Name: Elif Cemre Durgut

ID: 26493

Sign: Elif

c : cost
w : wage
p : probability
q : output

- PROBLEM SET 1 -

1) a+b)

High effort $\rightarrow$ $w^H = w_0 + c(e^H)$

$w^H = 0.1 + 0.1$

$\boxed{w^H = 0.2}$

Low effort $\rightarrow$ $w^L = w_0 + c(e^L)$

$w^L = 0.1 + 0$

$\boxed{w^L = 0.1}$

Expected profits:

High effort: $H = p^H \cdot q^H + (1-p^H)q^L - w^H$

$H = \frac{3}{4} \cdot 20 + \frac{1}{4} \cdot 1 - 0.2$

$H = 15.25 - 0.2 = 15.05$ //

Low effort: $L = p^H \cdot q^L + (1-p^H) \cdot q^H - w^L$

$L = \frac{3}{4} \cdot 1 + \frac{1}{4} \cdot 20 - 0.1$

$L = 5.75 - 0.1 = 5.65$ //

Conclusion: $15.05 > 5.65$. So, the principal will hire an agent with $\boxed{\text{high effort}}$ //

c) incentive compatibility constraint:

$H \geq L$

$p^H \cdot w^H + (1-p^H)w^L - c(e^H) \geq p^H \cdot w^L + (1-p^H)w^H - c(e^L)$

$\frac{3}{4} \cdot w^H + \frac{1}{4} \cdot w^L - 0.1 \geq \frac{3}{4} \cdot w^L + \frac{1}{4} \cdot w^H - 0$

$\frac{w^H}{2} \geq \frac{w^L}{2} + 0.1 \implies \boxed{w^H - w^L = 0.2}$

d) participation constraint:

$p^H \cdot w^H + (1-p^H) \cdot w^L - c(e^H) \geq w_0$

$\frac{3}{4} \cdot w^H + \frac{1}{4} \cdot w^L - 0.1 \geq 0.1$

$\frac{3}{4} w^H + \frac{1}{4} w^L \geq 0.2 \implies \boxed{3w^H + w^L = 0.8}$

e) $w^H - w^L = 0.2$

$+ \; 3w^H + w^L = 0.8$

$4w^H = 1$

$\rightarrow w^H = 0.25$ //

$\rightarrow w^L = 0.05$ //

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2) a)

Firm 1 : $\Pi_1 = (200-2(q_1+q_2))q_1 - 8q_1$

$\Pi_1 = 192q_1 - 2q_1^2 - 2q_1 \cdot q_2$

$\max(\Pi_1) = 192 - 4q_1 - 2q_2 = 0$

$\boxed{96 - 2q_1 - q_2 = 0}$

Firm 2 : $\Pi_2 = (200-2(q_1+q_2))q_2 - 10q_2$

$\Pi_2 = 190q_2 - 2q_1 \cdot q_2 - 2q_2^2$

$\max(\Pi_2) = 190 - 2q_1 - 4q_2 = 0$

$\boxed{95 - q_1 - 2q_2 = 0}$

b) $96 - 2q_1 - q_2 = 0$ substitute

$95 - q_1 - 2q_2 = 0 \Rightarrow q_1 = 95 - 2q_2$

$96 - 2(95 - 2q_2) - q_2 = 0$

$96 - 190 + 4q_2 - q_2 = 0$

$3q_2 = 94$

$q_2 = 31.\overline{3} \wedge q_1 = 32.\overline{3}$ → In fact, they should be integers!

market price at the equilibrium $\Rightarrow P = 200 - 2(31.\overline{3} + 32.\overline{3}) = 72.\overline{6}$

c) Price will be lower and quantities will be higher because there will be more competition.

Question 3 → next page!

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3)

		player 2	
	L	M	R
player 1 A	6,9	-5,0	0,0
B	9,-3	5,1	0,0
C	0,-4	0,0	1,5

a) There is no dominated strategy for player 2. Because:

if player 1 plays A : player 2 will play L

if " " B : " " " M

if " " C : " " " C

A is dominated strategy for player 1. Because:

if player 2 plays L : player 1 will play B

if " " " M : " " " " B

if " " " R : " " " " C

} player 1 will never play A. → we delete row A.

b)

	L	M	R
A	6,9	-5,0	0,0
B	9,-3	5,1	0,0
C	0,-4	0,0	1,5

Pure NE = $\{(B,M),(C,R)\}$

c)

	L → x	M → y	R → 1-x-y	player 1
p ← A	6,9	-5,0	0,0	⟷ $6x-5y$
q ← B	9,-3	5,1	0,0	⟷ $-3x+y$
1-p-q ← C	0,-4	0,0	1,5	⟷ $-4x+5-5x-5y$ $-9x-5y+5$
player 2	$9p-3q-4(1-p-q)$	q	$5(1-p-q)$	

$\underline{L=M}$

$9p-3q-4+4p+4q=q$

$13p+q-4=q$

$\boxed{p=\frac{4}{13}}$

$\underline{M=R}$

$q=5-5p-5q$

$6q=5-5p$

$6q=5-\frac{20}{13}$

$\boxed{q=\frac{15}{26}}$

$\underline{L=R}$

$13p+q-4=5-5p-5q$

$18p+6q=9$

$18\cdot\frac{4}{13}+6\cdot\frac{15}{26}=9$ ✓ it satisfied.

$\underline{A=B}$

$6x-5y=-3x+y$

$9x=6y$

$\underline{B=C}$

$-3x+y=5-9x-5y$

$6x+6y=5$

$6x+9x=5$

$15x=5$

$\boxed{x=\frac{1}{3}}$

$\boxed{y=\frac{1}{2}}$

$\underline{A=C}$

$6x-5y=-9x-5y+5$

$15x=5$

$x=\frac{1}{3}$ ← satisfied

Conclusion: There is a mixed strategy with

$p=4/13$ $q=15/26$

$x=1/3$ $y=1/2$