

Erratum: Causal Compatibility Inequalities Admitting Quantum Violations in the Triangle Structure

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I. THE WAGON-WHEEL INEQUALITY

Our article claimed that inequality (12), reproduced here,

$$\begin{aligned}
 & I_{\text{WagonWheel}} : \\
 & +P_{A_l B_l}(11) - P_{A_l B_l C_l C_r}(1111) + P_{A_l B_l}(00)P_{C_l C_r}(11) + P_{C_l C_r}(01)P_{C_l C_r}(10) \\
 & - P_{C_l C_r}(11)P_{A_l A_r B_l B_r C_l C_r}(000000) - P_{C_l C_r}(11)P_{A_l A_r B_l B_r C_l C_r}(010100) \\
 & - P_{C_l C_r}(10)P_{A_l A_r B_l B_r C_l C_r}(001001) - P_{C_l C_r}(10)P_{A_l A_r B_l B_r C_l C_r}(011101) \\
 & - P_{C_l C_r}(01)P_{A_l A_r B_l B_r C_l C_r}(100110) - P_{C_l C_r}(01)P_{A_l A_r B_l B_r C_l C_r}(110010) \\
 & + P_{C_l C_r}(00)P_{A_l A_r B_l B_r C_l C_r}(101111) + P_{C_l C_r}(00)P_{A_l A_r B_l B_r C_l C_r}(111011) \\
 & \leq 0,
 \end{aligned} \tag{1}$$

is a valid causal compatibility inequality for the triangle structure. Additionally, we claimed that the distribution in equation (8), namely Fritz's distribution P_F , achieves a violation of the above inequality with a numerical value of $\frac{1}{16} \not\leq 0$. Both of these reported claims are false.

One avenue for correcting the above inequality would be to simply reverse the direction of the inequality sign ($\leq \rightarrow \geq$), or equivalently flip the sign of each of the coefficients. While the inequality resulting from such a correction would indeed be a valid causal compatibility inequality for the triangle structure, it has the undesirable feature of being *satisfied* by the Fritz's distribution P_F with a numerical value of $\frac{1}{16} \geq 0$.

Therefore, it is our recommendation to replace the incorrect inequality above with the following inequality:

$$\begin{aligned}
 & I_{\text{WagonWheel}} : \\
 & -P_{A_l B_l}(10) + P_{A_l B_l C_l C_r}(1010) - P_{A_l B_l}(01)P_{C_l C_r}(10) - P_{C_l C_r}(00)P_{C_l C_r}(11) \\
 & - P_{C_l C_r}(01)P_{A_l A_r B_l B_r C_l C_r}(100110) - P_{C_l C_r}(01)P_{A_l A_r B_l B_r C_l C_r}(110010) \\
 & + P_{C_l C_r}(00)P_{A_l A_r B_l B_r C_l C_r}(101111) + P_{C_l C_r}(00)P_{A_l A_r B_l B_r C_l C_r}(111011) \\
 & + P_{C_l C_r}(10)P_{A_l A_r B_l B_r C_l C_r}(001001) + P_{C_l C_r}(10)P_{A_l A_r B_l B_r C_l C_r}(011101) \\
 & + P_{C_l C_r}(11)P_{A_l A_r B_l B_r C_l C_r}(000000) + P_{C_l C_r}(11)P_{A_l A_r B_l B_r C_l C_r}(010100) \\
 & \leq 0.
 \end{aligned} \tag{2}$$

Note that inequality (2) is related to inequality (1) by (i) flipping the signs of every coefficient, and (ii) flipping the bit-values for both B_r and C_l . This substitute inequality is indeed a valid causal compatibility inequality for the triangle structure and moreover is violated by the Fritz distribution with a numerical violation of $\frac{1}{16}(\sqrt{2} - 1) \not\leq 0$.

It is also worth noting that the qualitative features of the wagon-wheel inequality discussed in the “Numerical optimization results” section of the published article were actually obtained from our proposed substitute inequality (2), not the incorrectly published inequality (1), and therefore those comments do not require any alteration. Inequality (1) was published in place of inequality (2) due to an error made by the first author during the final drafting stages of the manuscript.

II. FIGURE 1

The diagram appearing in Figure 1 of our article is incorrectly a duplicate of the diagram appearing in Figure 3. Instead, Figure 1 should have been the following:

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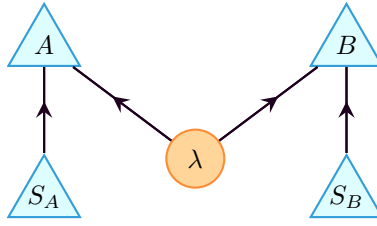


FIG. 1: The Bell structure consisting of two observers A, B together with measurement settings S_A and S_B respectively. The shared latent variable is labeled λ .

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