UTA Method

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Abstract

This report contains the summary of the chapter **UTA Methods** from the Book : Multiple Criteria Decision Analysis. This document was made during my internship at LAMSADE in the summer of 2017.

Contents

Chapter 1

Introduction

In 1982, E. Jacquet-Lagrze and J. Siskos proposed a decision mehtod called UTA. This method is proposed by the Multi-Attribute Utility Theory (MAUT) that build a utility function based on the DM[1] preferences.
Assuming that the decision is given, the UTA method will find the rational basis for the decision being made. Or how can we use the DM's preference leading to the exact same or "similar" decision.

The UTA method is used to solve a multi-criteria problem. It build a utility function based on the preferences of the DM and it consist in solving a linear program (LP).

UTA methods adpot the aggregation-disaggregation principles. The aggregation is where the model is known and the preferneces are unknown, while the disaggregation is where the model is based on a given preferences

[1]Decision Maker

Chapter 2

Method

2.1 Principles and Notation

2.1.1 Weighted form

A weighted form of the UTA Method has the following form:

$$u(g) = \sum_{i=1}^{n} p_i u_i(g_i) \tag{2.1}$$

subject to normalization constraints:

$$\begin{cases} \sum_{i=1}^{n} p_i = 1 \\ u_i(g_{i*}) = 0, u_i(g_{i*}) = 1, \forall i = 1, 2, ..., n \end{cases} \tag{2.2}$$

where $u_i, i = 1, 2, ..., n$ are non decreasing real valued functions, and p_i is the weight of u_i.

2.1.2 Unweighted form

An unweighted form of the UTA Method has the following form:

$$u(g) = \sum_{i=1}^{n} u_i(g_i) \tag{2.3}$$

subject to normalization constraints:

$$\begin{cases} \sum_{i=1}^{n} u_i(g_{i*}) = 1 \\ u_i(g_{i*}) = 0, \forall i = 1, 2, ..., n \end{cases} \tag{2.4}$$

2.2 Development

The value of each alternative $a \in A_R$ may be written as:

$$u^{'}[g(a)] = \sum_{i=1}^{n} u_i[g_i(a)] + \sigma(a) \forall a \in A_R \tag{2.5}$$

where $\sigma(a)$ is a potential error relative to $u^{'}[g(a)]$.

We use the linear interpolation to estimate the marginal value function. For each criteria, the interval $[g_{i*}, g_i^*]$ is cut into $(\alpha_i - 1)$ equals interval, and the end points g_i^j are given by the formula:

$$g_i^j = g_{i*} + \frac{j-1}{\alpha_i - 1}(g_i^* - g_{i*}) \forall j = 1, 2, ..., \alpha_i \tag{2.6}$$

The marginal value of an action a is calculated by a linear interpolation

$$u_i[g_i(a)] = u_i(g_i^j) + \frac{g_i(a) - g_i^j}{g_i^{j+1} - g_i^j}[u_i(g_i^{j+1}) - u_i(g_i^j)] \tag{2.7}$$

The set of reference action $A_R = a_1, a_2, ..., a_m$ is arranged where a_1 is the best action and a_m is the worst action. Which means :

$$\begin{cases} \Delta(a_k, a_{k+1}) \geq \delta \\ \Delta(a_k, a_{k+1}) = 0 \end{cases} \tag{2.8}$$

where $\Delta(a_k, a_{k+1}) = u^{'}[g(a_k)] - u^{'}[g(a_{k+1})]$ and δ is a small positive number.

The marginal value $u_i(g_i)$ must satisfy these constraints:

$$u_i(g_i^{j+1}) - u_(g_i^j) \geq s_i \forall j = 1, 2, ..., \alpha_i - 1, i = 1, 2, ..., n \tag{2.9}$$

where $s_i \geq 0$ and being indifference for each criteria.

The marginal value functions are estimated by means of the Linear Programm with (2.3), (2.4), (2.8), (2.9) as constraints, and an objective function depending on the $\sigma(a)$:

$$[min]F = \sum_{a \in A_R} \sigma(a)$$

subject to :

$$\begin{cases} \Delta(a_k, a_{k+1}) \geq \delta \quad or \quad \Delta(a_k, a_{k+1}) = 0 \\ u_i(g_i^{j+1}) - u_(g_i^j) \geq 0 \quad \forall \quad i \quad and \quad j \\ \sum_{i=1}^{n} u_i(g_i^*) = 1 \\ u_i(g_i^*) = 0 \quad u_i(g_i^j) \geq 0 \quad \sigma(a) \geq 0 \quad \forall a \in A_R \quad \forall i and j \end{cases} \tag{2.10}$$

The analysis found by this LP is a post-optimality analysis.

2.3 UTASTAR

The UTASTAR is an improved version of the UTA.

Chapter 3

Variants

Chapter 4

Applications and UTA-Based DSS

Bibliography

[1] Salvatore Greco, Matthias Ehrgott, Jose Rui Figueira *Multiple Criteria Decision Analysis*. State of the Art Surveys.