Appendix E: Other possibilities for the form of the observational data and causal hypotheses

A causal hypothesis is deemed **compatible** with a given distribution over observed variables if there is some causal model in the set defined by the causal hypothesis which yields this distribution,

$$H_G \quad \text{is compatible with} \quad D^{\text{obs}} \quad \text{iff} \quad \exists M \in H_G : P(\mathsf{ObservedNodes}[G]) \in D^{\text{obs}}, \tag{A-1}$$

where $P(\mathsf{ObservedNodes}[G])$ is fixed by the causal model M through Eqs. (Markov ??) and (MarkovObserved ??).

In broad strokes, the inflation DAG technique is a way of mapping a causal inference problem of this sort to a new such problem where the observational and causal inputs of the new problem are determined by the observational and causal inputs of the original problem and where compatibility between the observational and causal inputs of the original problem implies compatibility between the observational and causal inputs of the new problem. The technique is useful because, as we show, simple witnesses of incompatibility in the new problem yield nontrivial witnesses of incompatibility in the original problem.

The inflation DAG technique can accommodate many different forms for the observational constraints and for the causal hypothesis that appear in the original problem. It also imposes a restriction on the forms for the observational constraints and the causal hypothesis appearing in the new problem. It is therefore useful to pause and consider the range of possibilities for the two inputs of a causal decision problem.

Possibilities for the form of the observational constraints include a specification of:

O1 a joint distribution over the observed variables.

O2 a confidence interval around a joint distribution over the observed variables

O3 conditional independence relations among the observed variables

O3 marginals of the joint distribution for certain subsets of the observed variables

In addition to this sort of variety, one can imagine that the statistical dependences among a set of variables may be specified not by a joint distribution but by covariance matrices or the values of entropic quantities such as mutual information. Combinations of these possibilities are also possible.

The specification of the joint distribution, example O1, is the most restrictive form that the observational constraints can take. Specifying a region, example O2, provides a means of expressing uncertainty about the joint distribution. Specifying conditional independences, example O3, is the form of observational data that has been most thoroughly exploited in the development of tools for causal inference. If one specifies marginals, as in example O4, then the causal inference problem becomes a version of the marginals problem (described in the introduction), but where the space of joint distributions from which the marginals may arise is constrained by the causal hypothesis.

The inflation DAG technique can be applied for *any* of these types of observational constraints. Nonetheless. the particular concrete applications of the inflation DAG technique that we will describe in detail in this article will consider problems where the joint distribution is specified, i.e., constraints of the form of O1. The new causal inference problem to which this original problem is mapped by inflation, however, is one where the constraints concern marginals, i.e., they are of the form of O4.

It is also useful to exhibit some of the possibilities for the form that the causal hypothesis may take:

H1 the full set of causal models for a particular DAG G

H2 the full set of causal models for a particular DAG G excluding those that yield conditional independence relations beyond those implies by d-separation in G

H3 the set of causal models for a particular DAG G wherein there are constraints on the manner in which particular nodes causally depend on their parents and/or constraints on the cardinality of the set of values for particular latent variables

H4 the set of causal models for a particular DAG G wherein the manner in which a particular node causally depends on its parents is constrained to be equivalent, under some mapping between nodes, to the manner in which another node causally depends on its parents

In all cases, we have left implicit the trivial constraint that the causal hypothesis must include only those DAGs for which the set of observed nodes coincides with the set of variables described in the observational data.

The motivation for considering hypotheses of the form of H2 rather than H1 is the principle that a causal explanation should not be fine-tuned [reference][WoodSpekkens ?]. If, for some observational data, a causal hypothesis of type H1 is compatible but one of type H2 is not, then it means that the conditional independences in the observational data are not a consequence of the causal structure, but rather are a consequence of the choice of parameters. Such an explanation can be criticized on the grounds that it is fine-tuned. Finding a causal hypothesis of type H2 that is compatible with the observational data implies that one has a non-fine-tuned explanation of that data. The fine-tuning issue will come up in **??** where we discuss quantum causal models. Note, not there yet...

An example of a causal hypothesis of type H3 is that of an additive noise model: if an observed variable Y has an observed variable X and a latent variable U as parents, then the noise is deemed additive if $Y = \alpha X + \beta U$ for some scalars α and β [provide references]. Clearly, the conditional probability distributions $P_{Y|XU}$ that can be achieved in such an additive noise model are a subset of the valid conditional distributions. More general constraints on the conditional distributions have also been explored alongside constraints on the sizes of the latent variables [cite Lee-Spekkens].

A causal hypothesis of type H4 involves a novel and unusual sort of constraint, which has not, to our knowledge been studied previously. We include it on our list because it is the sort of causal hypothesis that appears in the new causal problem that is defined by our inflation DAG technique. Insofar as it is usually assumed that the manner in which one node in a DAG causally depends on its parents should be completely independent of the manner in which another depends on its parents—sometimes described as the assumption of *autonomy* of different causal mechanisms [Pearl]—it is quite conceivable that hypotheses of type H4 have *no* significance besides the role that they play in the inflation DAG technique.[12]

The inflation map takes a causal hypothesis of type H1 and maps it to a causal hypothesis of type H4.

[12] We could say something here about how this suggests that one does better to think of the inflation DAG in terms of counterfactuals and twin diagrams. .