Gebze Technical University
Computer Engineering

MAT 214
Numerical Analysis
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HOMEWORK 01 REPORT

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Q1) Kod kısmı dosyanın içerisine ayrıca koyulmuştur. Çıktıları ise hem burada hem de outpur.txt içindedir.

A) 3x − e^x = 0 for 1 ≤ x ≤ 2

ITERATION	ABSOLUTE ERROR	RELATIVE ERROR
1	0.5000000000	0.3333333333
2	0.2500000000	0.1428571429
3	0.1250000000	0.0769230769
4	0.0625000000	0.0400000000
5	0.0312500000	0.0204081633
6	0.0156250000	0.0103092784
7	0.0078125000	0.0051813472
8	0.0039062500	0.0025839793
9	0.0019531250	0.0012903226
10	0.0009765625	0.0006455778
11	0.0004882813	0.0003228931
12	0.0002441406	0.0001614726
13	0.0001220703	0.0000807298
14	0.0000610352	0.0000403633
15	0.0000305176	0.0000201820
16	0.0000152588	0.0000100909
17	0.0000076294	0.0000076295

Approximate root is found 1.5121383667 after 17 iterations.
Theoretically required number of iterations is 17.

B) 2x + 3cos x − e^x = 0 for 0 ≤ x ≤ 1

ITERATION	ABSOLUTE ERROR	RELATIVE ERROR
1	0.5000000000	1.0000000000
2	0.2500000000	0.3333333333
3	0.1250000000	0.1428571429
4	0.0625000000	0.0666666667
5	0.0312500000	0.0322580645
6	0.0156250000	0.0158730159
7	0.0078125000	0.0078740157
8	0.0039062500	0.0039215686
9	0.0019531250	0.0019569472
10	0.0009765625	0.0009775171
11	0.0004882813	0.0004885198
12	0.0002441406	0.0002442002
13	0.0001220703	0.0001220852
14	0.0000610352	0.0000610389
15	0.0000305176	0.0000305185
16	0.0000152588	0.0000152590
17	0.0000076294	0.0000076295

After 17 iterations approximate root is found 0.9999923706.
Theoretically required number of iterations is 17.

C) x2 − 4x + 4 − ln x = 0 for 1 ≤ x ≤ 2

ITERATION	ABSOLUTE ERROR	RELATIVE ERROR
1	0.5000000000	0.3333333333
2	0.2500000000	0.2000000000
3	0.1250000000	0.0909090909
4	0.0625000000	0.0434782609
5	0.0312500000	0.0222222222
6	0.0156250000	0.0109890110
7	0.0078125000	0.0055248619
8	0.0039062500	0.0027700831
9	0.0019531250	0.0013831259
10	0.0009765625	0.0006910850
11	0.0004882813	0.0003456619
12	0.0002441406	0.0001728608
13	0.0001220703	0.0000864230
14	0.0000610352	0.0000432133
15	0.0000305176	0.0000216071
16	0.0000152588	0.0000108035
17	0.0000076294	0.0000054018

After 17 iterations approximate root is found 1.4123916626.
Theoretically required number of iterations is 17.

C) x2 − 4x + 4 − ln x = 0 for 2 ≤ x ≤ 4

ITERATION	ABSOLUTE ERROR	RELATIVE ERROR
1	1.0000000000	0.3333333333
2	0.5000000000	0.1428571429
3	0.2500000000	0.0769230769
4	0.1250000000	0.0400000000
5	0.0625000000	0.0204081633
6	0.0312500000	0.0103092784
7	0.0156250000	0.0051282051
8	0.0078125000	0.0025575448
9	0.0039062500	0.0012771392
10	0.0019531250	0.0006389776
11	0.0009765625	0.0003193868
12	0.0004882813	0.0001597189
13	0.0002441406	0.0000798658
14	0.0001220703	0.0000399313
15	0.0000610352	0.0000199653
16	0.0000305176	0.0000099825

After 16 iterations approximate root is found 3.0570983887.
Theoretically required number of iterations is 18.

D) x + 1 − 2 sin(πx) = 0 for 0 ≤ x ≤ 0.5

ITERATION	ABSOLUTE ERROR	RELATIVE ERROR
1	0.2500000000	1.0000000000
2	0.1250000000	1.0000000000
3	0.0625000000	0.3333333333
4	0.0312500000	0.1428571429
5	0.0156250000	0.0769230769
6	0.0078125000	0.0370370370
7	0.0039062500	0.0188679245
8	0.0019531250	0.0095238095
9	0.0009765625	0.0047393365
10	0.0004882813	0.0023752969
11	0.0002441406	0.0011862396
12	0.0001220703	0.0005927682
13	0.0000610352	0.0002962963
14	0.0000305176	0.0001481262
15	0.0000152588	0.0000740576
16	0.0000076294	0.0000370302
17	0.0000038147	0.0000185147
18	0.0000019073	0.0000092575

After 18 iterations approximate root is found 0.2060337067.
Theoretically required number of iterations is 16.

D) x + 1 − 2 sin(πx) = 0 for 0.5 ≤ x ≤ 1

ITERATION	ABSOLUTE ERROR	RELATIVE ERROR
1	0.2500000000	0.3333333333
2	0.1250000000	0.2000000000
3	0.0625000000	0.0909090909
4	0.0312500000	0.0476190476
5	0.0156250000	0.0232558140
6	0.0078125000	0.0114942529
7	0.0039062500	0.0057142857
8	0.0019531250	0.0028653295
9	0.0009765625	0.0014306152
10	0.0004882813	0.0007158196
11	0.0002441406	0.0003580380
12	0.0001220703	0.0001789869
13	0.0000610352	0.0000895015
14	0.0000305176	0.0000447487
15	0.0000152588	0.0000223749
16	0.0000076294	0.0000111873
17	0.0000038147	0.0000055936

After 18 iterations approximate root is found 0.6819725037.
Theoretically required number of iterations is 16.

Teorik olarak gereken iterasyon sayısını bulurken şu formüle kullandım.

$$|p_n - p| \leq \frac{b-a}{2^n}, \quad \text{when} \quad n \geq 1.$$

Bu yöntem sonucu elde ettiğim verilere göre bu yöntem çok yavaş bir yakınsaklığa sahip. Ama hatalı sonuç bulmuyor.

Q2) Solve exercise 5 section 2.2 and calculate the theoretical number of iterations required according to Corollary2.5.

Use a fixed-point iteration method to determine a solution accurate to within $\mathbf{10^{-2}}$ for $\boldsymbol{x^4 - 3\,x^2 - 3 = 0}$ on [1, 2]. Use $\boldsymbol{p_0 = 1}$

$x^4 = 3\,x^2 + 3$

$x = \sqrt[4]{3\,x^2 + 3} = \mathrm{g(x)}$

$p_1 = g(p_0)$ $\quad \mathrm{p_0} = 1$ verildi.

$p_1 = g(1) \quad = \sqrt[4]{6} \cong 1.56508458$

Formüle göre **|g'(x)| $\leq$ k $\leq$ 1** olmalıdır.

$\mathrm{g(x)} = \sqrt[4]{3\,x^2 + 3}$ ve $\mathrm{g'(x)} = \frac{9}{2}(x^3 + x) > 0$ $\quad$ x $\geq$ 0 için geçerlidir.

$|p_n - p_{n-1}| \leq \frac{\mathrm{k^n}}{1-\mathrm{k}} |\mathrm{p_0} - \mathrm{p_1}|$

k sayısını 0.4 alalım.

$\mathrm{g'(x)} = \frac{9}{2}(x^3 + x) \leq 0.4$ $\qquad$ x < 0.05 için

$0.01 \leq \frac{0.4^n}{1-0.4}$ | 1 – 1.53508458|

$0.0088482329 \leq 0.4^{\mathrm{n}}$

$\frac{\ln(0.0088482329)}{\ln(0.4)} \leq \mathrm{n}$

$5.159 \leq \mathrm{n}$

n = 6

Q3) Exercise 4.a - Let $f(x) = -x^3 - \cos x$. With $p_0 = -1$ and $p_1 = 0$, find p3.
(With Secant method)

Sekand iterasyonu şu şekilde tanımlanıştır.

$$\mathrm{p_n} = p_{n-1} - \frac{f(p_{n-1})(p_{n-1} - p_{n-2})}{f(p_{n-1}) - f(p_{n-2})}$$

Soruda verilenlere göre

$f(p_0) = f(-1) = 1 - \cos(-1)$ olur.

$f(p_1) = f(0) = -1$ olur.

Formüle göre;

$$p_2 = p_1 - \frac{f(p_1)(p_1 - p_0)}{f(p_1) - f(p_0)} = 0 - \frac{f(0) + (1+0)}{f(0) - f(-1)} = \frac{-cos0}{(-cos0) - (-(-1^3) - \cos(-1))}$$

$$= \frac{1}{-2 + \cos(-1)} = \frac{1}{-2 + 0.5403} \cong -0.6851$$

$$p_3 = p_2 - \frac{f(p_2)(p_2 - p_1)}{f(p_2) - f(p_1)} = -0.6851 - \frac{f(-0.6851)(-0.6851 - 0)}{f(-0.6851) - f(0)}$$

$$= -0.6851 - \frac{-0{,}6851(-(-0.6851)^3 - \cos(-0.6851))}{-(-0.6851)^3 - \cos(-0.6851) + \cos(0)}$$

$\cong -1.25208$

$f(-1.252) = 1.649$ sonucunu elde ederiz.

Q3) Exercise 4.b - Let $f(x) = -x^3 - \cos x$. With $p_0 = -1$ and $p_1 = 0$, find p3.
(With False Position method)

Bir önceki soruda sekant yöntemini kullanarak işlem yaparken $f(p2)f(p1) > 0$ olduğunu gözlemledik. Böylece sekant yönteminde kullandığımız p_2 ve p_0 ı false positon yöntemini uygulamak için kullanabiliriz yani p_3 ü $\left(p_0, f(p_0)\right)$ birleştiren çizginin kesişim noktası olarak seçelim ve $(p_2, f(p_2))$ deki indisleri p_0 ve p_1 olarak değiştirelim.

Böylelikle

$f(p_0) = f(-1) = 1 - \cos(-1)$ olur.

$f(p_1) = f(0) = -1$ olur.

Formüle göre;

$$p_2 = p_1 - \frac{f(p_1)(p_1 - p_0)}{f(p_1) - f(p_0)} = 0 - \frac{f(0) + (1 + 0)}{f(0) - f(-1)} = \frac{-cos0}{(-cos0) - (-(-1^3) - \cos(-1))}$$

$$= \frac{1}{-2 + \cos(-1)} = \frac{1}{-2 + 0.5403} \cong -0.6851$$

$$p_3 = p_2 - \frac{f(p_2)(p_2 - p_0)}{f(p_2) - f(p_0)} = -0.6851 - \frac{\left(\frac{-1}{\cos(-1) - 2^3)} - \cos\left(\frac{1}{\cos(-1) - 2}\right)\right)\left(\frac{1}{\cos(-1) - 2}\right)}{\frac{-1}{(\cos(-1) - 2)^3} - \cos\left(\frac{-1}{(cos(-1) - 2)^3}\right) + 1}$$

$\cong -0.841355$

Use Newton's method to find solutions accurate to within 10−4 for the following problems.

Q3) Exercise 5.a $\boldsymbol{x^3 - 2\,x^2 - 5 = 0\,,}$ **[1,4]**

Üstteki denklemi 0 yapan x değeri yaklaşık olarak <u>2.69065</u> **tir. Biz bunu 2 alalım ve newton methodunu uygulamaya başlayalım.**

$p_0 = 2$

$p_{n_{+1}} = p_n - \frac{f(p_n)}{f\prime(p_n)}$ ve $f'(x) = 3x^2 - 4x$

$p_1 = 2 - \frac{2^3 - 2*\,2^2 - 5}{3*2^2 - 4*2} = 2 + \frac{5}{4} = 3.25$

$p_2 = 3.25 - \frac{3.25^3 - 2*\,3.25^2 - 5}{3*\,3.25^2 - 4*3.25} \cong 2.8110$

$p_3 = 2.8110 - \frac{2.8110^3 - 2*\,2.8110^2 - 5}{3*\,2.8110^2 - 4*2.8110} \cong 2.69798$

$\mathrm{p}_4 = 2.69798 - \frac{2.69798^3 - 2*\,2.69798^2 - 5}{3*\,2.69798^2 - 4*2.69798} \cong 2.69067715$

Dolayısıyla mutlak hata $|p_4 - p| < 10^{-4}$ yani 4 iterasyon sonra istenilen çözüme ulaşır.

Q3) Exercise 5.b $\boldsymbol{x^3 + 3\,x^2 - 1 = 0}$ **, [-3, -2]**

Üstteki denklemi verilen aralıkta 0 yapan x değeri yaklaşık olarak - 2.8793852 **tir. Biz bunu -3 olarak alalım ve newton methodunu uygulamaya başlayalım.**

$p_0 = -3$

$p_{n_{+1}} = p_n - \frac{f(p_n)}{f\prime(p_n)}$ ve $f'(x) = 3x^2 + 6x$

$p_1 = (-3) - \frac{(-3)^3+3*(-3)^2-1}{3*(-3^2)+6*(-3)}$ ≅ -2.88888

$p_2 - 2.888 - \frac{(-2.888)^3 - 3* -2.888^2-1}{3* -2.888^2+6 * -2.888}$ ≅ -2.8794515

$p_3 = -2.8794 - \frac{(-2.8794)^3 - 3* -2.8794^2-1}{3* -2.8794^2+6 * -2.8794}$ ≅ -2.8793852

Dolayısıyla mutlak hata $|p_4 - p| < 10^{-4}$ yani 3 iterasyon sonra istenilen çözüme ulaşır.

Q3) Exercise 5.c x – cos(x)= 0 , [0, $\frac{\pi}{2}$]

Üstteki denklemi verilen aralıkta 0 yapan x değeri yaklaşık olarak $\underline{0.7391128909}$ **tir. Biz bunu 0 olarak alalım ve newton methodunu uygulamaya başlayalım.**

$p_0 = 0$

$p_{n_{+1}} = p_n - \frac{f(p_n)}{f\prime(p_n)}$ ve $f'(x) = 1 + \sin(x)$

$p_1 = 0 - \frac{0-\cos(0)}{1+\sin(0)} = 1$

$p_2 = 1 - \frac{1-\cos(1)}{1+\sin(1)} \cong 0.7503$

$p_3 = 0.7503 - \frac{0.7503-\cos(0.7503)}{1+\sin(0.7503)} \cong .73911289091136$

Dolayısıyla mutlak hata $|p_4 - p| < 10^{-4}$ yani 3 iterasyon sonra istenilen çözüme ulaşır.

Q3) Exercise 5.d x – 0.8 – 0.2sin(x) = 0 , [0, $\frac{\pi}{2}$]

Üstteki denklemi verilen aralıkta 0 yapan x değeri yaklaşık olarak
x = 0.9643338 **tür. Biz bunu 0 olarak alalım ve newton methodunu uygulamaya başlayalım.**

$p_0 = 0$

$p_{n_{+1}} = p_n - \frac{f(p_n)}{f\prime(p_n)}$ ve $f'(x) = 1 - 0.2\cos(x)$

$p_1 = 0 - \frac{0-0.8-0.2\sin(0)}{1-0.2\cos(0)} = 1$

$p_2 = 1 - \frac{1-0.8-0.2\sin(1)}{1-0.2\cos(1)} \cong 0.9644$

$p_3 = 9644 - \frac{9644-0.8-0.2\sin(9644)}{1-0.2\cos(9644)} \cong 0.9643338890103$

Dolayısıyla mutlak hata $|p_4 - p| < 10^{-4}$ yani 3 iterasyon sonra istenilen çözüme ulaşır.