7.1.2

a) ε-productions

$S \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$
$A \to C$
$B \to S \mid A$
$C \to S$

NOTE:

$C \to \varepsilon \Rightarrow A \to \varepsilon \Rightarrow B \to \varepsilon$

b) unit productions

$S \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$
$A \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$
$B \to 0A0 \mid 00 \mid 1B1 \mid 11 \mid BB$

NOTE:

$A \to C \to S \to \cdots$
$B \to S \to \cdots$
$B \to A \to C \to S \to \cdots$

c) useless states → remove S production.

d)
$A \to FD \mid DD \mid GE \mid EE \mid BB$
$B \to FD \mid DD \mid GE \mid EE \mid BB$
$D \to 0$
$E \to 1$
$F \to DA$
$G \to EB$

7.1.4

a) ε-productions

$S \to AAA$
$A \to aA \mid a$

b) unit productions → No!

c) useless states → No!

d)
$S \to CA$
$A \to DA \mid a$
$C \to AA$
$D \to a$

7.4.3

b) baaab

$S \to AB \to BAB \to bAB \to bBAB \to bCCAB$
$\qquad\qquad\qquad\qquad\qquad \downarrow$
$\qquad\qquad\qquad\qquad baaab \in L$

c) aabab

$S \to BC \to CCC \to aaAB \to aaBAB \to aabab \in L$

First question:
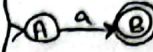
Right linear:
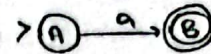
left linear:
• Take the reverse

$A \to aB$
$A \to \varepsilon$

• Draw

• Reverse

7.2.1

b) $\{a^n b^n c^i \mid i \le n\}$
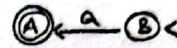
$z = a^{n+1} b^{n+1} c^n \in L \qquad z = uvwxy$

$|vwx| \le n$ so

i) $vwx = a^k$ or b^k or c^k where $0 < k \le n$

ii) $vwx = a^i b^J$ or $vwx = b^i c^J$ where $0 < i+J \le n$

According to PL:

$q := |vx| > 0$ so,

if (i) holds, $uwy = a^{n+1-q} b^{n+1} c^n \qquad n+1-q \ne n+1 \quad \notin L$

if (ii) holds, $uwy = a^{n+1-q_1} b^{n+1-q_2} c^n \notin L$ since $q_1 + q_2 = q > 0$

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c) $\{0^p \mid p \text{ is a prime}\}$

consider some prime $p \ge n+2$, Let $w = 1^p$

$z = uvwxy \nearrow \begin{array}{l} |vwx| \le n \\ |vx| > 0 \end{array}$

$|vwx| = m \qquad |vx| = q \qquad |uwy| = p-q$

$uv^i wx^i y = p-q + i \cdot q = p-q + (p-q)q = (q+1)(p-q)$
$\qquad\qquad\qquad\qquad\quad \underbrace{\qquad}_{i = p-q}$

$\downarrow$
it has
two factors
$\hookrightarrow$ not a prime → $\notin L$