Algorithmic Robotics
COMP/ELEC/MECH 450/550
Homework 3

DUE: October 12^{th} at the beginning of class (1pm). Submit your answers as a PDF to Canvas.

Please read the honor code and the additions described in the course syllabus. Present your work and your work only. You must *explain* all of your answers. Answers without explanation will be given no credit.

1. **(30 points)** An assumption when using a PRM based approach for motion planning is that the configuration space is not *pathologically* difficult.

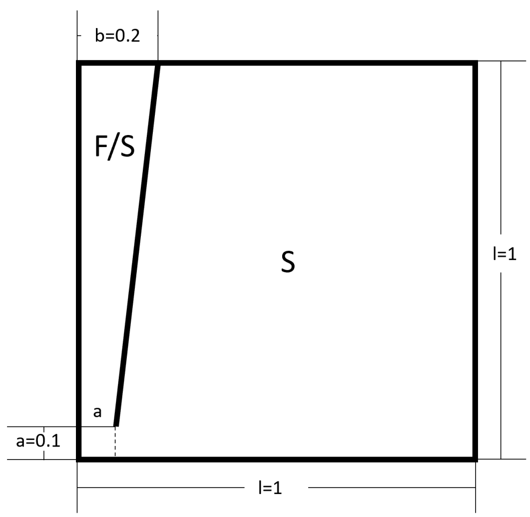

Figure 1: The Configuration Space for Problem 1

For the configuration space shown in Figure 1:

(a) **(5 points)** Estimate the ε-goodness of the space, i.e., what is the largest ε such that this space is ε-good?

(b) **(5 points)** Alice the world-renowned roboticist claims that the 0.5-lookout of S is a 0.4 fraction of S, e.g., $\beta = 0.5, \alpha = 0.4$. Do you think this is true? Why or why not?

(c) **(10 points)** For a space with low $(\varepsilon, \alpha, \beta)$-expansiveness, what kind of sampling techniques would you expect to work well? Explain.

(d) **(10 points)** Imagine that you extrude this configuration space to 3 dimensions. Essentially, this is a 3D configuration space composed of stacked 2D slices, each slice being the configuration space shown in Figure 1. How will the $(\varepsilon, \alpha, \beta)$ values be affected by this change? Will they increase, decrease or remain unchanged? Explain.

2. **(25 points)** The Astrobee[1] is a free-flying robot system developed to perform intravehicular activity work on the International Space Station (ISS). It is a cube-shaped robot whose state can be described by the vector $x = [p, v, q, \omega] \in \mathbb{R}^{13}$, where p is the position of a reference point in the robot body, q is a quaternion representation of the robot's orientation, v is the robot's velocity and ω its angular velocity. The Astrobee has a propulsion system capable of applying force $F \in \mathbb{R}^3$ in any direction and torque $M \in \mathbb{R}^3$ about any axis. The dynamics of the system can be written as:

$$\dot{p} = v, m\dot{v} = F, \dot{q} = \frac{1}{2}\Omega(\omega)q, J\dot{\omega} = M - S(\omega)J\omega$$

where m is the mass, $\Omega(\omega)$ and $S(\omega)$ are angular velocity matrices and J is the inertia matrix.

(a) **(5 points)** What is the configuration space of the Astrobee robot?

(b) **(5 points)** What is the control space of the robot?

(c) **(5 points)** Is this a non-holonomic or a holonomic robot? Why?

The engineers at the Rice Aeronautics and Space Administration (RASA) have changed the propulsion system so that the robot does not rotate anymore, i.e., its state is described by $x = [p, v] \in \mathbb{R}^6$. Describe the type of motion that would generate the following velocity constraints and specify whether the system is holonomic or non-holonomic:

(d) **(5 points)** $v_x^2 + v_y^2 + v_z^2 = 1$

(e) **(5 points)** $v_x^2 + v_y^2 \leq 1, v_z = 0$

3. **(20 points)** Recall the visibility graph method. Similar to the PRM, the visibility graph also captures the continuous space using a graph structure. Compare these two methods. For each method, provide at least one scenario in which it would work well while the other would not. Justify your answer.

4. **(25 points)** You have just been hired as Motion Planner expert in company that develops a variety of customized robotic solutions. Your first task is deciding what planners to use in the following situations:

   - 3D robotic arm with 7 revolute joints moving within an environment where all the objects are static.
   - A mobile manipulator that consists of a differential-drive mobile base and a 8 degree-of-freedom robot arm. The manipulator will only move when the base is at certain pre-defined locations and mostly in free-space. You can specify different planners for the base and for the manipulator.
   - A mobile robot in the shape of a disc (like a Roomba). It is important that paths do not go too close to the obstacles (large clearance).
   - A car-like robot in a fixed environment. For this robot you have a highly efficient steering function available.

   For each situation, specify what planner(s) you will use and explain why. You can choose from all the planners discussed in class.

[1] https://arc.aiaa.org/doi/pdf/10.2514/6.2018-2517