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Math 201 – Assignment 2 (Worksheet Week 9)

Problem 1:

a) $S=[(1,0,0),(0,1,0),(0,0,1)]$

$T=[(1,2,3),(0,1,4),(0,0,1)]$

$$[Id]_{T,S}=\left[\,[(1,0,0)]_T \;\vdots\; [(0,1,0)]_T \;\vdots\; [(0,0,1)]_T\,\right]$$

$(1,0,0)=a(1,2,3)+b(0,1,4)+c(0,0,1)$

$a=1$
$2a+b=0 \rightarrow b=-2$
$3a+4b+c=0 \rightarrow c=5$

$$\Rightarrow [(1,0,0)]_T=\begin{bmatrix}1\\-2\\5\end{bmatrix}$$

$(0,1,0)=a(1,2,3)+b(0,1,4)+c(0,0,1)$

$a=0$
$2a+b=1 \rightarrow b=1$
$3a+4b+c=0 \rightarrow c=-4$

$$\Rightarrow [(0,1,0)]_T=\begin{bmatrix}0\\1\\-4\end{bmatrix}$$

$(0,0,1)=a(1,2,3)+b(0,1,4)+c(0,0,1)$

$a=0$
$2a+b=0 \rightarrow b=0$
$3a+4b+c=1 \rightarrow c=1$

$$\Rightarrow [(0,0,1)]_T=\begin{bmatrix}0\\0\\1\end{bmatrix}$$

$$[Id]_{T,S}=\begin{bmatrix}1&0&0\\-2&1&0\\5&-4&1\end{bmatrix}_{3\times3} \;//$$

b) $[v]_T=[Id]_{T,S}\,[v]_S$

$$v=(3,-2,14) \longrightarrow [v]_S=\begin{bmatrix}3\\-2\\14\end{bmatrix}_{3\times1}$$

$$[v]_T=\begin{bmatrix}1&0&0\\-2&1&0\\5&-4&1\end{bmatrix}\begin{bmatrix}3\\-2\\14\end{bmatrix}=\begin{bmatrix}3\\-8\\37\end{bmatrix}_{3\times1} \;//$$

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math 201- Assignment 2 (worksheet week 8)

Problem 2:

Standard basis $\rightarrow$ $B = \left[\begin{pmatrix}1&0\\0&0\end{pmatrix}, \begin{pmatrix}0&1\\0&0\end{pmatrix}, \begin{pmatrix}0&0\\1&0\end{pmatrix}, \begin{pmatrix}0&0\\0&1\end{pmatrix}\right]$

$$T\left[\begin{pmatrix}1&0\\0&0\end{pmatrix}\right] = \begin{bmatrix}2&3\\0&4\end{bmatrix}\begin{bmatrix}1&0\\0&0\end{bmatrix} - \begin{bmatrix}1&0\\0&0\end{bmatrix}\begin{bmatrix}3&0\\0&k\end{bmatrix} = \begin{bmatrix}2&0\\0&0\end{bmatrix} - \begin{bmatrix}3&0\\0&0\end{bmatrix} = \begin{bmatrix}-1&0\\0&0\end{bmatrix}$$

$$T\left[\begin{pmatrix}0&1\\0&0\end{pmatrix}\right] = \begin{bmatrix}2&3\\0&4\end{bmatrix}\begin{bmatrix}0&1\\0&0\end{bmatrix} - \begin{bmatrix}0&1\\0&0\end{bmatrix}\begin{bmatrix}3&0\\0&k\end{bmatrix} = \begin{bmatrix}0&2\\0&0\end{bmatrix} - \begin{bmatrix}0&k\\0&0\end{bmatrix} = \begin{bmatrix}0&2-k\\0&0\end{bmatrix}$$

$$T\left[\begin{pmatrix}0&0\\1&0\end{pmatrix}\right] = \begin{bmatrix}2&3\\0&4\end{bmatrix}\begin{bmatrix}0&0\\1&0\end{bmatrix} - \begin{bmatrix}0&0\\1&0\end{bmatrix}\begin{bmatrix}3&0\\0&k\end{bmatrix} = \begin{bmatrix}3&0\\4&0\end{bmatrix} - \begin{bmatrix}0&0\\3&0\end{bmatrix} = \begin{bmatrix}3&0\\1&0\end{bmatrix}$$

$$T\left[\begin{pmatrix}0&0\\0&1\end{pmatrix}\right] = \begin{bmatrix}2&3\\0&4\end{bmatrix}\begin{bmatrix}0&0\\0&1\end{bmatrix} - \begin{bmatrix}0&0\\0&1\end{bmatrix}\begin{bmatrix}3&0\\0&k\end{bmatrix} = \begin{bmatrix}0&3\\0&4\end{bmatrix} - \begin{bmatrix}0&0\\0&k\end{bmatrix} = \begin{bmatrix}0&3\\0&4-k\end{bmatrix}$$

$$[T]_B = \begin{bmatrix}-1&0&3&0\\0&2-k&0&3\\0&0&1&0\\0&0&0&4-k\end{bmatrix}$$

Theorem: $T: V \to V$ is injective if and only if $[T]_B$ is invertible where B is any basis of V.

1) According to theorem, $[T]_B$ should be invertible.
So, columns should be linearly independent.

(i) $2-k \neq 0$

$k \neq 2$

if $k=2$, then

$C_2 = \begin{bmatrix}0\\0\\0\\0\end{bmatrix} \rightarrow$ L.D. X

(ii) $4-k \neq 0$

$k \neq 4$

If $k=4$, then

$C_4 = \begin{bmatrix}0\\3\\0\\0\end{bmatrix} \rightarrow$ can be written in terms of C_2.

L.D. X

$\rightarrow$ And also, when we try to solve $A.x=0$ for k values other than 2 and 4, it gives $[0,0,0,0] \Rightarrow$ trivial sol.

$\rightarrow$ injective

2) $\underbrace{\dim(\mathrm{Ker}(T))}_{0} + \underbrace{\dim(\mathrm{Im}(T))}_{4} = \underbrace{\dim(M_{2\times2}(\mathbb{R}))}_{4}$

$\mathrm{Im}(T) = M_{2\times2}(\mathbb{R}) \Rightarrow T$ is surjective

isomorphism $\rightarrow$ linear ✓, injective ✓, surjective ✓

$\boxed{\text{Answer} = k \in \mathbb{R} - \{2,4\}}$