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PSM- Maturín

① Gauss - Seidel:

Diagonalizando:

$10x + 3y + z = 14$
$5x - 10y + 3z = -5$
$x + 3y + 10z = 14$

$x_{k+1} = \frac{1}{10}(14 - 3y_k - z_k)$

$y_{k+1} = \frac{1}{-10}(-5 - 5x_{k+1} - 3z_k)$

$z_{k+1} = \frac{1}{10}(14 - 5x_{k+1} - 3y_{k+1})$

$(x_0, y_0, z_0) = (0, 0, 0)$

1ra Iteración:

$x_1 = \frac{1}{10}[14 - 3(0) - (0)] = 1,4$

$y_1 = -\frac{1}{10} \cdot [-5 - (1,4) - 3.10)] = 1,2$

$z_1 = \frac{1}{10} \cdot [14 - (1,4) - 3.(1,2)] = 0,9$

2da Aproximación:

$x_2 = \frac{1}{10} \cdot [14 - 3.(1,2) - (0,9)] = 0,95$

$y_2 = -\frac{1}{10} \cdot [-5 - 5(0,95) - 3(0,9)] = 1,245$

$z_2 = \frac{1}{10} \cdot [14 - (0,95) - 3.(1,245)] = 0,9315$

3ra Aproximación:

$x_3 = \frac{1}{10} \cdot [14 - 3(1,245) - (0,9315)] = 0,9334$

$y_3 = -\frac{1}{10} [-5 - 5(0,9334) - 3(0,9315)] = 1,2461$

$z_3 = \frac{1}{10} \cdot [14 - (0,9334) - 3(1,2461)] = 0,9328$

4ta Aproximación:

$x_4 = \frac{1}{10} \cdot [14 - 3.(1,2461) - (0,9328)] = 0,9329$

$y_4 = -\frac{1}{10} \cdot [-5 - 5(0,9329) - 3(0,9328)] = 1,2463$

$z_4 = \frac{1}{10} \cdot [14 - (0,9329) - 3(1,2463)] = 0,9328$

Sol:

$x \approx 0,9329 \approx 0,93$

$y \approx 1,2463 \approx 1,25$

$z \approx 0,9328 \approx 0,93$

Gauss Jacobi.

Ecuaciones de recurrencia:

$$x_{k+1} = \frac{1}{10}(14 - 3y_k - z_k)$$

$$y_{k+1} = -\frac{1}{10} \cdot (-5 - 5x_k - 3z_k)$$

$$z_{k+1} = \frac{1}{10} \cdot (14 - x_k - 3y_k)$$

Iniciando en $(x_0, y_0, z_0) = (0, 0, 0)$

Aproximaciones:

1ra:

$x_1 = \frac{1}{10} \cdot [14 - 3(0) - (0)] = 1,4$

$y_1 = -\frac{1}{10}[-5 - 5(0) - 3(0)] = 0,5$

$z_1 = \frac{1}{10}[14 - (0) - 3(0)] = 1,4$

2da:

$x_2 = \frac{1}{10}[14 - 3(0,5) - (1,4)] = 1,11$

$y_2 = -\frac{1}{10}[-5 - 5 \cdot (1,4) - 3(1,4)] = 1,62$

$z_2 = \frac{1}{10} \cdot [14 - (1,4) - 3(0,5)] = 1,11$

3ra:

$x_3 = \frac{1}{10} \cdot [14 - 3 \cdot (1,62) - (1,11)] = 0,803$

$y_3 = -\frac{1}{10}[-5 - 5(1,11) - 3(1,11)] = 1,388$

$z_3 = \frac{1}{10} \cdot [14 - (1,11) - 3 \cdot (1,62)] = 0,803$

4ta:

$x_4 = \frac{1}{10} \cdot [14 - 3(1,388) - (0,803)] = 0,9033$

$y_4 = -\frac{1}{10} \cdot [-5 - 5(0,803) - 3(0,803)] = 1,1424$

$z_4 = \frac{1}{10} \cdot [14 - (0,803) - 3(1,388)] = 0,9033$

5ta:

$x_5 = \frac{1}{10} \cdot [14 - 3(1,1424) - (0,9033)] = 0,967$

$y_5 = -\frac{1}{10}[-5 - 5(0,9033) - 3(0,9033)] = 1,2226$

$z_5 = \frac{1}{10} \cdot [14 - 0,9033 - 3(1,1424)] = 0,967$

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El resto de los cálculos se muestran a continuación:

6ta:

$x_6 = 0{,}9365$

$y_6 = 1{,}2736$

$z_6 = 0{,}9365$

7ma:

$x_7 = 0{,}9243$

$y_7 = 1{,}2492$

$z_7 = 0{,}9243$

8vo

$x_8 = 0{,}9328$

$y_8 = 1{,}2394$

$z_8 = 0{,}9328$

9no:

$x_9 = 0{,}9349$

$y_9 = 1{,}2462$

$z_9 = 0{,}9349$

10ma:

$x_{10} = 0{,}9326$

$y_{10} = 1{,}2479$

$z_{10} = 0{,}9326$

11vas

$x_{11} = 0{,}9324$

$y_{11} = 1{,}2461$

$z_{11} = 0{,}9324$

ultima:

$x_{12} = 0{,}9329$

$y_{12} = 1{,}2459$

$z_{12} = 0{,}9329$

Sol: $x \approx 0{,}9329 \approx 0{,}93$

$y \approx 1{,}2459 \approx 1{,}25$

$z \approx 0{,}9329 \approx 0{,}93$

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(2)

$$0{,}3x_1 - 0{,}2x_2 + 10x_3 = 71{,}4$$
$$3x_1 - 0{,}1x_2 - 0{,}2x_3 = 7{,}85$$
$$0{,}1x_1 + 7x_2 - 0{,}3x_3 = -19{,}3$$

Diagonalizando: y cambiando: $\begin{cases} x_1 = x \\ x_2 = y \\ x_3 = z \end{cases}$

$$3x - 0{,}1y - 0{,}2z = 7{,}85$$
$$0{,}1x + 7y - 0{,}3z = -19{,}3$$
$$0{,}3x - 0{,}2y + 10z = 71{,}4$$

Ecuaciones de recurrencia:

$$x_{k+1} = \frac{1}{3}\cdot(7{,}85 + 0{,}1y_k + 0{,}2z_k)$$
$$y_{k+1} = \frac{1}{7}\cdot(-19{,}3 - 0{,}1x_{k+1} + 0{,}3z_k]$$
$$z_{k+1} = \frac{1}{10}\cdot(71{,}4 - 0{,}3x_{k+1} + 0{,}2y_{k+1})$$

Inicial $(x_0, y_0, z_0) = (0,0,0)$

1ra Aproximación:

$$x_1 = \frac{1}{3}\cdot(7{,}85 + 0{,}1\cdot(0) + 0{,}2(0)] = 2{,}6167$$
$$y_1 = \frac{1}{7}\cdot(-19{,}3 - 0{,}1(2{,}6167) + 0{,}3(0)] = -2{,}7945$$
$$z_1 = \frac{1}{10}\cdot(71{,}4 - 0{,}3\cdot(2{,}6167) + 0{,}2\cdot(-2{,}7945))] = 7{,}0056$$

2da Aproximación:

$$x_2 = \frac{1}{3}\cdot[7{,}85 + 0{,}1(-2{,}7945) + 0{,}2(7{,}0056)] = 2{,}9906$$
$$y_2 = \frac{1}{7}\cdot[-19{,}3 - 0{,}1\cdot(2{,}9906) + 0{,}3\cdot(7{,}0056)] = -2{,}4996$$
$$z_2 = \frac{1}{10}\cdot[71{,}4 - 0{,}3(2{,}9906) + 0{,}2(-2{,}4996)] = 7{,}0003$$

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3ra. Aproximación:

$x_3 = \frac{1}{3}[7{,}85 + 0{,}1(-2{,}4996) + 0{,}2(7{,}0003)] = 3$

$y_3 = \frac{1}{7}\cdot(-19{,}3 - 0{,}1(3) + 0{,}3(7{,}0003)] = -2{,}5$

$z_3 = \frac{1}{10}\cdot[71{,}4 - 0{,}3(3) + 0{,}2(-2{,}5)] = 7$

Sol: $x = 3,\ y = -2{,}5,\ z = 7$

$x_1 = 3,\ x_2 = -2{,}5,\ x_3 = 7$

3 b

$$\begin{aligned} x + y + az &= a^2 \\ x + ay + z &= a \\ ax + y + z &= 1 \end{aligned} \rightarrow \left(\begin{array}{ccc:c} 1 & 1 & a & a^2 \\ 1 & a & 1 & a \\ a & 1 & 1 & 1 \end{array}\right) \begin{array}{l} R_2 = R_2 - R_1 \\ R_3 = R_3 - aR_1 \end{array}$$

$$= \left(\begin{array}{ccc:c} 1 & 1 & a & a^2 \\ 0 & a-1 & 1-a & a-a^2 \\ 0 & 1-a & 1-a^2 & 1-a^3 \end{array}\right) R_2 = \frac{R_2}{a-1} = \left(\begin{array}{ccc:c} 1 & 1 & a & a^2 \\ 0 & 1 & -1 & -a \\ 0 & 1-a & 1-a^2 & 1-a^3 \end{array}\right) R_3 = R_3 - (1-a)R_2$$

$$= \left(\begin{array}{ccc:c} 1 & 1 & a & a^2 \\ 0 & 1 & -1 & -a \\ 0 & 0 & \begin{matrix} 1-a^2 \\ -a+a^2 \end{matrix} & \begin{matrix} 1-a^3 \\ -a^2+a^3 \end{matrix} \end{array}\right) R_3 = \frac{R_3}{1-a} = \left(\begin{array}{ccc:c} 1 & 1 & a & a^2 \\ 0 & 1 & -1 & -a \\ 0 & 0 & 1 & 1+a \end{array}\right) \begin{array}{l} R_1 = R_1 - aR_3 \\ R_2 = R_2 + R_3 \end{array}$$

$$= \left(\begin{array}{ccc:c} 1 & 1 & 0 & -a \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1+a \end{array}\right) R_1 = R_1 - R_2 = \left(\begin{array}{ccc:c} 1 & 0 & 0 & -1-a \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1+a \end{array}\right)$$

Sol: $x = -(1+a)$
$y = 1$
$z = 1+a$

Este sistema de ecuaciones es compatible ya que tiene una solución única para un valor de "a"

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ⓐ El sistema no es compatible porque tiene más Incognitas que ecuaciones

Sol:

$$\begin{aligned} x-y+2z-T&=-1\\ 4x-2y+6z+3T-4U&=3\\ 2x+4y-2z+4T-7U&=4\\ x+y-3T-u&=-3 \end{aligned} = \left(\begin{array}{ccccc|c} 1&-1&2&-1&0&-1\\ 4&-2&6&3&-4&3\\ 2&4&-2&4&-7&4\\ 1&1&0&-3&-1&-3 \end{array}\right)$$

$$= \left(\begin{array}{ccccc|c} 1&-1&2&-1&0&-1\\ 1&1&0&-3&-1&-3\\ 4&-2&6&3&-4&3\\ 2&4&-2&4&-7&4 \end{array}\right) \begin{array}{l} R_2=R_2-R_1\\ R_3=R_3-4R_1\\ R_4=R_4-2R_1 \end{array}$$

$$= \left(\begin{array}{ccccc|c} 1&-1&2&-1&0&-1\\ 0&-2&-2&-2&-1&-2\\ 0&2&-2&7&-4&7\\ 0&6&-6&6&-7&8 \end{array}\right) \begin{array}{l} R_3=R_3+R_2\\ R_4=R_4+3R_2 \end{array}$$

$$= \left(\begin{array}{ccccc|c} 1&-1&2&-1&0&-1\\ 0&-2&-2&-2&-1&-2\\ 0&0&-4&5&-5&5\\ 0&0&-12&0&-10&0 \end{array}\right) R_4=R_4-3R_3$$

$$= \left(\begin{array}{ccccc|c} 1&-1&2&-1&0&-1\\ 0&-2&-2&-2&-1&-2\\ 0&0&-4&5&-5&5\\ 0&0&0&-15&5&-15 \end{array}\right) \begin{array}{l} R_2/-2\\ R_3/-4\\ R_4/-15 \end{array}$$

$$= \left(\begin{array}{ccccc|c} 1&-1&2&-1&0&-1\\ 0&1&1&1&1/2&1\\ 0&0&1&-5/4&5/4&-5/4\\ 0&0&0&1&-1/3&1 \end{array}\right) \begin{array}{l} R_1=R_1+R_4\\ R_2=R_2-R_4\\ R_3=R_3+5/4R_4 \end{array}$$

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$$= \left(\begin{array}{ccccc|c} 1 & -1 & 2 & 0 & -1/3 & 0 \\ 0 & 1 & 1 & 0 & 5/6 & 0 \\ 0 & 0 & 1 & 0 & 5/6 & 0 \\ 0 & 0 & 0 & 1 & -1/3 & 1 \end{array}\right) \begin{array}{l} R_1 = R_1 - 2R_3 \\ R_2 = R_2 - R_3 \end{array}$$

$$= \left(\begin{array}{ccccc|c} 1 & -1 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 5/6 & 0 \\ 0 & 0 & 0 & 1 & -1/3 & 1 \end{array}\right) R_1 = R_1 + R_2$$

$$= \left(\begin{array}{ccccc|c} 1 & 0 & 0 & 0 & -2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 5/6 & 0 \\ 0 & 0 & 0 & 1 & -1/3 & 1 \end{array}\right)$$

Sol: sea $u = w$

$x = 2w$

$y = 0$

$z = -5/6\, w$

$T = 1 + 1/3\, w$

$u = w$

El sistema de ecuaciones no es compatible ya que tiene infinitos soluciones dependiendo del valor que tome (u) que es una variable.

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