Analyzing the choice of transportation

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Abstract

This document contains the problem of analysing the choice of transportation. This problem is from the chapter UTA Methods of the Book: Multiple Criteria Decision Analysis. This document was made during my internship at LAMSADE in the summer of 2017.

A DM wants to analyse the choice of transportation. The DM is interstered in the following criteria

1. price
2. time (min)
3. comfort (possibility to have a seat)

The evaluation of the previous criteria is presented in the following table:

Means of transportation	Price	Time	Comfort	Ranking of the DM
RER	3	10	+	1
METRO (1)	4	20	++	2
METRO (2)	2	20	0	2
BUS	6	40	0	3
TAXI	30	30	+++	4

DM's preferences: $RER \succ Metro1 \approx Metro2 \succ Bus \succ Taxi$

First of all, we should specify the scale [1] for each criteria.

- Price $\rightarrow$ $[30, 16, 2]$
- Time $\rightarrow$ $[40, 30, 20, 10]$
- Comfort $\rightarrow$ $[0, +, ++, +++]$

According to this formula: $v(g(a)) = \sum_{i=1}^{n} v_i(g_i(a))$, the value of each alternative may be written:

- $v[g(RER)] = 0.07v_1(16) + 0.93v_1(2) + v_2(10) + v_3(+)$
- $v[g(METRO1)] = 0.14v_1(16) + 0.86v_1(2) + v_2(20) + v_3(++)$
- $v[g(METRO2)] = v_1(2) + v_2(20) + v_3(0) = v_1(2) + v_2(20)$
- $v[g(BUS)] = 0.29v_1(16) + 0.71v_1(2) + v_2(40) + v_3(0) = 0.29v_1(16) + 0.71v_1(2)$
- $v[g(TAXI)] = v_1(30) + v_2(30) + v_3(+++) = v_2(30) + v_3(+++)$

We have that $v_1(30) = v_2(40) = v_3(0) = 0$.
Since the marginal value $u_i(g_i)$ can be expressed in terms of variables w_{ij}: $u_i(g_i^j) = \sum_{t=1}^{j-1} w_{it}$, the value of each alternative can be written:

- $v[g(RER)] = w_{11} + 0.93w_{12} + w_{21} + w_{22} + w_{23} + w_{31}$
- $v[g(METRO1)] = w_{11} + 0.86w_{12} + w_{21} + w_{22} + w_{31} + w_{32}$
- $v[g(METRO2)] = w_{11} + w_{12} + w_{21} + w_{22}$
- $v[g(BUS)] = w_{11} + 0.71w_{12}$
- $v[g(TAXI)] = w_{21} + w_{31} + w_{32} + w_{33}$

For each pair of consecutive alternatives, we express the difference between them:

- $\Delta(RER, METRO1) = 0.07w_{12} + w_{23} - w_{32} \geq \delta$
- $\Delta(METRO1, METRO2) = -0.14w_{12} + w_{31} + w_{32} = 0$
- $\Delta(METRO2, BUS) = 0.29w_{12} + w_{21} + w_{22} \geq \delta$
- $\Delta(BUS, TAXI) = w_{11} + 0.71w_{12} - w_{21} - w_{31} - w_{32} - w_{33} \geq \delta$

Having $\delta = 0.05$, we can solve the following LP:

Objective:

$$Minimize \sum_{a \in A} \sigma_a^+ + \sigma_a^- \tag{1}$$

Subject to:

$$\begin{cases} 0.07w_{12} + w_{23} - w_{32} - \sigma_{RER}^+ + \sigma_{RER}^- + \sigma_{METRO1}^+ - \sigma_{METRO1}^- \geq \delta \\ -0.14w_{12} + w_{31} + w_{32} - \sigma_{METRO1}^+ + \sigma_{METRO1}^- + \sigma_{METRO2}^+ - \sigma_{METRO2}^- = 0 \\ 0.29w_{12} + w_{21} + w_{22} - \sigma_{METRO2}^+ + \sigma_{METRO2}^- + \sigma_{BUS}^+ - \sigma_{BUS}^- \geq \delta \\ w_{11} + 0.71w_{12} - w_{21} - w_{31} - w_{32} - w_{33} - \sigma_{BUS}^+ + \sigma_{BUS}^- + \sigma_{TAXI}^+ - \sigma_{TAXI}^- \geq \delta \\ w_{11} + w_{12} + w_{21} + w_{22} + w_{23} + w_{31} + w_{32} + w_{33} = 1 \end{cases} \tag{2}$$

So by using the com.google.ortools library, we can solve the Linear Program above with $\sigma = 0.05$.
This Linear Program solution is coded in Java class ChoiceTransportation.
By executing the class ChoiceTransportation, you will have the following result:

[1] the interval $[g_{i*}, g_i^*]$ is cut into equal intervals

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Problem solved in 17 milliseconds
Optimal objective value = 0.0
w31 = 0.0
w11 = 0.5
w22 = 0.05
w33 = 0.4
w21 = 0.0
w32 = 0.0
w12 = 0.0
w23 = 0.05
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An optimal solution is $w_{11} = 0.5$, $w_{22} = 0.05$, $w_{23} = 0.05$, $w_{33} = 0.4$ with $\sum_{a \in A} \sigma_a^+ + \sigma_a^- = 0$. The utilities found for each alternative are as follows:

- $v(g(RER)) = 0.6$
- $v(g(METRO1)) = 0.55$
- $v(g(METRO2)) = 0.55$
- $v(g(BUS)) = 0.5$
- $v(g(TAXI)) = 0.4$

Those utilities are consistent with the DM's preference ranking.