# Proof Corrections for AN11411
## Causal Compatibility Inequalities Admitting of Quantum Violations in the Triangle Structure
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**Specific Questions and Comments to Address for This Paper:**

1. TODO: Email E. Wolfe

2. The authors agree completely with all associated modifications made to the article pursuant to the policy on claims of priority and novelty of contributions.

3. The authors have checked the proof completely and have no corrections to make regarding the removal of italics or boldface.

4. The authors approve of all substitutions made regarding the use of slash between words throughout the document.

5. Thank you for pointing out this one sentence paragraph. The proof matches exactly how the authors would/will correct this mistake.

6. Noted and approved.

7. Regarding figures 4, 5, 6 and 7, the incorrect figures were used in the proof. The discrepancy between the figure files and appearance in the submitted article were a result of using the a clipping/trimming option of `\includegraphics` command of the `graphicx` package in order to trim whitespace and superfluous figure titles. The usage of the this option is as follows:

   ```
   \includegraphics[trim={<left> <lower> <right> <upper>},clip]{figure_example.pdf}
   ```

   Attached to this proof correction are "corrected" figure replacements for figures 4, 5, 6 and 7 which have the cropping already applied. As such, the figures should now appear correctly in the article and will match the figure files exactly. The figure files are respectively named:

   ```
   figure_fritz_dist_plotted_bits.pdf
   figure_noisy_yet_non_local_fritz.pdf
   figure_plotted_dist_I_1_max_violation.pdf
   figure_plotted_dist_I_3_max_violation.pdf
   ```

   Regarding figures 4 and 5 only, the colored gradient legends have been removed and replaced with the explicit legends. In the original manuscript, these explicit legends were included adjacent to the figure as an equation. These legends are now a part of the attached figure files, as they should be.

8. The deletion of `\cdot` and addition of `\times` in equation (17) is appropriate.

9. Broyden-Fletcher-Goldfarb-Shanno is the correct "de-initialism" of BFGS.

10. Thanks for noticing that the original sentence was missing something. However, the use of "include" here would be inappropriate. Ultimately, our original definition was unnecessarily complicated and confusing. The authors recommend the following substitution: "...in fact Fig. 6 and Fig. 4 share their possibilistic structure. A distribution's possibilistic structure is the subset of events which it assigns a non-zero probability. That Fig. 6 and Fig. 4 have the same..."

11. This reference is correct and its link is accurate.

12. The reference for [13] is correct in the proof. Updated publishing information for [13]:

```
@inproceedings{evans2012graphical,
  author={R. J. Evans},
  booktitle={2012 IEEE International Workshop on Machine Learning for Signal Processing},
  title={Graphical methods for inequality constraints in marginalized DAGs},
  year={2012},
  volume={},
  number={},
  pages={1-6},
  doi={10.1109/MLSP.2012.6349796},
  ISSN={2378-928X},
  month={Sept},
}
```

13. Updated publishing information for [13]:

```
@article{Steudel_2010,
  title={Information-theoretic inference of common ancestors},
  author={Steudel, Bastian and Ay, Nihat},
  journal={Entropy},
  volume={17},
  number={4},
  pages={2304--2327},
  year={2015},
  url={http://www.mdpi.com/1099-4300/17/4/2304/htm},
  publisher={Multidisciplinary Digital Publishing Institute}
}
```

14. The reference for [20] is correct in the proof. Updated publishing information for [20]:

```
@article{Weilenmann_2016,
  title={Non-Shannon inequalities in the entropy vector approach to causal structures},
  author={Weilenmann, Mirjam and Colbeck, Roger},
  journal={Quantum},
  volume={2},
  pages={57},
  year={2018},
  url={https://quantum-journal.org/papers/q-2018-03-14-57/},
  publisher={Verein zur F{\"o}rderung des Open Access Publizierens in den Quantenwissenschaften}
}
```

15. Publishing information for [21] is accurate and up-to-date.

16. Publishing information for [22] is accurate and up-to-date.

17. Updated publishing information for [23]:

```
@article {evans2015margins,
  author = {Evans, Robin J.},
  title = {{Graphs for Margins of Bayesian Networks}},
  journal = {Scandinavian J. Stat.},
  volume = {43},
  number = {3},
  issn = {1467-9469},
  url = {https://arxiv.org/abs/1408.1809},
  doi.hide = {10.1111/sjos.12194},
  pages = {625--648},
  keywords = {causal model, directed acyclic graph, latent variable},
  year = {2016},
  note = {10.1111/sjos.12194},
}
```

18. Publishing information for [24] is accurate and up-to-date. Also appeared in Conference Proceedings in 2012:

```
@inproceedings{richardson2012nested,
  title={Nested Markov properties for acyclic directed mixed graphs},
  author={Richardson, Thomas S and Robins, James M and Shpitser, Ilya},
  booktitle={Proceedings of the Twenty-Eighth Conference on Uncertainty in Artificial Intelligence},
  pages={13--13},
  year={2012},
  organization={AUAI Press}
}
```

19. Publishing information for [29] is accurate and up-to-date.

20. Publishing information for [31] is accurate and up-to-date.

21. Publishing information for [34] is accurate, accessed today and available at:

    `https://wiki.math.ntnu.no/_media/ma2501/2014v/cholesky.pdf`

22. The reference for [47] is correct in the proof.

23. The reference for [48] is correct in the proof.

24. The reference link for [51] is no longer directed to the correct documentation page as version 6.5 is no longer supported. An updated link and reference for version 8.0 is:

```
@Misc{Gurobi_2016,
  author  = {{Gurobi Optimizer}},
  title   = {{FarkasDual}},
  year    = {2018},
  url     = {https://www.gurobi.com/documentation/8.0/refman/farkasdual.html},
  urldate = {2018-08-01},
}
```

25. The reference for [52] is correct in the proof.

26. The reference for [53] is correct in the proof.

27. The reference for [54] is correct in the proof.

FQ.

Q.
- The references for [6,15,24] are correct in the proof.
- These private communications should be dated: November 2016 for [27] and August 2017 for both [41] and [42].
- The reference for [59] is mostly correct; this technical report was "published" by the Cambridge University Engineering Dept. An access url is `https://infoscience.epfl.ch/record/169768/`.

**Other Corrections to Address for This Paper:**

- The justification of items within Tables I and II are inconsistent.

- In equation (C1), the addition of the `\times` is incorrect. The pertinent portion of equation (C1) is intended to be a comma separated list.

- Page 16 line 1097, there is a citation to [59] typo; the surrounding sentence should read "A popular tool for linear quantifier elimination is Fourier-Motzkin elimination [21,55,58]." with [59] omitted.

- Page 1 line 50, should read "...inequalities can be understood as consequences...". Currently, the word "be" is missing.

- Page 3 line 169, there is a math typo: the descendants of a node $\Delta[\mathcal{G}]n$ should be $\mathsf{De}_{\mathcal{G}}(n)$. Specifically, `\mathsf{De}_{\mathcal{G}}(n)` produces the desired effect.

- Page 3 line 198, there is a typo in an inline equation. The surrounding sentence should read: "...existence or non-existence of a set of causal parameters for $\mathcal{G}$ such that $P_{\mathcal{N}_O} = \sum_{n \notin \mathcal{N}_O} P_{\mathcal{N}}$ where $P_{\mathcal{N}}$ is again given by Eq. (1). The complementary...".

- Page 5 line 381, there is a small typo; "variable" should be "variables". The surrounding sentence should read: "...crucial distinction is that $S_A$ and $S_B$ are random variables representing...".

- Page 7 line 511, "there" should be omitted. "Second, it is known that the inequalities...".

- In several locations, the proof correctly replaces the usage of "Section" with its shorthand "Sec.". Should this correction also be applied to usage of "Appendix", i.e. "Appendix" becomes "App." where applicable?