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Problem 1

1) Truth table of $P \vee Q \wedge R \Leftrightarrow Q \to \neg R$ ($\wedge$ has higher precedence than $\vee$)

P	Q	R	$\neg R$	$Q \to \neg R$	$Q \wedge R$	$P \vee Q \wedge R$	$P \vee Q \wedge R \Leftrightarrow Q \to \neg R$
T	T	T	F	F	T	T	F
T	T	F	T	T	F	T	T
T	F	T	F	T	F	T	T
T	F	F	T	T	F	T	T
F	T	T	F	F	T	T	F
F	T	F	T	T	F	F	F
F	F	T	F	T	F	F	F
F	F	F	T	T	F	F	F

2) x: All people

y: All people

$P(x,y)$: y is a child of x

$$\exists x \exists y \exists z \, (P(x,y) \wedge P(x,z)) \wedge \forall t \, \underbrace{((t \neq y \vee t \neq z) \to \neg P(x,z))}_{\text{it denotes exactly 2 child}}$$

x: malinda in this case.

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Problem 2:

It is uncountable because it is not possible to set one-to-one correspondence between S and $\mathbb{Z}^+$

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Problem 3:

1) minlcm = lcm(2,10) = 10
i = 1
J = 2
if (10<10) → false
J = 3
if (6<10), then minlcm = 6
J = 4
if (18<6) : false
J = 5
if (10<6) : false
i = 2
J = 3
if (30<6) : false
J = 4
if (90<6) : false
J = 5
if (10<6) : false
i = 3
J = 4
if (18<6) : false
J = 5
if (30<6) : false
i = 4
J = 5
if (45<6) : false

Final output ⇒ minlcm = 6 //

According to i:

2) 1^{st} iteration → n-1 comparisons

2^{nd} iteration → n-2 comparisons

⋮

$(n-1)^{th}$ iteration → 1 comparison

Total comparison: $1+2+3+\dots+(n-2)+(n-1)$

$$= \frac{(n-1).n}{2} \; //$$

3) $f(x) = \dfrac{x(x-1)}{2}$

$$|f(x)| = \left|\frac{x^2}{2} - \frac{x}{2}\right| \leq \left|\frac{x^2}{2}\right| + \left|\frac{x}{2}\right|$$

$$= \frac{x^2}{2} + \frac{1}{2}\underbrace{|x|}_{}$$

$x \leq x^2$ for $x \geq 1$

$$\leq \frac{x^2}{2} + \frac{x^2}{2} = x^2$$

$$|f(x)| \leq x^2$$

$f(x)$ is $O(x^2)$ with $C=1$ and $k=1$

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Problem 4:

gcd (36, 132) = ?

According to Euclidean Algorithm:

$132 = 36.3 + 24 \Rightarrow 24 = 132 - 3.36$

$36 = 24.1 + 12 \Rightarrow 12 = 36 - 1.24$

$24 = 12.2 + 0$

$gcd(132, 36) = 12$

If we apply backward substitution:

$12 = 36 - 1.24 = 36 - 1(132 - 3.36)$

$= 36 - 1.132 + 3.36$

$= 4.36 - 1.132$

$\boxed{12 = 4.36 - 1.132}$

$\boxed{s = 4}$ $\boxed{t = -1}$

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Problem 5:

b_n : number of all binary strings of length n which have even 0's and even 1's.

$\underbrace{\_\ \_\ \_\ \cdots\ \_}_{n \text{ positions}}$

case (i):

$\underbrace{\_\ \_\ \_\ \_\ \cdots}_{n-2}\ \underline{0}\ \underline{0}$ There exists b_{n-2} strings ending with 00

case (ii)

$\underbrace{\_\ \_\ \_\ \cdots}_{n-2}\ \underline{1}\ \underline{1}$ There exists b_{n-2} strings ending with 11

$$b_n = \begin{cases} 2b_{n-2} & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$$ with initial condition $b_2 = 2$

for all $n \geq 1$.

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Problem 6 :

Let the power of x denotes digits.

Then,

$$G(x) = (x^0 + x^1 + x^2 + x^3 + x^4 + x^5 + x^6 + x^7 + x^8 + x^9)^5 \rightarrow \text{we should find } x^{17}$$

$$G(x) = \left(\frac{1-x^{10}}{1-x}\right)^5 = (1-x^{10})^5 \cdot \frac{1}{(1-x)^5}$$

$$\boxed{\frac{1}{(1-x)^n} = \sum_{k=0}^{\infty} \binom{n+k-1}{k} \cdot x^k}$$

using this formula

$$= \left(\binom{5}{0}x^0 - \binom{5}{1}x^{10} + \binom{5}{2}x^{20} - \binom{5}{3}x^{30} + \binom{5}{4}x^{40} - \binom{5}{5} \cdot x^{50}\right) \sum_{k=0}^{\infty} \binom{4+k}{k} \cdot x^k$$

Now, we should find the terms include x^{17}.

$$\binom{5}{0} \cdot x^0 \cdot \binom{4+17}{17} \cdot x^{17} - \binom{5}{1} \cdot x^{10} \cdot \binom{4+7}{7} \cdot x^7$$

$$= x^{17}\left(\binom{21}{17} - 5 \cdot \binom{11}{7}\right) = x^{17}(5985 - 5.330)$$

$$= 4335 . x^{17}$$

$\downarrow$

There exists 4335 such integers. //

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Problem 7:

To be an equivalence relation; it should reflexive, symmetric and transitive.

i) Reflexive : $(a,a) \in R$ for all $a \in S$

The same simple graphs are isomorphic to each other. $\Rightarrow$ reflexivity ✓

(Their degree sequences and adjacency matrix are the same because they are the same graphs)

ii) Symmetric : if $(a,b) \in R$ then $(b,a) \in R$ for all $a,b \in S$

In isomorphism the order (G_1, G_2) or (G_2, G_1) does not matter.

If adjacency matrix of G_1 = Adjancency matrix of G_2

$\Downarrow$ then

Adjacency matrix of G_2 = Adjacency matrix of G_1

They are isomorphic $\Rightarrow$ Symmetry ✓

iii) Transitive: if $(a,b),(b,c) \in R$, then $(a,c) \in R$.

If adjacency matrix of G_1 = adjacency matrix of G_2

and

adjacency matrix of G_2 = adjacency matrix of G_3,

then

adjacency matrix of G_1 = adjacency matrix of G_3 based on bijection.

G_1 and G_3 are isomorphic $\Rightarrow$ transitivity ✓

By (i), (ii) and (iii); this is an an equivalence relation //

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Problem 8:

If H_1 and H_2 are Eulerian graphs, then their vertices have even degree.

To make H Eulerian graph, all vertices of H should have even degree, as well.

we need at least 4 edges because if we add

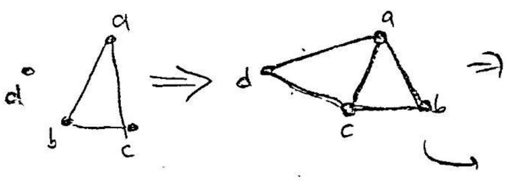
i) 1 edge, vertices' degrees are not even anymore

ii) 2 " " " "

iii) 3 " " " "

i) and iii) are obvious because they are odd and they break the evenness of a vertex.
(being false)

ii) adding 2 edges also breaks evenness. Because for ex;



$\Rightarrow$ a, b, c are not even any more

(a,d) and (d,c) are new edges.

So, we should add at least 4 edges

2 of them starts at the same vertex and ends

2 of them starts at another same vertex and ends.