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UTA Method

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July 16, 2017

Abstract

This report contains a summary about the variant of the UTA Method, UTASTAR. This document was made during my internship at LAMSADE in the summer of 2017.

Contents

Chapter 1

Introduction

In decision theory a basic problem involving several criterias, concerns the way that the final decision should be made. However, this problem is posed in the opposite way: assuming that the decision is given, how is it possible to find the rational basis for the decision being made? Or how is it possible to assess the model leading to exactly the same decision as the actual one or the most "similar" decision.

The following steps represent the methodology of decision-making problems:

1. object of the decision, definition of the set of potential actions and the determination of a problem statement
2. modeling a consistent family of criteria
3. defining a global preference model
4. decision-aid or decision support

Example: Buying a new car

Let's consider we are trying to figure out which car to buy. Using the methodology represented above, we state that the objective of this problem is **buying a new car**. After stating the objective, we can list the potential of actions that represent the list of cars that we may buy:

- Peugeot 208 GTi
- Nissan Sentra
- Citroen C4
- Peugeot 308 berline

After listing the list of potential cars, we can define a list of criteria that we will base our decision on. When defining the list of criteria you should always remember that the potential actions should be evaluated by those criterias, so the criteria must be easy to evaluate and should be logical. Let's say we will base our purchase on the following criteria:

- price (in Euro)
- comfort (0, +, ++, +++) *0 being not comfortable and +++ very comfortable*
- safety (1, 2, 3, 4, 5) *1 being not safety and 5 safe*

During the decision process, we will determinate the global preferences of the potential actions:

1. Citroen C4
2. Peugeot 208 GTi
3. Peugeot 308 berline
4. Nissan Sentra

Once the global preference is defined, we can start the decision support.

In this document, we will study a multi-criteria decision analysis mehtods UTA, a method proposed in 1982 that build a utility function based on the preference of the DM. More precisely we will conduct our research over the new version of UTA: UTASTAR. The method is illustrated on an example.

Chapter 2

UTA Method

One of the multi-criteria decision analysis methods is the UTA method, which was proposed by E. Jacquet-Lagrze and J. Siskos in 1982. This method is proposed by the Multi-Attribute Utility Theory (MAUT) that build a utility function based on the DM[1] preferences.

The UTA method is used to solve a multi-criteria problem by building a utility function based on the preferences of the DM and solving a linear program (LP). It adopt the aggregation-disaggregation principles: where the model is based on a given preferences.

The UTASTAR, a variant of the UTA method, has been considered a better algorithm than UTA. Better result were found using the UTASTAR algorithm. So this is why we will focus on this method rather than the UTA method.

The aim of the UTASTAR method is to estimate a set of additive utility functions which are as consistent as possible with the decision maker's preferences.

At the begining of the problem, the DM should present the following information

- rank of the actions
- give the criteria he want to base his decision on
- evaluate the action compared to the criterion

Once those information are presented, the UTASTAR algorithm can be executed.

[1] Decision Maker

2.1 Principles and Notation

Let's call $A = a, b, c, ...$ the set of potential actions and $g_1, g_2, g_3, ..., g_n$ the family of criteria. Where $g_i(a)$ represent the funtion of an action(alternative)a on the criteria g_i with $a \in A_R$.

We define g_{i*} as the least preferred criteria: $g_{i*} = min_{a \in A} g_i(a)$ and g_i^* as the most preferred criteria: $g_i^* = max_{a \in A} g_i(a)$. So the interval for each criteria g_i is: $[g_{i*}, g_i^*]$.

If we want to evaluate two actions, for example a and b, on only one criteria g_i we have the following relations:

$$\begin{cases} a \succ b \Leftrightarrow g_i(a) > g_i(b) & preference \\ a \sim b \Leftrightarrow g_i(a) = g_i(b) & indifference \end{cases} \tag{2.1}$$

The criteria aggregation model in UTASTAR has the following form:

$$v(g(a)) = \sum_{i=1}^{n} v_i(g_i(a)) \tag{2.2}$$

subject to normalization constraints:

$$\begin{cases} \sum_{i=1}^{n} v_i(g_i^*) = 1 \\ v_i(g_{i*}) = v_i(g_i^1) = 0, \forall i = 1, 2, ..., n \end{cases} \tag{2.3}$$

where $v_i, i = 1, 2, ..., n$ are non decreasing real valued function.

In UTASTAR we have

$$w_{ij} = v_i(g_i^{j+1}) - v_i(g_i^j) \geq 0 \quad \forall i \quad j \tag{2.4}$$

Which will allow us to write:

$$v_i(g_i^j) = \sum_{t=1}^{j-1} w_{it} \quad \forall i = 1, 2, ..., n \quad and \quad j = 2, 3, ..., \alpha_i - 1 \tag{2.5}$$

With the evaluation of an action a $g(a) = [g_1(a), g_2(a), ..., g_n(a)]$, we have the following relation:

$$\begin{cases} v[g(a)] > v[g(b)] \Leftrightarrow a \succ b \\ v[g(a)] \sim v[g(b)] \Leftrightarrow a = b \end{cases} \tag{2.6}$$

2.2 Development

The updated version of UTA, UTASTAR, propose a double error function for each action: $\sigma^{+}(a)$ and $\sigma^{-}(a)$. So the value of each alternative $a \in A_R$ can be written:

$$v^{'}[g(a)] = \sum_{i=1}^{n} v_i[g_i(a)] - \sigma^{+}(a) + \sigma^{-}(a) \quad \forall a \in A_R \tag{2.7}$$

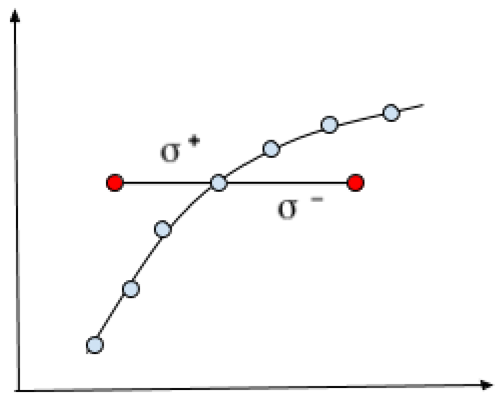

Figure 2.1: Error function in UTASTAR

For each criteria, the interval $[g_{i*}, g_i^*]$ is cut into $(\alpha_i - 1)$ equals interval, and the end points g_i^j are given by the formula:

$$g_i^j = g_{i*} + \frac{j-1}{\alpha_i - 1}(g_i^* - g_{i*}) \forall j = 1, 2, ..., \alpha_i \tag{2.8}$$

The marginal value of an action a is calculated by a linear interpolation

$$v_i[g_i(a)] = v_i(g_i^j) + \frac{g_i(a) - g_i^j}{g_i^{j+1} - g_i^j}[v_i(g_i^{j+1}) - v_i(g_i^j)] \tag{2.9}$$

The set of reference action $A_R = a_1, a_2, ..., a_m$ is arranged where a_1 is the best action and a_m is the worst action. Which indicate that we have two possible situations:

- $a_k \succ a_{k+1}$ preference
- $a_k \sim a_{k+1}$ indifference

So if we have that $\Delta(a_k, a_{k+1}) = v^{'}[g(a_k)] - v^{'}[g(a_{k+1})]$ and δ is a small positive number we will obtain the following relations:

$$\begin{cases} \Delta(a_k, a_{k+1}) \geq \delta \\ \Delta(a_k, a_{k+1}) = 0 \end{cases} \tag{2.10}$$

The marginal value functions are estimated by means of the Linear Programm with (2.2), (2.3), (2.10) as constraints, and an objective function depending on the σ^+ and σ^-:

$$[min]z = \sum_{k=1}^{m}[\sigma^+(a_k) + \sigma^-(a_k)]$$

subject to:

$$\begin{cases} \Delta(a_k, a_{k+1}) \geq \delta \quad or \quad \Delta(a_k, a_{k+1}) = 0 \\ \sum_{i=1}^{n}\sum_{j=1}^{\alpha_i - 1} w_{ij} = 1 \\ w_{ij} \geq 0, \quad \sigma^+(a_k) \geq 0, \quad \sigma^-(a_k) \geq 0, \quad \forall i, jandk \end{cases} \tag{2.11}$$

Chapter 3

Example

The implementation of UTASTAR algorithm is illustrated by the example from the Introduction chapter : **buying a new car**.
The DM is interested only in the following criteria:

- price (in Euro)
- comfort (0, +, ++, +++) *0 being not comfortable and +++ very comfortable*
- safety $(1, 2, 3, 4, 5)$ *1 being not safety and 5 safe*

The evaluation of the previous criteria is presented in the following table:

Cars	Price	Comfort	Safety	Ranking of the DM
Nissan Sentra (ns)	17 000	+++	4	1
Citroen C4 (c4)	15 000	++	2	2
Peugeot 208 GT (p208)	25 000	+	3	3
Peugeot 308 berline (p308)	18 500	0	3	4

First of all, we should specify the scale [1] for each criteria.

- Price $\Rightarrow$ $[g_{1*}, g_1^*] = [25\,000, 20\,000, 15\,000]$
- Comfort $\Rightarrow$ $[g_{2*}, g_2^*] = [0, +, ++, +++]$
- Safety $\Rightarrow$ $[g_{3*}, g_3^*] = [1, 3, 5]$

According to this formula: $v(g(a)) = \sum_{i=1}^{n} v_i(g_i(a))$, the value of each alternative may be written:

- $v(g(ns)) = 0.4v_1(15\,000) + 0.6v_1(20\,000) + v_2(+++) + 0.5v_3(3) + 0.5v_3(5)$

[1] the interval $[g_{i*}, g_i^*]$ is cut into equal intervals

- $v(g(c4)) = v_1(15\,000) + v_2(++) + 0.5v_3(1) + 0.5v_3(3) = v_1(15\,000) + v_2(++) + 0.5v_3(3)$
- $v(g(p208)) = v_1(25\,000) + v_2(+) + v_3(3) = v_2(+) + v_3(3)$
- $v(g(p308)) = 0.3v_1(15\,000) + 0.7v_1(20\,000) + v_2(0) + v_3(3) = 0.3v_1(15\,000) + 0.7v_1(20\,000) + v_3(3)$

We have that $v_1(25\,000) = v_2(0) = v_3(1) = 0$.
Since the marginal value $u_i(g_i)$ can be expressed in terms of variables w_{ij}: $u_i(g_i^j) = \sum_{t=1}^{j-1} w_{it}$, the value of each alternatie can be written:

- $v(g(ns)) = w_{11} + 0.4w_{12} + w_{21} + w_{22} + w_{23} + w_{31} + 0.5w_{32}$
- $v(g(c4)) = w_{11} + w_{12} + w_{21} + w_{22} + 0.5w_{31}$
- $v(g(p208)) = w_{21} + w_{31}$
- $v(g(p308)) = w_{11} + 0.3w_{12} + w_{31}$

For each pair of consecutive alternatives, we express the difference between them:

- $\Delta(ns, c4) = -0.6w_{12} + w_{23} + 0.5w_{31} + 0.5w_{32} - \sigma_{ns}^+ + \sigma_{ns}^- + \sigma_{c4}^+ - \sigma_{c4}^-$
- $\Delta(c4, p208) = w_{11} + w_{12} + w_{22} - 0.5w_{31} - \sigma_{c4}^+ + \sigma_{c4}^- + \sigma_{p208}^+ - \sigma_{p208}^-$
- $\Delta(p208, p308) = w_{21} - w_{11} - 0.3w_{12} - \sigma_{p208}^+ + \sigma_{p208}^- + \sigma_{p308}^+ - \sigma_{p308}^-$

Having $\delta = 0.05$, we can solve the following LP:

Objective:

$$Minimize \quad \sum_{a \in A} \sigma_a^+ + \sigma_a^- \tag{3.1}$$

Subject to:

$$\begin{cases} -0.6w_{12} + w_{23} + 0.5w_{31} + 0.5w_{32} - \sigma_{ns}^+ + \sigma_{ns}^- + \sigma_{c4}^+ - \sigma_{c4}^- \geq 0.05 \\ w_{21} - w_{11} - 0.3w_{12} - \sigma_{p208}^+ + \sigma_{p208}^- + \sigma_{p308}^+ - \sigma_{p308}^- \geq 0.05 \\ -0.1w_{12} - w_{21} - w_{22} - w_{23} - 0.5w_{32} - \sigma_{p308}^+ + \sigma_{p308}^- + \sigma_{ns}^+ - \sigma_{ns}^- \geq 0.05 \\ w_{11} + w_{12} + w_{21} + w_{22} + w_{23} + w_{31} + w_{32} = 1 \end{cases} \tag{3.2}$$

An optimal solution is $w_{12} = 0.34$, $w_{21} = 0.152$, $w_{31} = 0.51$ with $\sum_{a \in A} \sigma_a^+ + \sigma_a^- = 0$. The utilities found for each alternative are as follows:

- $v(g(ns)) = 0.798$
- $v(g(c4)) = 0.747$
- $v(g(p208)) = 0.662$
- $v(g(p308)) = 0.62$

Those utilities are consistent with the DM's preference ranking.

Chapter 4

Conclusion

The UTA method build a utility function based on the preferences of the DM and it consist in solving a linear program (LP) to solve a multi-criteria problem.

This method will elaborate a model of preferences which is as similiar as possible to the DM's preferences.

An improved version of the UTA is the UTASTAR. In UTA we used a single error $\sigma(a)$ in UTASTAR we use a double positive error function. The updated version has performed better than the regular method.

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