Math 204 – Final – May 23, 2020

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Time allowed is 140 minutes. There are 8 problems worth a total of 80 points.

Please spend at least 5 minutes to read the following instructions.

Reminders:

1. You MUST write your Full name **with capital letters** on the top left corner of and student ID on top right corner with signature under it, on **EACH PAGE**. See the template attached. If you submit a page without your **name, student ID** and **signature**, then it will **NOT** be graded.

2. **Write solution of each exercise on a SEPARATE page. You must submit your solution on SUcourse in pdf format as a single file.**

3. You can use **CamScanner** (a free application) on your mobile device/tablet to scan the homework. It automatically creates one file in pdf format.

4. Next to every question, there is a suggested time given that is enough to solve that question.

5. You should not get any form of help from anyone else. In the suspicion of cheating/copying solutions from lecture notes/book/websites, the students will be asked to take an oral exam.

6. You should stop writing your solution at 11:20 am and start preparing the pdf file to submit on SUcourse. You have **40 minutes** to scan all the pages and upload them as a single pdf file on SUcourse. **Deadline to submit your solution is 12:00 pm.**

7. If you want to submit your solution after 12:00, then you must write an email to

   aqureshi@sabanciuniv.edu

   immediately explaining the reason that why you want to submit late. Please note that if you make submission after 12:00, then 20 points will be deducted from your final score. Note that IN ANY CASE, you CAN NOT make submission after 12:30.

You MUST show your work and give explanation for each solution.
Writing the final answer directly will not earn you any points.

Problem 1: (10 points)

1. (10 minutes) Write the truth table of

$$P \vee Q \wedge R \iff Q \to \neg R.$$

2. (5 minutes) Let $P(x, y)$ denote "y is a child of x". The domain for x and y is all people in the world. Translate the following statement into Logical expression by using suitable quantifier, logic operator and the predicate $P(x, y)$.

Malinda has exactly two children.

Problem 2: (10 points)

(15 minutes)An infinite ternary string is a string $a_1 a_2 a_3 \ldots$, where $a_i \in \{0, 1, 2\}$. Let S be the set of all infinite ternary strings. Is S countably infinite or uncountable? If it is countable infinite then give a precise one-to-one correspondence between S and Z^+. If it is uncountable then give a precise argument.

Problem 3: (15 points)

(20 minutes) Recall that for any positive integer $a, b \in \mathbb{Z}$, $lcm\ (a, b)$ denotes the least common multiple of a and b. Consider the following algorithm.

```
procedure minimumLCM (a_1, ..., a_n:  integers, n ≥ 2)
minlcm:=lcm (a_1, a_2)
for  i := 1 to n-1
for  j := i+1 to n
   if  ( (lcm (a_i, a_j) < minlcm ) then  minlcm := lcm (a_i, a_j)
return minlcm
```

1. Run the algorithm on the following input, stating the value of the minlcm at each iteration of **i** , and the final output of the algorithm.
   2 10 6 9 5

2. What is the total number of comparisons made in one run of the algorithm on an input of n integers?

3. Give a big-oh estimate for the time complexity of the algorithm in terms of the **comparisons** made.

Problem 4: (10 points)

(15 minutes) Use the **Euclidean Algorithm** to compute the integers s and t such that

$$36s + 132t = 12.$$

(Note: You need to write all steps of Euclidean Algorithm. Using calculator or applying brute force will not earn you any point.)

Problem 5: (10 points)

(20 minutes) For all $n \geq 0$, let b_n be the number of all binary strings of length n which have even number of 0's and even number of 1's. Find the recurrence relation for b_n with suitable initial conditions.

Problem 6: (10 points)

(15 minutes) How many 5 digit positive integers exist with sum of their digits equals to 17?

(Note: An example of a "5 digit positive integer" is 23891 and sum of the digits of 23891 is 2+3+8+9+1=23)

Problem 7: (8 points)

(15 minutes) Let $\mathcal{S}$ be the collection of all simple graphs. We define a relation R on S as "for any $G_1, G_2 \in S$, we have $(G_1, G_2) \in R$ if and only if G_1 is isomorphic to G_2". Show that R is an equivalence relation.

Problem 8: (7 points)

(15 minutes)Let H be a simple graph with exactly two connected components, namely, H_1 and H_2. If both H_1 and H_2 are Eulerian graphs, then compute the minimum number of edges that need to be added to H such that new graph becomes a simple Eulerian graph?

(Note: If your solution is based on some particular examples, then it will not earn you any points.)