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Subject: CAAM 420/520 – Homework 4

Problem 1 (a) Regions B, C, D, G, and J are sent by the fully-embedded ranks.

(b) Regions A and L are received by the fully-embedded ranks.

(c) Communication and computation cannot be overlapped in this case. The dependency of the stencil dictates that the regions must be processed in the following steps.

i. B, E, H, I, J
ii. C, F, G
iii. D

To compute the regions in step 1, we must post a blocking receive to obtain regions A and L. We can then compute the regions in step 1, 2, and 3 in sequence. Once the block has been completely computed, then we can post a blocking send of the inner halo given by regions B, C, D and D, C, G. As a result, we cannot overlap communication and computation in this case.

Problem 2 (a) We have two arrays, `u_old`, which stores the solution at time step $t^{(k)}$, and `u_new`, which stores the solution at time step $t^{(k+1)}$ and is uninitialized at the beginning of each time step. We can post an asynchronous receive before computing the inner halo, since we can just reference values from `u_old` with $t^{(k)}$ to perform the finite difference update for the inner halo. We cannot post an asynchronous send until the inner halo is computed and stored in `u_new`, thus the send will still depend on the computation of the inner halo (Regions E, F, G, H, J, K, L and M) when we have two arrays.

(b) Using row-major indexing, the outer halos given by regions D and N will need to be packed into separately allocated receive buffers. Similarly, corresponding the inner halos given by regions E, H, K and G, J, M will need to be packed into separately allocated send buffers as well. Contrarily, the outer halos given by regions B and P can simply be referenced by pointer for the receive portion of the halo exchange. These halos will contain extra data, as a result of the indexing used in this problem. Additionally, the inner halos given by regions E, F, G and K, L, M can be referenced by pointer for the send portion of the halo exchange. These halos will also contain extra data.

(c) With only one array being acted upon, we must process a ranks region in the following order to ensure correctness while having the opportunity to hide communication.

i. Compute the inner halo. (Regions E, F, G, H, J, K, L, and M)
ii. Post an asynchronous send of the inner halo and pack send data for the directions that require it. (Regions E, F, G, H, J, K, L, and M)
iii. Post an asynchronous receive of the outer halo. (Regions B, D, N, and P)
iv. Compute the innermost region. (Region I)

v. Wait for the send and receive operations to finish and unpack receive data for the directions that require it.

To maximally hide communication, we will first perform step 1, and then perform steps 2, 3, and 4 simultaneously before performing step 5 last.