Algorithmic Robotics
COMP/ELEC/MECH 450/550
Homework 4

DUE: November 2nd by 1pm. Submit your answers as a PDF to Canvas.

Please read the honor code and the additions that are described in the Course

1. **(30 points)**

   The Astrobee[1] is a free-flying robot system developed to perform intravehicular activity work on the International Space Station (ISS). It is a cube-shaped robot whose state can be described by the vector $x = [p, v, q, \omega] \in \mathbb{R}^{13}$, where p is the position of a reference point in the robot body, q is a quaternion representation of the robot's orientation, v is the robot's velocity and ω its angular velocity. The Astrobee has a propulsion system capable of applying force $F \in \mathbb{R}^3$ in any direction and torque $M \in \mathbb{R}^3$ about any axis. The dynamics of the system can be written as:

   $$\dot{p} = v, m\dot{v} = F, \dot{q} = \frac{1}{2}\Omega(\omega)q, J\dot{\omega} = M - S(\omega)J\omega$$

   where m is the mass, $\Omega(\omega)$ and $S(\omega)$ are angular velocity matrices and J is the inertia matrix.

   (a) (5 points) What is the configuration space of the Astrobee robot?

   (b) (5 points) What is the control space of the robot?

   (c) (5 points) Is this a non-holonomic or a holonomic robot? Why?

   The engineers at the Rice Aeronautics and Space Administration (RASA) have changed the propulsion system so that the robot does not rotate anymore, i.e., its state is described by $x = [p, v] \in \mathbb{R}^6$. Describe the type of motion that would generate the following velocity constraints and specify whether the system is holonomic or non-holonomic:

   (d) (5 points) $v_x^2 + v_y^2 + v_z^2 = 1$

   (e) (10 points) $v_x^2 + v_y^2 \leq 1, v_z = 0$

2. **(20 points)** Generally, sampling-based planning approaches are concerned with finding a feasible path. Recently, many optimizing planners have been proposed, such as:

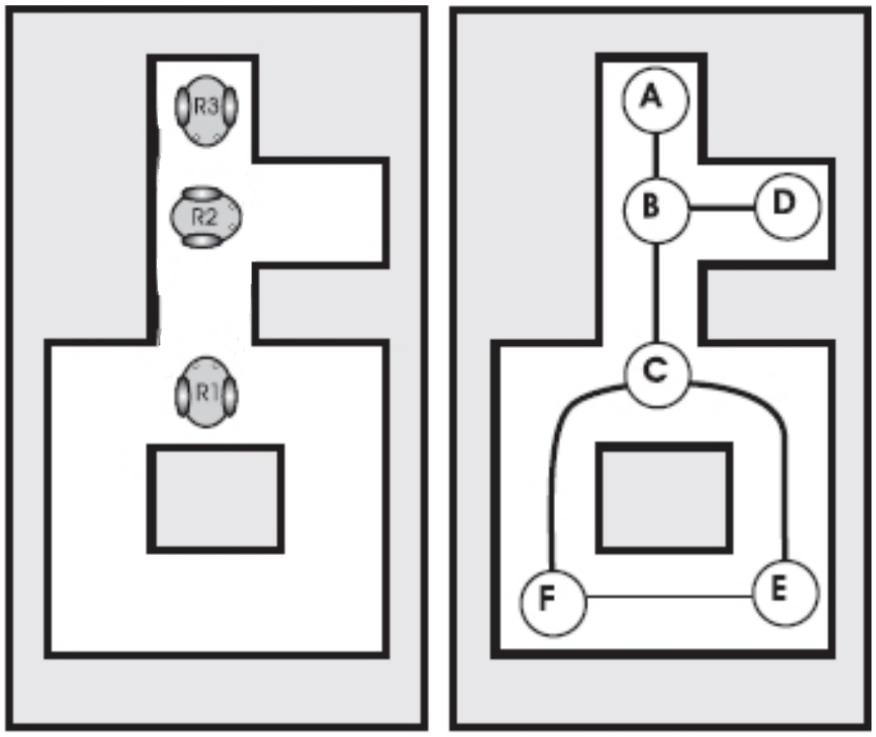
   - asymptotically optimal planners, which are sampling-based planners that optimize for a cost, and
   - optimizing planners, which refine and perturb an initial trajectory to find some "optimal" motion.

   (a) What is the core idea behind RRT*? That is, explain what modifications are done to RRT in order to make it asymptotically optimal. What methods are changed, and what changes are they?

[1] https://arc.aiaa.org/doi/pdf/10.2514/6.2018-2517

(b) Bob the world-famous roboticist decides to use RRT* to plan the motion of a second-order car. Can he use RRT*? Explain why yes or why no or what the difficulties may be.

3. **(25 points)** A multi-robot planning problem is shown in Figure 1. On the left, a depiction of three robots (R_1, R_2, and R_3) is shown at their starting configuration. On the right, the "graph-based" map for the space is shown. The map is a set of discrete locations (A through F) that the robots can occupy. A robot can transition from one location (e.g., A to B) if there is not another robot at the destination location and there is a path free of robots from start to goal. This transition is described by the action MOVE(R_n, L_1, L_2), which moves the robot R_n from location L_1 to location L_2. In this problem, the three robots start at locations (C, B, A) respectively, and the goal configuration is to have the robots reach locations (B, C, F) respectively, as shown in the graph-based map.



(a) Original planning problem. (b) Graph-based map.

Figure 1: Multi-Robots in a Constrained Environment

(a) Provide a sequence of actions in the form MOVE(R_n, L_1, L_2) such that planning problem is solved. Provide a larger sequence that achieves the same result.

(b) There are two broad categories of multi-robot motion planning algorithms: decentralized and centralized planners. Decentralized planners consider each robot separately and must find a way to coordinate the robots. This can be done for example by assigning priorities to the robots and moving at each time the robot withe the highest priority. Centralized planners consider all the robots as one "super" robot and define a suitable configuration space for that robot. What approach would be more suitable for this problem, a decentralized or a centralized method? Why?

(c) Imagine an environment with 100 of these robots in an open space. Which approach would be more suitable, a decentralized or a centralized method? Why?

4. **(25 points)** Recall from the class the "blockworld" domain. In this problem, there are a set of blocks on a table environment that can be stacked on top of each other. Below is the PDDL[2] description of this domain:

```
(define (domain blocksworld)
(:predicates (clear ?x) (ontable ?x) (handempty) (holding ?x) (on ?x ?y)
)

(:action pickup
:parameters (?ob)
:precondition (and (clear ?ob) (ontable ?ob) (handempty))
:effect (and (holding ?ob) (not (clear ?ob)) (not (ontable ?ob))
(not (handempty))))

(:action putdown
:parameters  (?ob)
:precondition (and (holding ?ob))
:effect (and (clear ?ob) (handempty) (ontable ?ob)
(not (holding ?ob))))

(:action stack
:parameters  (?ob ?underob)
:precondition (and  (clear ?underob) (holding ?ob))
:effect (and (handempty) (clear ?ob) (on ?ob ?underob)
(not (clear ?underob)) (not (holding ?ob))))

(:action unstack
:parameters  (?ob ?underob)
:precondition (and (on ?ob ?underob) (clear ?ob) (handempty))
:effect (and (holding ?ob) (clear ?underob) (not (on ?ob ?underob))
(not (clear ?ob)) (not (handempty)))))
```

(a) Give a valid task plan in the "blockworld" domain to solve the problem below. Write out each step (what predicates are true, i.e. `(ontable c)`) and what action is taken at each step.

```
(define (problem sussman-anomaly)
(:domain blocksworld)
(:objects a b c d)
(:init (ontable a) (ontable b)
(on c a)
(clear b) (clear c)
(holding d))
(:goal (and (on b c)
(on a b)
(on d a))))
```

(b) Name one limitation of encoding a possible planning problem in PDDL. Is there a planning problem that cannot be encoded into PDDL? Give an example, or justify why you could encode any feasible problem.

[2]The Planning Domain Definition Language https://en.wikipedia.org/wiki/Planning_Domain_Definition_Language was not discussed in depth during the lectures. However, we expect the description shown here to be self-explanatory and simple enough to answer the question in this homework.