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ID: 26493

Lecture Section: B

Sign: Elif

Recitation Section: B10

Question 1:

$\longrightarrow$ Transitive: for all $a, b, c \in R$, if $(a,b), (b,c) \in R$, then $(a,c) \in R$.

$\longrightarrow$ Asymmetry: for all $a, b \in A$, if $(a,b) \in R$ and $(b,a) \in R$, then $a = b$.

i) R is not transitive.

Counter example: $(a,b) = (0000\underline{1}00000, 0000\underline{1}00100) \in R$

↳ only 5^{th} bits are the same

AND

$(b,c) = (0000100\underline{1}00, 0000000\underline{1}00) \in R$

↳ only 8^{th} bits are the same.

$\Downarrow$

$(a,c) = (0000\underline{1}00\underline{0}00, 0000\underline{0}00\underline{1}00) \notin R$

↳ Neither 5^{th} nor 8^{th} are the same.

↳ $(a,c) \notin R$.

ii) R is not antisymmetric.

Counterexample: $(a,b) = (0000\underline{1}00001, 0000\underline{1}00000) \in R$

$(b,a) = (0000\underline{1}00000, 0000\underline{1}00001) \in R$

However, $a \neq b$ (Their 10^{th} bits are different)

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Question 2

A relation R on A is called equivalence relation if R is

- reflexive
- symmetric
- transitive

i) reflexive ✓

$(a,a) \in R$ for every element $a \in A$. Because $a \equiv a \pmod{13}$, $13 \mid a-a \Rightarrow 13 \mid 0$.

ii) symmetric ✓

if $(a,b) \in R$, then $(b,a) \in R$ for all $a,b \in A$. Because:

$(a,b) \in R \leftrightarrow a \equiv \pm b \pmod{13} \longleftrightarrow 13 \mid a-b$ OR $13 \mid a+b \leftrightarrow 13 \mid b-a$ OR $13 \mid b+a$

$\leftrightarrow b \equiv \pm a \pmod{13}$

iii) transitive ✓

for all $a,b,c \in R$, if $(a,b),(b,c) \in R$, then $(a,c) \in R$. Because:

$(a,b),(b,c) \in R \rightarrow a \equiv \pm b \pmod{13}$, $b \equiv \pm c \pmod{13}$ $\rightarrow 13 \mid a-b$, $13 \mid b-c$ $\rightarrow a-b = 13k$, $b-c = 13m$ $\rightarrow a-c = (a-b)+(b-c) = 13k + 13m = 13(k+m)$

$\rightarrow 13 \mid a-c \rightarrow a \equiv \pm c \pmod{13}$

(I've considered $a \equiv b \pmod{13}$, $b \equiv c \pmod{13}$ Because $\pm$ means OR.)

Equivalence class of 3:

$$[3]_R = \{ b \in \mathbb{Z} \mid (3,b) \in R \}$$
$$= \{ b \in \mathbb{Z} \mid 3 \equiv \pm b \pmod{13} \}$$
$$= \{ b \in \mathbb{Z} \mid 13 \mid 3 \pm b \}$$
$$= \{ \ldots, -23, -16, -10, -3, 3, 10, 16, 23, \ldots \}$$
$$= \{ 13m \pm 3 \mid m \in \mathbb{Z} \}$$

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Question 3: How many distinct simple graphs exists with 5 vertices $\{a,b,c,d,e\}$?

Solution: Out of 5 elements (vertices), we should choose 2 of them.
(Because an edge consists of two vertices)

$$\binom{5}{2} = \frac{5.4}{2} = 10 \longrightarrow E(G) \text{ should be subset of this set with 10 pairs.}$$

For each pair, we have 2 options: take it / leave it. $\Rightarrow$ 2^{10} many different simple graphs exist with 5 vertices.