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CSCI 3302
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Homework 1

1.) Each wheel (4 wheels total) has 4 DOF able to rotate clockwise, counter clockwise & translational move of forward and backwards. For all wheels total this would be 16 degrees of freedom. Including the bar we push on, this would make 20 DOF with movement forward, backwards, up, & down. The control of the bar we're pushing is what allows us to mow an entire lawn in the x-y plane.

2.) For an orientable robot in the x-y plane we must consider translational, & rotational movement. This includes up, down, left, right, & roll pitch yaw.

3.) a.)

we know $\cos\theta = \vec{v}_1 \cdot \vec{v}_2 \xrightarrow{\text{dot product}} x_1y_1 + x_2y_2 + x_3y_3$

$\vec{v}_1 = \{0.966, 0.2588, 0\} - \vec{x}$

$\vec{v}_2 = \{-.2588, .966, 0\} - y$

$= (.966)(-.2588) + (.2588)(.966) + \overset{\phi}{\phi}$

$= \phi$

$\cos\theta = \phi$

$\cos\theta$ is ϕ at $\boxed{\pi/2 \text{ or } 90^\circ}$

b.) The vector in order to be perpendicular to both $\vec{v}_1$ & $\vec{v}_2$ we need cross product

$\vec{v}_1 \times \vec{v}_2 =$

$$\begin{vmatrix} .966 & .2588 & 0 \\ -.2588 & .966 & 0 \end{vmatrix}$$

$\hat{i}\,[\overset{0}{(.2588)(0) - (0)}] - \hat{j}\,[\overset{0}{(.966)(0) - (-.2588)(0)}] + \hat{k}\,[(.966)(.966) - (.2588)(-.2588)]$

$= 0 + 0 + \hat{k} = \boxed{[0, 0, 1]\} \text{ vector } \vec{v}_3 \text{ required}}$

4.) Assuming: $X_A\, Y_A\, Z_A$ & $X_B\, Y_B\, Z_B$

$$ {}^{A}_{B}R = \begin{pmatrix} X_A X_B & Y_A X_B & Z_A X_B \\ X_A Y_B & Y_A Y_B & Z_A Y_B \\ X_A Z_B & Y_A Z_B & Z_A Z_B \end{pmatrix} $$

b.) $X^B = [0,1,0]$ in $\{A\}$ ← Frame

swap A & B

We need to cross & X^B

switched

$$ {}^{B}_{A}R \times \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} = \begin{pmatrix} 0 & X_A & 0 \\ 0 & Y_A & 0 \\ 0 & Z_A & 0 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = [X_A\ Y_A\ Z_A] $$

Same but A & B swapped

c.) $$ {}^{B}_{A}R = \begin{pmatrix} X_B X_A & Y_B X_A & Z_B X_A \\ X_B Y_A & Y_B Y_A & Z_B Y_A \\ X_B Z_A & Y_B Z_A & Z_B Z_A \end{pmatrix} $$

5.) Where Φ is the steering angle,
Translational velocity ; $V = (r)(w)$ X comp : $V_x = V\cos\Phi$ Y comp : $V_y = V\sin\Phi$
Angular velocity ; $w = \left(\frac{V}{L}\right)\sin\Phi$