Algorithmic Robotics
COMP/ELEC/MECH 450/550
Homework 3

DUE: October 12$^{\text{th}}$ at the beginning of class (1pm). Submit your answers as a PDF to Canvas.

Please read the honor code and the additions described in the course syllabus. Present your work and your work only. You must *explain* all of your answers. Answers without explanation will be given no credit.

1. **(30 points)** An assumption when using a PRM based approach for motion planning is that the configuration space is not *pathologically* difficult.

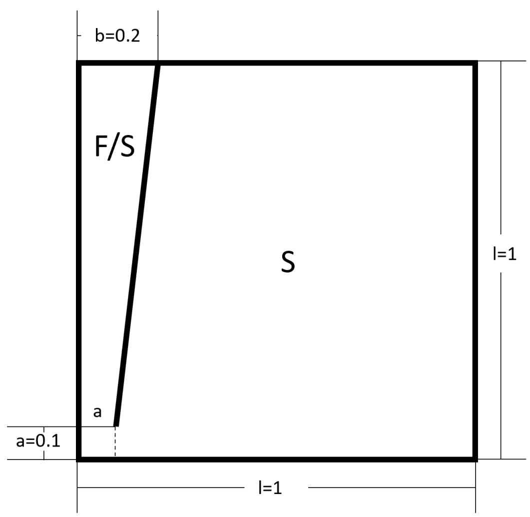


Figure 1: The Configuration Space for Problem 1

For the configuration space shown in Figure 1:

(a) **(5 points)** Estimate the ε-goodness of the space, i.e., what is the largest ε such that this space is ε-good?

(b) **(5 points)** Alice the world-renowned roboticist claims that the 0.5-lookout of S is a 0.4 fraction of S, e.g., $\beta = 0.5, \alpha = 0.4$. Do you think this is true? Why or why not?

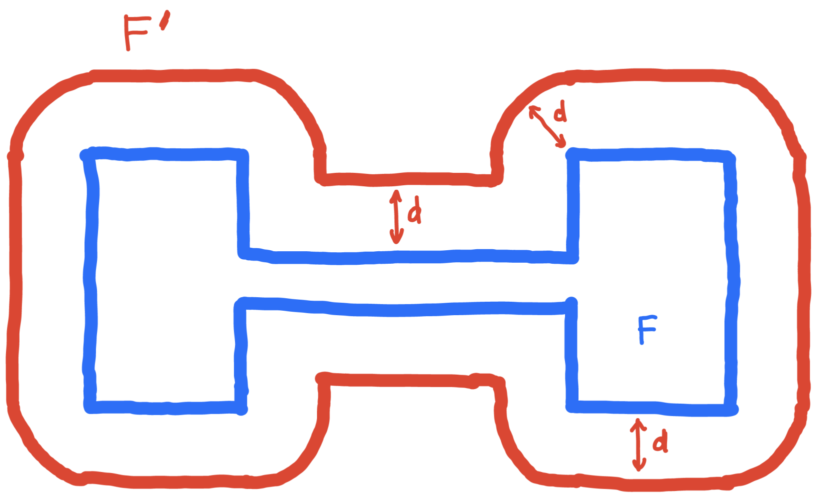
(c) **(10 points)** For a space with low $(\varepsilon, \alpha, \beta)$-expansiveness, what kind of sampling techniques would you expect to work well? Explain.

(d) **(10 points)** Imagine that you extrude this configuration space to 3 dimensions. Essentially, this is a 3D configuration space composed of stacked 2D slices, each slice being the configuration space shown in Figure 1. How will the $(\varepsilon, \alpha, \beta)$ values be affected by this change? Will they increase, decrease or remain unchanged? Explain.

2. **(30 points)**

A student taking this class has come up with a new idea of sampling for PRM and wants your help to assess the feasibility of implementation and the potential problems of the idea. Read all the questions before starting.

The idea is as follows: Let F be the free configuration space. The new roadmap construction algorithm, `new-roadmap` starts by constructing a roadmap R' in a dilated free space F' obtained by allowing the space F to "expand" by d. The picture below shows an example of a dilated free space. It is assumed that there exists a collision checking routine that returns TRUE if a configuration is in F (the original free space). It is also assumed that there is a collision checking routine that returns TRUE if a configuration is in F' (the dilated free space). The rough idea is to first construct R' and then "push" this roadmap gently (move its nodes and edges) to obtain a roadmap in F.



(a) (10 points) Suppose that you have a roadmap R' in F'. Describe one way to "push" its nodes to F ? We want one reasonable solution.

(b) (10 points) Describe one way to "push" the edges of your roadmap to F. We want one reasonable solution.

(c) (10 points) The student further suggests to apply the idea successively. That is first dilate the space with a large δ_1, then a smaller one δ_2, then a smaller one, etc K times. Each time the roadmap is pushed from the space dilated with δ_k to the space dilated with δ_{k+1}. Assume suitable collision checking procedures are provided. What do you think of this idea? Is it totally bogus? Could it help create good roadmaps in some cases and when might it be useful. Here we want a discussion. There are no right or wrong answers. Justify your answers.

3. **(20 points)** Recall the visibility graph method. Similar to the PRM, the visibility graph also captures the continuous space using a graph structure. Compare these two methods. For each method, provide at least one scenario in which it would work well while the other would not. Justify your answer.

4. **(20 points)** You have just been hired as Motion Planner expert in company that develops a variety of customized robotic solutions. Your first task is deciding what planners to use in the following situations:

- 3D robotic arm with 7 revolute joints moving within an environment where all the objects are static.
- A mobile manipulator that consists of a differential-drive mobile base and a 8 degree-of-freedom robot arm. The manipulator will only move when the base is at certain pre-defined locations and mostly in free-space. You can specify different planners for the base and for the manipulator.
- A mobile robot in the shape of a disc (like a Roomba). It is important that paths do not go too close to the obstacles (large clearance).
- A car-like robot in a fixed environment. For this robot you have a highly efficient steering function available.

For each situation, specify what planner(s) you will use and explain why. You can choose from all the planners discussed in class.