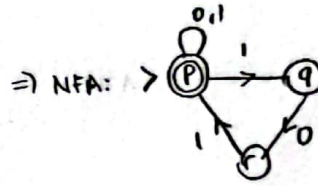1) $E = (1+ (0+101)^*)^* = (1^* + (0+101)^*)^*$

E-NFA:

$\Rightarrow$ NFA:

	0	1
→*P	P	P,q
P,q	P,r	P,q
P,r	P	P,q

P → A
Pq → B
Pr → C

DFA:

---
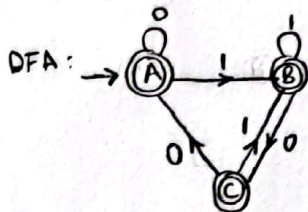
2) Eliminate 2:

Eliminate 3:

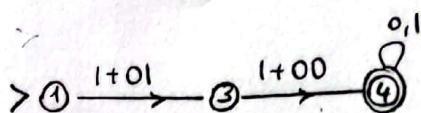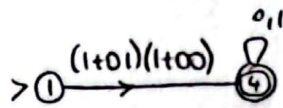$RE = (1+01)(1+00)(0+1)^*$

PUMPING LEMMA

3) a) i) Let N be the number of states of a DFA that accepts L

ii) Let's choose $w = 0^n.1.1.0^n$ $|w| = 2n+2 \geq n$
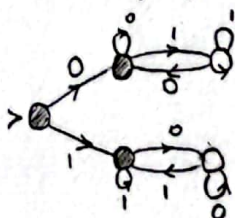
iii) $x.y = 0^p$

$y = 0^q$ s.t. $q > 0$

$x = 0^{p-q}$

iv) $xy^iz = 0^{p-q}.0^{qi}.0^{n-p}.1.1.0^n$

for $i = 0$: $xy^0z = 0^{n-q}.1.1.0^n \notin L$ because $n-q \neq n$ since $q > 0$.

→ This proves that the language is NOT regular by Pumping Lemma.

b) This language is regular and NFA can be written.

6)

4.1.2)

by Pumping Lemma:

b) Let N be the number of states of DFA that accepts L

$w = 0^{N^3}$

$w = x \cdot y \cdot z \qquad |w| = N^3$

$0 < |y| \le N \longrightarrow N^3 < |xyyz| < N^3 + N$

Let's see if $|xyyz|$ might be a perfect cube.

we know that 0^{N^3} is perfect cube and the next perfect cube would be:

$t = 0^{(N+1)^3} \qquad |t| = (N+1)^3 = N^3 + 3N^2 + 3N + 1$

$\downarrow$

this is greater than the upper limit for $|xyyz|$.

L is not a regular language

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c) by Pumping Lemma:

N : # of states of DFA

$w = 0^{2^n} = xyz$

$x = 0^p$

$y = 0^q \qquad p + q \le n, \ q > 0$

$xy^iz = 0^p \cdot 0^{qi} \cdot 0^{2^n - p - q} = 0^{2^n - q + qi}$

$\longrightarrow$

For this to be in L:

$2^n - q + qi$ must be a power of 2.

for $i = 2$

$2^n < 2^n + q \le 2^n + n < 2^n + 2^n = 2^{n+1} \implies 2^n < 2^n + q < 2^{n+1}$

$\qquad \downarrow \qquad\quad \downarrow \qquad\quad \downarrow \qquad\qquad\qquad\qquad \downarrow$

$\qquad q > 0 \quad p + q \le n \quad n < 2^n \qquad\qquad\qquad$ not in L.

next power of 2

$\uparrow$

$\downarrow$ not regular //

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h) $w \cdot 1^n$, $|w| = n$

by Pumping Lemma:

N : # of states of DFA

$w = 0^n . 1^n = xyz$

$xy = 0^p$

$y = 0^q, \ x = 0^{p-q}$ where $q > 0, p \le n$

$xy^iz = 0^{p-q} . 0^{qi} . 0^{n-p} . 1^n$

for $i = 0 \longrightarrow xz = 0^{n-q} . 1^n \longrightarrow n - q \ne n$ because $q > 0$

$\longrightarrow$ It is not in L $\longrightarrow$ not regular //