Proof Problems

Exercise M:

Laws for map:

map-empty law: (map f '()) = '()

map-cons law: (map f (cons y ys)) = (cons (f y) (map f ys))

Laws for curry

curry law: (((curry f) x) y)=(f x y)

uncurry law: ((uncurry f) x y) = ((f x) y)

Laws for o

o law: ((o f g) x) = (f (g x))

Claim: (o ((curry map) f) ((curry map) g)) == ((curry map) (o f g))

Proof by Equational Reasoning:

For this proof, I assumed that the LHS and RHS expressions took in two types of inputs: an empty list or a non-empty list (cons x xs). Thus, I broke my proof up into those two cases:

((o ((curry map) f) ((curry map) g)) '()) == (((curry map) (o f g)) '()) for Case 1

((o ((curry map) f) ((curry map) g)) (cons x xs)) == (((curry map) (o f g)) (cons x xs)) for Case 2

For Case 1, I prove that the LHS and RHS evaluate to the same value.

Case 1: input: empty list

LHS: ((o ((curry map) f) ((curry map) g)) '())
= {o law}
(((curry map) f) (((curry map) g) '()))

= {curry law}

(((curry map) f) (map g '()))

= {curry law}

(map f (map g '()))

= {map-empty law}

(map f '())

= { map-empty law)}

'()

RHS: (((curry map) (o f g)) '())

= {curry law}

(map (o f g) '())

= {map-empty law}

'()

LHS == RHS

#t

For Case 2, I prove that the LHS and RHS are equal by modifying the LHS to become the RHS.

Case 2: input (cons x xs)

LHS: ((o ((curry map) f) ((curry map) g)) (cons x xs))

= {o law}

(((curry map) f) (((curry map) g) (cons x xs)))

= {curry law}

(((curry map) f) (map g (cons x xs)))

= {map cons law}

(((curry map) f) (cons (g x) (map g xs)))

= {curry law}

(map f (cons (g x) (map g xs)))

= {map cons law}

(cons (f (g x)) (map f (map g xs)))

= {reverse curry law}

(cons (f (g x)) (((curry map) f) (map g xs)))

= {reverse curry law}

(cons (f (g x)) (((curry map) f) (((curry map) g) xs)))
= {reverse o law}
(cons (f (g x)) ((o ((curry map) f) ((curry map) g) xs)))
= {apply inductive hypothesis}
(cons (f (g x)) (((curry map) (o f g)) xs))
= {curry law}
(cons (f (g x)) (map (o f g) xs))
= {reverse o law}
(cons (f (g x)) (map (f (g x)) xs))
= {reverse map-cons law}
(map (f (g x)) (cons x xs))
= {o law}
(map (o f g) (cons x xs))
= {reverse curry law}
(((curry map) (o f g)) (cons x xs)) ;; RHS
#t

<u>Conclusion</u>: Our claim is true!