Analyzing the choice of transportation

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July 3, 2017

Abstract

This document contains the problem of analysing the choice of transportation. This problem is from the chapter UTA Methods of the Book: Multiple Criteria Decision Analysis. This document was made during my internship at LAMSADE in the summer of 2017.

A DM wants to analyse the choice of transportation. The DM is interstered in the following criteria

1. price
2. time (min)
3. comfort (possibility to have a seat)

The evaluation of the previous criteria:

Means of transportation	Price	Time	Comfort	Ranking of the DM
RER	3	10	+	1
METRO (1)	4	20	++	2
METRO (2)	2	20	0	2
BUS	6	40	0	3
TAXI	30	30	+++	4

DM's preferences: $RER \succ Metro1 \approx Metro2 \succ Bus \succ Taxi$

1 Scale for each criteria

For each criteria, the interval $[g_i^*, g_{i*}]$ is cut into $(\alpha_i - 1)$ equal intervals. So in this case we have:

- Price $\rightarrow$ $[30, 16, 2]$
- Time $\rightarrow$ $[40, 30, 20, 10]$
- Comfort $\rightarrow$ $[0, +, ++, +++]$

2 Marginal value by linear interpolation

The marginal value is calculated by a linear interpolation. In this case we have:

- $v[g(RER)] = 0.07v_1(16) + 0.93v_1(2) + v_2(10) + v_3(+)$
- $v[g(METRO1)] = 0.14v_1(16) + 0.86v_1(2) + v_2(20) + v_3(++)$
- $v[g(METRO2)] = v_1(2) + v_2(20) + v_3(0) = v_1(2) + v_2(20)$
- $v[g(BUS)] = 0.29v_1(16) + 0.71v_1(2) + v_2(40) + v_3(0) = 0.29v_1(16) + 0.71v_1(2)$
- $v[g(TAXI)] = v_1(30) + v_2(30) + v_3(+++) = v_2(30) + v_3(+++)$

3 Replace v_i with w_{ij}

- $v[g(RER)] = w_{11} + 0.93w_{12} + w_{21} + w_{22} + w23 + w_{31}$
- $v[g(METRO1)] = w_{11} + 0.86w_{12} + w_{21} + w_{22} + w_{31} + w_{32}$
- $v[g(METRO2)] = w_{11} + w_{12} + w_{21} + w_{22}$
- $v[g(BUS)] = w_{11} + 0.71w_{12}$
- $v[g(TAXI)] = w_{21} + w_{31} + w_{32} + w_{33}$

4 Difference between each pair of consecutive actions

- $\Delta(RER, METRO1) = 0.07w_{12} + w_{23} - w_{32} + \sigma_{RER} - \sigma_{METRO1} \geq \delta$
- $\Delta(METRO1, METRO2) = -0.14w_{12} + w_{31} + w_{32} + \sigma_{METRO1} - \sigma_{METRO2} = 0$
- $\Delta(METRO2, BUS) = 0.29w_{12} + w_{21} + w_{22} + \sigma_{METRO2} - \sigma_{BUS} \geq \delta$
- $\Delta(BUS, TAXI) = w_{11} + 0.71w_{12} - w_{21} - w_{31} - w_{32} - w_{33} + \sigma_{BUS} - \sigma_{TAXI} \geq \delta$

5 Linear Program

Main objectif: $[min]F = \sum_{a \in A_R} \sigma(a)$
subject to :

$$\Delta(RER, METRO1) \geq \delta$$
$$\Delta(METRO1, METRO2) = 0$$
$$\Delta(METRO2, BUS) \geq \delta$$
$$\Delta(BUS, TAXI) \geq \delta$$
$$\sum_{i=1}^{n} u_i(g_i^*) = 1$$

With $[min]F = \sum_{a \in A_R} \sigma(a)$ as the main objectif, we have the following linear program to solve:

Desc	w_{11}	w_{12}	w_{21}	w_{22}	w_{23}	w_{31}	w_{32}	w_{33}	Result
$\Delta(RER, METRO1) \geq \delta$	0	0.07	0	0	1	0	-1	0	$\geq \delta$
$\Delta(METRO1, METRO2) = 0$	0	-0.14	0	0	0	1	1	0	$= 0$
$\Delta(METRO2, BUS) \geq \delta$	0	0.29	1	1	0	0	0	0	$\geq \delta$
$\Delta(BUS, TAXI) \geq \delta$	1	0.71	-1	0	0	-1	-1	-1	$\geq \delta$
$\sum_{i=1}^{n} u_i(g_i^*) = 1$	1	1	1	1	1	1	1	1	$= 1$

So by using the com.google.ortools library, we can solve the Linear Program above with $\sigma = 0.05$. This Linear Program solution is coded in Java class ChoiceTransportation.

```
Minimize
 Obj: +1 eRER_ +1 eRER__1 +1 eMETRO1_ +1 eMETRO1__2 +1 eMETRO2_ +1 eMETRO2__3 +1 e
Subject to
 auto_c_000000000: +0.07000000000000001 w12 +1 w23 -1 w32 -1 eRER_ +1 eRER__1 +1 e
 auto_c_000000001: -0.14 w12 +1 w31 +1 w32 -1 eMETRO1_ +1 eMETRO1__2 +1 eMETRO2_
 auto_c_000000002: +0.29 w12 +1 w21 +1 w22 -1 eMETRO2_ +1 eMETRO2__3 +1 eBUS_ -1 e
 auto_c_000000003: +1 w11 +0.71 w21 -1 w22 -1 w31 -1 w32 -1 w33 +1 eTAXI_ -1 eTAX
 auto_c_000000004: +1 w11 +1 w12 +1 w21 +1 w22 +1 w23 +1 w31 +1 w32 +1 w33  = 1
```

An optimal solution has been found of the LP with $\sigma = 0.05$. The objective was accomplished with $[min]F = \sum_{a \in A_R} \sigma(a) = 0$ and $w_{11} = 0.5$, $w_{22} = 0.05$, $w_{23} = 0.05$, $w_{33} = 0.4$.

```
Problem solved in 286 milliseconds
Optimal objective value = 0.0
w31 = 0.0
w11 = 0.5
w22 = 0.05
w33 = 0.4
w21 = 0.0
w32 = 0.0
w12 = 0.0
w23 = 0.05
```