

1.

a. Decimal: 370

Binary: $370 \div 2$

$$\begin{array}{r} 185 \\ 0 \\ 92 \\ 1 \\ 46 \\ 0 \\ 23 \\ 1 \\ 11 \\ 1 \\ 5 \\ 1 \\ 2 \\ 1 \\ 1 \\ 0 \end{array} \Rightarrow (101110010)_2$$

Octal: $\frac{101110010}{5 \ 6 \ 2} \Rightarrow (568)_8$

Hexadecimal: $\frac{101110010}{1 \ 7 \ 2} \Rightarrow (172)_{16}$

b. Decimal: 81.8125

Binary: $81 \div 2$

$$\begin{array}{r} 40 \\ 1 \\ 20 \\ 0 \\ 10 \\ 0 \\ 5 \\ 1 \\ 2 \\ 1 \\ 1 \\ 0 \end{array} \quad \begin{array}{l} 0.8125 \cdot 2 = 1.625 \\ 0.625 \cdot 2 = 1.25 \\ 0.25 \cdot 2 = 0.5 \\ 0.5 \cdot 2 = 1 \end{array} \Rightarrow (1010001.1101)_2$$

$\underbrace{1010001}_{1010001} \quad \underbrace{1101}_{1101}$

Octal: $\frac{1010001.1101}{1 \ 2 \ 1 \ 6 \ 4} \Rightarrow (121.64)_8$

Hexadecimal: $\frac{1010001.1101}{5 \ 1 \ 13} \Rightarrow (51.D)_{16}$

c. Decimal: 0.78125

Binary: $0.78125 \cdot 2 = 1.5625$

$$\begin{array}{l} 0.5625 \cdot 2 = 1.125 \\ 0.125 \cdot 2 = 0.250 \\ 0.250 \cdot 2 = 0.5 \\ 0.5 \cdot 2 = 1 \end{array} \Rightarrow (0.11001)_2$$

Octal: $0.\frac{11001}{6 \ 2} \Rightarrow (0.62)_8$

Hexadecimal: $0.\frac{11001}{12 \ 8} \Rightarrow (0.C8)_{16}$

2.

Decimal	Binary	Octal	Hexa
153	10011001	231	99
150	10010110	226	96
95	1011111	137	5f
114	1110010	162	72

3.

a. $[-2^{4-1}, 2^{4-1}-1] = [-8, 7]$

b.

	$\begin{array}{r} 0101 \\ 0010 \\ \hline 0111 \end{array}$	$\begin{array}{r} 0101 \\ 1110 \\ \hline 10011 \end{array}$	$\begin{array}{r} 1101 \\ 1111 \\ \hline 11100 \end{array}$	$\begin{array}{r} 1010 \\ 1001 \\ \hline 10011 \end{array}$	$\begin{array}{r} 0111 \\ 0110 \\ \hline 1101 \end{array}$	$\begin{array}{r} 0110 \\ 1010 \\ \hline 10000 \end{array}$
Overflow	NO	Yes	NO	Yes	Yes	Yes

4.

	integer part	fraction part
$\frac{1057}{2048} \cdot 2 = \frac{1057}{1024}$	1 (1)	$\frac{33}{1024}$
$\frac{33}{1024} \cdot 2 = \frac{33}{512}$	0 (2)	$\frac{33}{512}$
$\frac{33}{512} \cdot 2 = \frac{33}{256}$	0 (3)	$\frac{33}{256}$
$\frac{33}{256} \cdot 2 = \frac{33}{128}$	0 (4)	$\frac{33}{128}$
$\frac{33}{128} \cdot 2 = \frac{33}{64}$	0 (5)	$\frac{33}{64}$
$\frac{33}{64} \cdot 2 = \frac{33}{32}$	1 (6)	$\frac{1}{32}$
$\frac{1}{32} \cdot 2 = \frac{1}{16}$	0 (7)	$\frac{1}{16}$
$\frac{1}{16} \cdot 2 = \frac{1}{8}$	0 (8)	$\frac{1}{8}$

Real $\rightarrow \frac{1057}{2048} = 0.516113281$

8-bit fraction = $(0.1000100)_2$
 version
 \downarrow to decimal
 0.515625

Error = real - 0.515625 = 0.000488...

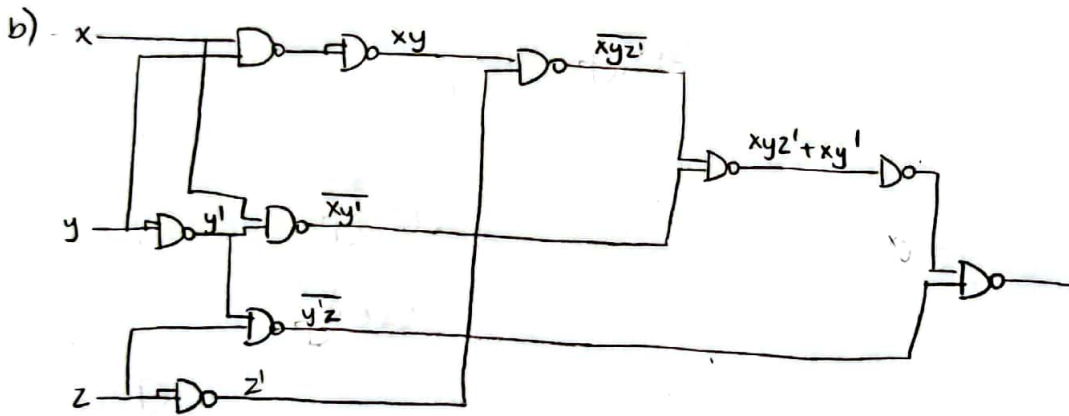
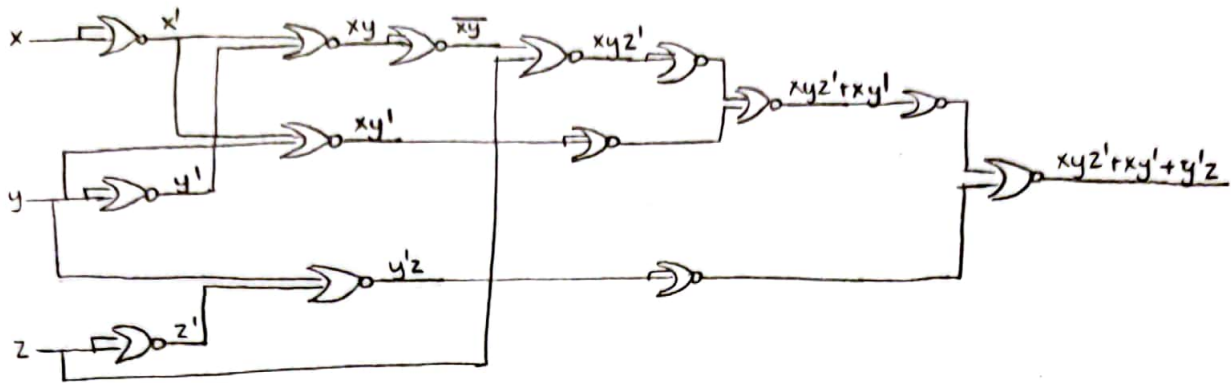
$\frac{1}{8} \cdot 2^3 = 1 \Rightarrow 3$ more bits (11 in total) is needed to fully represent.

Full version would be $(0.1000100001)_2$

5. $F(x, y, z, t) = x'y' + yz + xz' \xrightarrow{\text{canonical form}} = x'y'(z+z') + yz(x+x') + xz'(y+y')$
 $= \underbrace{x'y'z}_{001} + \underbrace{x'y'z'}_{000} + \underbrace{xyz}_{111} + \underbrace{xy'z}_{011} + \underbrace{xyz'}_{110} + \underbrace{xy'z'}_{100}$
 $= \sum(0, 1, 3, 4, 6, 7) \Rightarrow$ sum of minterms
 $= \prod(2, 5) \Rightarrow$ product of maxterms

6. $F(x, y, z) = xyz' + xy' + y'z$

a. I first drew it with AND and OR gates, then converted them to NOR gates.



7. $y + x'z = (x' + y)(y + z)$

x	y	z	$y + x'z$	$x' + y$	$y + z$	$(x' + y)(y + z)$
0	0	0	0	1	0	0
0	0	1	1	1	1	1
0	1	0	1	1	1	1
0	1	1	1	1	1	1
1	0	0	0	0	0	0
1	0	1	0	0	1	0
1	1	0	1	1	1	1
1	1	1	1	1	1	1

They are the same.

8. $(a + b + c')(b'c + a'c')$

$$= ab'c + \underbrace{a \cancel{a} c'}_0 + \underbrace{b \cancel{b} c'}_0 + a'bc' + \underbrace{b' \cancel{c} c'}_0 + \underbrace{a' c' \cancel{c}}_{c'}$$

$$= ab'c + a'bc' + a'c' = ab'c + a'c' //$$

$$a'c'(b + 1) = a'c'$$

9. $a'b'c + ab'c' + ab'c + a'b'c'$

$$a'b'(\underbrace{c + c'}_1) + ab'(\underbrace{c' + c}_1) = a'b' + ab'$$

$$= b'(a' + a) = b' //$$

13. According to the fact in Slide 2, p.33 :

$$F = \Sigma(2, 3, 6) = \Pi(0, 1, 4, 5, 7) //$$

10. $a + (b' + ce' + bd'e)$

Replace + with . and . with + for dual.

$$\begin{aligned} &= a \cdot (b' \cdot (c + e') \cdot (b + d' + e)) \\ &= a \cdot ((b'c + b'e') \cdot (b + d' + e)) \\ &= \underbrace{ab'c'b}_0 + ab'cd' + ab'ce + \underbrace{ab'e'b}_0 + \underbrace{ab'e'd'}_0 + \underbrace{ab'e'e}_0 \\ &= ab'(cd' + ce + e'd') // \end{aligned}$$

11. $E = a + (\bar{b} + c + \bar{c} \cdot \bar{d})$

$$\begin{aligned} \bar{E} &= \bar{a} \cdot (\bar{b} + c + \bar{c} \cdot \bar{d}) \\ &= \bar{a} \cdot (b \cdot \bar{c} \cdot (c + d)) \\ &= \bar{a} b \bar{c} (c + d) = \underbrace{\bar{a} b \bar{c} c}_0 + \bar{a} b \bar{c} d \\ &= a' b c' d // \end{aligned}$$

12. $a + a'(b'c + bde)$

