

UTA Method

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Abstract

This report contains the summary of the chapter **UTA Methods** from the Book : Multiple Criteria Decision Analysis. This document was made during my internship at LAMSADE in the summer of 2017.

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Chapter 1

Introduction

In decision theory a basic problem involving several criterias, concerns the way that the final decision should be made. However, this problem is posed in the opposite way: assuming that the decision is given, how is it possible to find the rational basis for the decision being made? Or how is it possible to assess the model leading to exactly the same decision as the actual one or the most "similar" decision.

Those steps represent the methodology of decision-making problems :

1. Object of the decision, definition of the set of potential actions and the determination of a problem statement
2. Modeling a consistent family of criteria
3. Defining a global preference model
4. Decision-aid or decision support

Example : Buying a new car

Let's consider we are trying to figure out which car to buy. Using the methodology represented above, we state that the objective of this problem is **buying a new car**. After stating the objective, we can list the potential of actions that represent the list of cars that we may buy:

- Peugeot 208 GTi
- Nissan Sentra
- Citroen C4
- Peugeot 308 berline

After listing the list of potential cars, we can define a list of criteria that we will base our decision on. When defining the list of criteria you should always remember that the potential actions should be evaluated by those criterias, so the criteria must be easy to evaluate and should be logical. Let's say we will base our purchase on the following criteria :

- price (in Euro)
- comfort (0, +, ++, +++) *0 being not comfortable and +++ very comfortable*
- safety (1, 2, 3, 4, 5) *1 being not safety and 5 safe*

During the decision process, we will determinate the global preferences of the potential actions :

1. Citroen C4
2. Peugeot 208 GTi
3. Peugeot 308 berline
4. Nissan Sentra

Once the global preference is defined, we can start the decision support.

In this document, we will study a multi-criteria decision analysis mehtods UTA, a method proposed in 1982 that build a utility function based on the preference of the DM. More precisely we will conduct our research over the new version of UTA : UTASTAR. The method is illustrated on an example.

Chapter 2

UTA Method

One of the multi-criteria decision analysis methods is the UTA method, which was proposed by E. Jacquet-Lagrèze and J. Siskos in 1982. This method is proposed by the Multi-Attribute Utility Theory (MAUT) that build a utility function based on the DM¹ preferences.

The UTA method is used to solve a multi-criteria problem by building a utility function based on the preferences of the DM and solving a linear program (LP). It adopt the aggregation-disaggregation principles : where the model is based on a given preferences.

The UTASTAR has been considered a better algorithm than UTA. Better result were found using the UTASTAR algorithm. So this is why we will focus on this method rather than the UTA method.

At the begining of the problem, the DM should present the following information

- Rank of the actions
- Give the criteria he want to base his decision on
- Evaluate the action compared to the criterion

Once those information are presented, the UTA algorithm can be executed.

¹Decision Maker

2.1 Principles and Notation

Let's call $A = a, b, c, \dots$ the set of potential actions and $g_1, g_2, g_3, \dots, g_n$ the family of criteria. Where $g_i(a)$ represent the evaluation of an action a on the criteria g_i . Having the following values :

- Least preferred criteria : $g_{i*} = \min(g_i)$
- Most preferred criteria : $g_i^* = \max(g_i)$

So the interval for each criteria g_i is : $[g_{i*}, g_i^*]$.

If we want to evaluate two actions, for example a and b , on only one criteria g_i we have the following relations :

$$\begin{cases} a \succ b \Leftrightarrow g_i(a) > g_i(b) & \text{preference} \\ a \sim b \Leftrightarrow g_i(a) = g_i(b) & \text{indifference} \end{cases} \quad (2.1)$$

The criteria aggregation model in UTA has the following form :

$$u(g) = \sum_{i=1}^n u_i(g_i) \quad (2.2)$$

subject to normalization constraints:

$$\begin{cases} \sum_{i=1}^n u_i(g_i^*) = 1 \\ u_i(g_{i*}) = u_i(g_i^1) = 0, \forall i = 1, 2, \dots, n \end{cases} \quad (2.3)$$

where $u_i, i = 1, 2, \dots, n$ are non decreasing real valued function.

In UTASTAR we can write :

$$u_i(g_i^j) = \sum_{t=1}^{j-1} w_{it} \quad \forall i = 1, 2, \dots, n \quad \text{and} \quad j = 2, 3, \dots, \alpha_i - 1 \quad (2.4)$$

with $w_{ij} = u_i(g_i^{j+1}) - u_i(g_i^j) \geq 0 \quad \forall i, j$

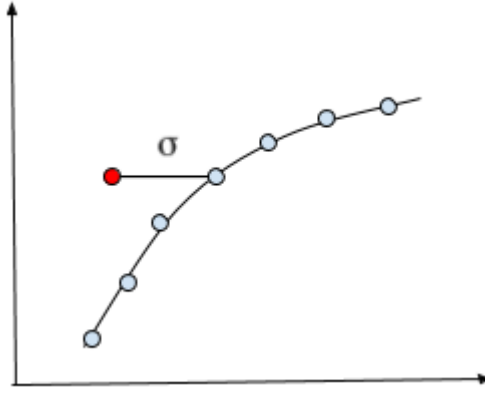
With the evaluation of an action a $g(a) = [g_1(a), g_2(a), \dots, g_n(a)]$, we have the following relation :

$$\begin{cases} u[g(a)] > u[g(b)] \Leftrightarrow a \succ b \\ u[g(a)] \sim u[g(b)] \Leftrightarrow a \sim b \end{cases} \quad (2.5)$$

2.2 Development

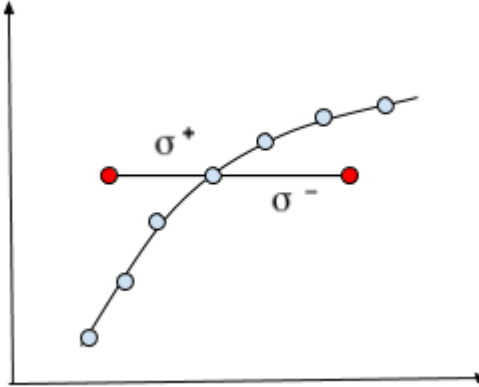
In UTA we have only one error function $\sigma(a)$ where $\sigma(a) \geq 0$. This error function represent the estimated difference between the utility $u[g(a)]$ and the utility that should be attributed to the action a . So the value of each alternative $a \in A_R$ can be written :

$$u' [g(a)] = \sum_{i=1}^n u_i[g_i(a)] + \sigma(a) \forall a \in A_R \quad (2.6)$$



The updated version of UTA, UTASTAR, propose a double error function for each action : $\sigma^+(a)$ and $\sigma^-(a)$. So the value of each alternative $a \in A_R$ can be written :

$$u' [g(a)] = \sum_{i=1}^n u_i[g_i(a)] - \sigma^+(a) + \sigma^-(a) \quad \forall a \in A_R \quad (2.7)$$



For each criteria, the interval $[g_{i*}, g_i^*]$ is cut into $(\alpha_i - 1)$ equals interval, and the end points g_i^j are given by the formula:

$$g_i^j = g_{i*} + \frac{j-1}{\alpha_i - 1}(g_i^* - g_{i*}) \forall j = 1, 2, \dots, \alpha_i \quad (2.8)$$

The marginal value of an action a is calculated by a linear interpolation

$$u_i[g_i(a)] = u_i(g_i^j) + \frac{g_i(a) - g_i^j}{g_i^{j+1} - g_i^j} [u_i(g_i^{j+1}) - u_i(g_i^j)] \quad (2.9)$$

The set of reference action $A_R = a_1, a_2, \dots, a_m$ is arranged where a_1 is the best action and a_m is the worst action. Which indicate that we have two possible situations :

- $a_k \succ a_{k+1}$ preference
- $a_k \sim a_{k+1}$ indifference

So if we have that $\Delta(a_k, a_{k+1}) = u'[g(a_k)] - u'[g(a_{k+1})]$ and δ is a small positive number we will obtain the following relations :

$$\begin{cases} \Delta(a_k, a_{k+1}) \geq \delta \\ \Delta(a_k, a_{k+1}) = 0 \end{cases} \quad (2.10)$$

The marginal value functions are estimated by means of the Linear Programm with (2.2), (2.3), (2.10) as constraints, and an objective function depending on the σ^+ and σ^- :

$$[min]z = \sum_{k=1}^m [\sigma^+(a_k) + \sigma^-(a_k)]$$

subject to :

$$\begin{cases} \Delta(a_k, a_{k+1}) \geq \delta \quad or \quad \Delta(a_k, a_{k+1}) = 0 \\ \sum_{i=1}^n \sum_{j=1}^{\alpha_i-1} w_{ij} = 1 \\ w_{ij} \geq 0, \quad \sigma^+(a_k) \geq 0, \quad \sigma^-(a_k) \geq 0, \quad \forall i, j \text{ and } k \end{cases} \quad (2.11)$$

The analysis found by this LP is a post-optimality analysis.

Chapter 3

Examples

3.1 Analyzing the choice of transportation

3.1.1 Problem

A DM wants to analyse the choice of transportation. The DM is interstered in the following criteria

1. price
2. time (min)
3. comfort (possibility to have a seat)

The evaluation of the previous criteria :

Means of transportation	Price	Time	Comfort	Ranking of the DM
RER	3	10	+	1
METRO (1)	4	20	++	2
METRO (2)	2	20	0	2
BUS	6	40	0	3
TAXI	30	30	+++	4

DM's preferences : $RER \succ Metro1 \approx Metro2 \succ Bus \succ Taxi$

3.1.2 Solution

Scale for each criteria

For each criteria, the interval $[g_i^*, g_{i*}]$ is cut into $(\alpha_i - 1)$ equal intervals. So in this case we have :

- Price $\rightarrow [30, 16, 2]$
- Time $\rightarrow [40, 30, 20, 10]$
- Comfort $\rightarrow [0, +, ++, +++]$

Marginal value by linear interpolation

The marginal value is calculated by a linear interpolation. In this case we have:

- $u[g(RER)] = 0.07u_1(16) + 0.93u_1(2) + u_2(10) + u_3(+)$
- $u[g(METRO1)] = 0.14u_1(16) + 0.86u_1(2) + u_2(20) + u_3(++)$
- $u[g(METRO2)] = u_1(2) + u_2(20) + u_3(0) = u_1(2) + u_2(20)$
- $u[g(BUS)] = 0.29u_1(16) + 0.71u_1(2) + u_2(40) + u_3(0) = 0.29u_1(16) + 0.71u_1(2)$
- $u[g(TAXI)] = u_1(30) + u_2(30) + u_3(+++) = u_2(30) + u_3(+++)$

Replace u_i with w_{ij}

- $u[g(RER)] = w_{11} + 0.93w_{12} + w_{21} + w_{22} + w_{23} + w_{31}$
- $u[g(METRO1)] = w_{11} + 0.86w_{12} + w_{21} + w_{22} + w_{31} + w_{32}$
- $u[g(METRO2)] = w_{11} + w_{12} + w_{21} + w_{22}$
- $u[g(BUS)] = w_{11} + 0.71w_{12}$
- $u[g(TAXI)] = w_{21} + w_{31} + w_{32} + w_{33}$

Difference between each pair of consecutive actions

- $\Delta(RER, METRO1) = 0.07w_{12} + w_{23} - w_{32} + \sigma_{RER} - \sigma_{METRO1} \geq \delta$
- $\Delta(METRO1, METRO2) = -0.14w_{12} + w_{31} + w_{32} + \sigma_{METRO1} - \sigma_{METRO2} = 0$
- $\Delta(METRO2, BUS) = 0.29w_{12} + w_{21} + w_{22} + \sigma_{METRO2} - \sigma_{BUS} \geq \delta$
- $\Delta(BUS, TAXI) = w_{11} + 0.71w_{12} - w_{21} - w_{31} - w_{32} - w_{33} + \sigma_{BUS} - \sigma_{TAXI} \geq \delta$

Linear Program

With $[min]F = \sum_{a \in A_R} \sigma(a)$ as the main objectif, we have the following linear program to solve:

Desc	w_{11}	w_{12}	w_{21}	w_{22}	w_{23}	w_{31}	w_{32}	w_{33}	Result
$\Delta(RER, METRO1) \geq \delta$	0	0.07	0	0	1	0	-1	0	$\geq \delta$
$\Delta(METRO1, METRO2) = 0$	0	-0.14	0	0	0	1	1	0	$= 0$
$\Delta(METRO2, BUS) \geq \delta$	0	0.29	1	1	0	0	0	0	$\geq \delta$
$\Delta(BUS, TAXI) \geq \delta$	1	0.71	-1	0	0	-1	-1	-1	$\geq \delta$
$\sum_{i=1}^n u_i(g_i^*) = 1$	1	1	1	1	1	1	1	1	$= 1$

3.2 Analyzing the choice of transportation

3.2.1 Problem

Lebanon is in need of a new location for the placement of a recycling center. The locations has been studied on the following criteria:

- Technical
- Economical activity
- Environmental
- Socio-Demographic
- Financial

A specialized team has selected 6 possible locations. The following table contains the evaluations of those locations on the previous criterias :

Location	Technical	Economical activity	Environmental	Socio-Demographic	Financial	Ranking
L1	3	3	3	1	3	6
L2	3	3	3	2	4	4
L3	3	4	3	3	3	2
L4	2	2	3	2	4	5
L5	5	3	4	4	3	1
L6	4	1	3	4	4	3

DM's preferences : $L5 \succ L3 \succ L6 \succ L2 \succ L4 \succ L1$

3.2.2 Solution

Scale for each criteria

For each criteria, the interval $[g_i^*, g_{i*}]$ is cut into $(\alpha_i - 1)$ equal intervals. So in this case we have :

- Technical $\rightarrow [1, 2, 3, 4, 5]$
- Economical activity $\rightarrow [1, 2, 3, 4, 5]$
- Environmental $\rightarrow [1, 2, 3, 4, 5]$
- Socio-Demographic $\rightarrow [1, 2, 3, 4, 5]$
- Financial $\rightarrow [1, 2, 3, 4, 5]$

Marginal value by linear interpolation

The marginal value is calculated by a linear interpolation. In this case we have:

- $u[g(L1)] =$
- $u[g(L2)] =$
- $u[g(L3)] =$
- $u[g(L4)] =$
- $u[g(L5)] =$
- $u[g(L6)] =$

Replace u_i with w_{ij}

- $u[g(L1)] =$
- $u[g(L2)] =$
- $u[g(L3)] =$
- $u[g(L4)] =$
- $u[g(L5)] =$
- $u[g(L6)] =$

Difference between each pair of consecutive actions

- $\Delta(L1, L2) \geq \delta$
- $\Delta(L2, L3) \geq \delta$
- $\Delta(L3, L4) \geq \delta$
- $\Delta(L4, L5) \geq \delta$
- $\Delta(L5, L6) \geq \delta$

Linear Program

Chapter 4

Conclusion

The UTA method build a utility function based on the preferences of the DM and it consist in solving a linear program (LP) to solve a multi-criteria problem.

This method will elaborate a model of preferences which is as similiar as possible to the DM's preferences.

An improved version of the UTA is the UTASTAR. In UTA we used a single error $\sigma(a)$ in UTASTAR we use a double positive error function. The updated version has performed better than the regular method.

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