Buying New Car

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Abstract

This document contains the problem of buying a new car from the summary of UTA. This document was made during my internship at LAMSADE in the summer of 2017.

A DM wants to buy a new car. The DM is interested only in the following criteria:

- price (in Euro)
- comfort (0, +, ++, +++) *0 being not comfortable and +++ very comfortable*
- safety $(1, 2, 3, 4, 5)$ *1 being not safety and 5 safe*

The evaluation of the previous criteria is presented in the following table:

Cars	Price	Comfort	Safety	Ranking of the DM
Nissan Sentra (ns)	17 000	+++	4	1
Citroen C4 (c4)	15 000	++	2	2
Peugeot 208 GT (p208)	25 000	+	3	3
Peugeot 308 berline (p308)	18 500	0	3	4

First of all, we should specify the scale [1] for each criteria.

- Price $\Rightarrow$ $[g_{1*}, g_1^*] = [25\,000, 20\,000, 15\,000]$
- Comfort $\Rightarrow$ $[g_{2*}, g_2^*] = [0, +, ++, +++]$
- Safety $\Rightarrow$ $[g_{3*}, g_3^*] = [1, 3, 5]$

According to this formula: $v(g(a)) = \sum_{i=1}^{n} v_i(g_i(a))$, the value of each alternative may be written:

- $v(g(ns)) = 0.4v_1(15\,000) + 0.6v_1(20\,000) + v_2(+++) + 0.5v_3(3) + 0.5v_3(5)$
- $v(g(c4)) = v_1(15\,000) + v_2(++) + 0.5v_3(1) + 0.5v_3(3) = v_1(15\,000) + v_2(++) + 0.5v_3(3)$
- $v(g(p208)) = v_1(25\,000) + v_2(+) + v_3(3) = v_2(+) + v_3(3)$
- $v(g(p308)) = 0.3v_1(15\,000) + 0.7v_1(20\,000) + v_2(0) + v_3(3) = 0.3v_1(15\,000) + 0.7v_1(20\,000) + v_3(3)$

We have that $v_1(25\,000) = v_2(0) = v_3(1) = 0$.
Since the marginal value $u_i(g_i)$ can be expressed in terms of variables w_{ij}: $u_i(g_i^j) = \sum_{t=1}^{j-1} w_{it}$, the value of each alternatie can be written:

- $v(g(ns)) = w_{11} + 0.4w_{12} + w_{21} + w_{22} + w_{23} + w_{31} + 0.5w_{32}$
- $v(g(c4)) = w_{11} + w_{12} + w_{21} + w_{22} + 0.5w_{31}$
- $v(g(p208)) = w_{21} + w_{31}$
- $v(g(p308)) = w_{11} + 0.3w_{12} + w_{31}$

For each pair of consecutive alternatives, we express the difference between them:

- $\Delta(ns, c4) = -0.6w_{12} + w_{23} + 0.5w_{31} + 0.5w_{32} - \sigma_{ns}^+ + \sigma_{ns}^- + \sigma_{c4}^+ - \sigma_{c4}^-$
- $\Delta(c4, p208) = w_{11} + w_{12} + w_{22} - 0.5w_{31} - \sigma_{c4}^+ + \sigma_{c4}^- + \sigma_{p208}^+ - \sigma_{p208}^-$
- $\Delta(p208, p308) = w_{21} - w_{11} - 0.3w_{12} - \sigma_{p208}^+ + \sigma_{p208}^- + \sigma_{p308}^+ - \sigma_{p308}^-$

Having $\delta = 0.05$, we can solve the following LP:

Objective:

$$Minimize \quad \sum_{a \in A} \sigma_a^+ + \sigma_a^- \tag{1}$$

Subject to:

$$\begin{cases} -0.6w_{12} + w_{23} + 0.5w_{31} + 0.5w_{32} - \sigma_{ns}^+ + \sigma_{ns}^- + \sigma_{c4}^+ - \sigma_{c4}^- \geq 0.05 \\ w_{21} - w_{11} - 0.3w_{12} - \sigma_{p208}^+ + \sigma_{p208}^- + \sigma_{p308}^+ - \sigma_{p308}^- \geq 0.05 \\ -0.1w_{12} - w_{21} - w_{22} - w_{23} - 0.5w_{32} - \sigma_{p308}^+ + \sigma_{p308}^- + \sigma_{ns}^+ - \sigma_{ns}^- \geq 0.05 \\ w_{11} + w_{12} + w_{21} + w_{22} + w_{23} + w_{31} + w_{32} = 1 \end{cases} \tag{2}$$

So by using the com.google.ortools library, we can solve the Linear Program above with $\sigma = 0.05$. This Linear Program solution is coded in Java class BuyingNewCar.
By executing the class BuyingNewCar, you will have the following result:

[1] the interval $[g_{i*}, g_i^*]$ is cut into equal intervals

```
Problem solved in 8 milliseconds
Optimal objective value = 0.0
w31 = 0.0
w11 = 0.0
w22 = 0.0
w21 = 0.15199999999999997
w32 = 0.5080000000000001
w12 = 0.3399999999999999
w23 = 0.0
ep308+ = 0.0
ep208- = 0.0
ep308- = 0.0
ep208+ = 0.0
ec4- = 0.0
ec4+ = 0.0
ens- = 0.0
ens+ = 0.0
```

An optimal solution is $w_{12} = 0.34$, $w_{21} = 0.152$, $w_{31} = 0.51$ with $\sum_{a\in A} \sigma_a^+ + \sigma_a^- = 0$. The utilities found for each alternative are as follows:

- $v(g(ns)) = 0.798$
- $v(g(c4)) = 0.747$
- $v(g(p208)) = 0.662$
- $v(g(p308)) = 0.62$

Those utilities are consistent with the DM's preference ranking.