *intuitionist*-minded logician would not accept $C\text{-R}^{\text{IN}}$ (for she would think that $\neg\neg\varphi \vdash \varphi$ does not hold, while also thinking that $C(\langle\neg\neg\varphi\rangle, \langle\neg\varphi\rangle)$ is false, and hence presumably that $\emptyset \vdash C(\langle\neg\neg\varphi\rangle, \langle\neg\varphi\rangle)$ does not hold). But it is well known that intuitionist logic is a non-starter as an approach to the semantic paradoxes, and all non-classical approaches I know of in fact retain a fully involutive negation.

developed for $I\text{-L}^{\text{IN}}$ and $I\text{-R}^{\text{IN}}$ in section II indicate that, when properly viewed as principles characterising the relevant features of the extension of \vdash as a relation of logical consequence, $C\text{-L}^{\text{IN}}$ and $C\text{-R}^{\text{IN}}$ enjoy the same naive compellingness as $V\text{-L}^{\text{IN}}$ and $V\text{-R}^{\text{IN}}$. Thus, we can defeasibly conclude that something is an adequate solution to paradox (V) only if it is an adequate solution to paradox (C). Unfortunately, however, paradox (C) does nowhere employ contraction, and so paradox (V) (let alone paradox (C)) cannot be adequately solved by simply rejecting that metarule.

IV. Naive Logical Properties and Structural Properties

In the official derivation of paradox (V) given in section I, the structural properties of *transitivity* and *monotonicity* are employed just as well as contraction is. In fact, in the official derivations of paradoxes (I₀) and (C) given in sections II and III respectively, those properties are employed even if contraction is not. Should we conclude that, while simply rejecting contraction is not an adequate solution to the paradoxes of naive logical properties, simply rejecting either transitivity or monotonicity is?¹⁰

¹⁰ While I do not know of any non-monotonic approach to the semantic paradoxes, a non-transitive approach with features that might have been thought to help in our context has recently been developed by Dave Ripley (see for example his “Conservatively Extending Classical Logic with Transparent Truth,” *The Review of Symbolic Logic*, V (2012): 354–78). Actually, I myself favour a non-transitive approach to another millenary riddle, the *sorites* paradox (although using a logic importantly different from Ripley’s, see for example my “A Model of Tolerance,” *Studia Logica*, XC (2008): 337–68; in fact, Ripley’s logic belongs to the family of logics that was first introduced in that paper). However, for reasons I cannot go into in this note, in the case of the semantic paradoxes I prefer the non-contractive approach mentioned in fn 6. I am going to argue in this section that, current appearances notwithstanding, the paradoxes of naive logical properties give no reason to prefer a non-transitive approach over a non-contractive one.

Not at all. As for monotonicity, notice that already the official derivation of paradox (V) manages to arrive, without monotonicity, at the conclusion that $\emptyset \vdash \varphi$ holds for arbitrary φ (i.e. that φ is a logical truth for arbitrary φ). I take it that that conclusion is already bad enough, so that rejection of monotonicity comes too late in the paradox to be of any help.

As for transitivity, notice that the rule converse to $I\text{-L}^{\text{IN}}$:

$$\frac{I(\langle \varphi \rangle) \vdash \emptyset}{\varphi \not\vdash \emptyset} \quad I\text{-L}^{\text{OUT}}$$

allows for an alternative, transitivity-free derivation of a paradox. With ι as in section II, suppose for *reductio* that $\iota \vdash \emptyset$ does not hold. Then, by $I\text{-L}^{\text{IN}}$, $I(\langle \iota \rangle) \vdash \emptyset$ holds. But, by definition of ' ι ', that is tantamount to $\iota \vdash \emptyset$ holding, and hence, by *reductio* (in our metatheoretical reasoning about \vdash), $\iota \vdash \emptyset$ holds. But, by definition of ' ι ', that is tantamount to $I(\langle \iota \rangle) \vdash \emptyset$ holding, and hence, by $I\text{-L}^{\text{OUT}}$, $\iota \vdash \emptyset$ does not hold. Contradiction (in our metatheoretical reasoning about \vdash).
(Let us label this paradox ‘paradox (I₁)’.)

Having as input $I\text{-L}^{\text{IN}}$ ’s output and as output $I\text{-L}^{\text{IN}}$ ’s input, $I\text{-L}^{\text{OUT}}$ is also a non-standard metarule, but in a way converse to $I\text{-L}^{\text{IN}}$, in that it is an implication from a certain rule’s holding to another rule’s failing to hold. For the same reason observed in section II, such feature should not be objectionable in our context. Indeed, considerations analogous to those developed for $I\text{-L}^{\text{IN}}$ and $I\text{-R}^{\text{IN}}$ in section II indicate that, when properly viewed as a principle characterising the relevant

features of the extension of \vdash as a relation of logical consequence, $I\text{-L}^{\text{OUT}}$ enjoys the same naive compellingness as $V\text{-L}^{\text{IN}}$ and $V\text{-R}^{\text{IN}}$ (in fact, while $I\text{-L}^{\text{IN}}$ can be thought of as a *completeness* principle of I -predications refuted by logic with respect to non-inconsistent sentences, $I\text{-L}^{\text{OUT}}$ can be thought of as the converse, even more compelling *soundness* principle). Thus, we can defeasibly conclude that something is an adequate solution to paradox (V) only if it is an adequate solution to paradox (I_1). Unfortunately, however, paradox (I_1) does nowhere employ transitivity, and so paradox (V) (let alone paradox (I_1)) cannot be adequately solved by simply rejecting that metarule.

Indeed, since paradox (I_1) only relies on the usual kind of self-reference, $I\text{-L}^{\text{IN}}$ and $I\text{-L}^{\text{OUT}}$, it follows that *simply rejecting any other structural (or, for that matter, operational) principle will not constitute an adequate solution* to the paradox. Once the full range of the paradoxes of naive logical properties is brought into view, what thus emerges is that such paradoxes cannot be adequately solved by simply rejecting any structural or operational principle (apart, that is, from the metarules for naive logical properties)—in other words, *they cannot be adequately solved by simply changing the logic.*

Is this the end of the road for a dialectical use of the paradoxes of naive logical properties aimed at arguing in favour of some non-classical approaches to the semantic paradoxes over others? Does this show that *any approach whatsoever to the semantic paradoxes must ultimately reject some naive principles for logical properties*, so that—contrary to what Beall and Murzi argue—it is no objection to a non-classical approach if it fails to validate some

such principle? Maybe so, or maybe, more worryingly but in keeping with the assumption made in section I, that should indicate that, since naive principles for *logical* properties have got to fail, naive principles for *semantic* properties (such as truth) are also cast into grave doubt. However, to mitigate a little such grim prospects, a countervailing thought is worth mentioning at the end of this note.

At least judging from the official derivations I have offered, all of paradoxes (I₀), (C) and (I₁) make use of *distinctively classical* moves in the *metatheoretical reasoning about* \vdash (I have in fact explicitly flagged these moves in each derivation). As things *de facto* stand, that is only fair play, since virtually all non-classical approaches to the semantic paradoxes still adopt what is (in effect) a *classical metatheory* in specifying what exactly their proposed *non-classical logic* is. However, the programme of specifying the relevant non-classical logic adopting instead a *non-classical metatheory* is an option that is sometimes mentioned as a possible avenue of inquiry,¹¹ although hardly ever recommended or pursued in any detail. If this option should be argued to be preferable on *conceptual* grounds, and be shown to be feasible on *technical* ones, then one might want to go back and reconsider the perplexing paradoxes of naive logical properties, to see whether, in such a radically new setting, they can be adequately solved by some non-classical (which would now mean both logically non-classical and *metatheoretically non-classical*) approaches, and,

¹¹ See for example Hartry Field, *Saving Truth from Paradox* (Oxford: Oxford University Press, 2008), p. 391.

consequently, whether they can be taken to favour some non-classical approaches over others.¹²

¹² I make some first steps in this direction, and in particular towards and in favour of a non-contractive logic specified in a non-contractive metatheory, in my “Naive Truth and Naive Logical Properties,” forthcoming in *The Review of Symbolic Logic*.