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Question 1:

Solution 1:

$$\underbrace{\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 4 \\ 1 & -1 & -1 \end{bmatrix}}_{A} \underbrace{\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}}_{x} = \underbrace{\begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix}}_{b}$$

To be able to use Cramer's Rule, matrix must be invertible. So, we should check invertibility of A using 8^{th} property det(A) should not be zero.

a) we can use short cut since A is 3x3 matrix:

$$\det(A) = (a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32}) - (a_{12}a_{21}a_{33} + a_{11}a_{23}a_{32} + a_{13}a_{22}a_{31})$$

$$= (2.2(-1) + (-1)4.1 + 1.1.(-1)) - ((-1).1.(-1) + 2.4.(-1) + 1.2.1)$$

$$= (-4-4-1) - (1-8+2) = -9 - (-5) = -9+5 = -4 \neq 0$$ So, A is invertible.

$\rightarrow$ Since A is invertible ($\det(A) \neq 0$), Cramer's Rule can be used to solve this linear system.

b) expansion along the 2^{nd} row

$$\det B_1 = \begin{vmatrix} 3 & -1 & 1 \\ 1 & 2 & 4 \\ 5 & -1 & -1 \end{vmatrix} \downarrow = \underset{\overset{\|}{1}}{a_{21}} C_{21} + \underset{\overset{\|}{2}}{a_{22}} C_{22} + \underset{\overset{\|}{4}}{a_{23}} C_{23}$$

$$\Rightarrow x_1 = \frac{\det B_1}{\det A} = \frac{-26}{-4} = \frac{13}{2}$$

$$= 1.(-1)^{2+1}.\det(m_{21}) + 2(-1)^{2+2}\det(m_{22}) + 4(-1)^{2+3}\det(m_{23})$$

$$= (-1).2 + 2.(-8) + 4(-1).2 = -16-8 = -26$$

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expansion along the 1^{st} row

$$\det B_2 = \begin{vmatrix} 2 & 3 & 1 \\ 1 & 1 & 4 \\ 1 & 5 & -1 \end{vmatrix} \downarrow = \underset{\overset{\|}{2}}{a_{11}} C_{11} + \underset{\overset{\|}{3}}{a_{12}} C_{12} + \underset{\overset{\|}{1}}{a_{13}} C_{13}$$

$$\Rightarrow x_2 = \frac{\det B_2}{\det A} = \frac{-23}{-4} = \frac{23}{4}$$

$$= 2.(-1)^{1+1}.\det(m_{11}) + 3(-1)^{1+2}.\det(m_{12}) + 1.(-1)^{1+3}.\det(m_{13})$$

$$= 2.(-21) + 3(-1).(-5) + 4 = -23$$

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expansion along the 1^{st} column

$$\det B_3 = \begin{vmatrix} 2 & -1 & 3 \\ 1 & 2 & 1 \\ 1 & -1 & 5 \end{vmatrix} \downarrow = \underset{\overset{\|}{2}}{a_{11}} C_{11} + \underset{\overset{\|}{1}}{a_{21}} C_{21} + \underset{\overset{\|}{1}}{a_{31}} C_{31}$$

$$\Rightarrow x_3 = \frac{\det B_3}{\det A} = \frac{17}{-4} = \frac{-17}{4}$$

$$= 2(-1)^{1+1}.\det(m_{11}) + 1.(-1)^{2+1}.\det(m_{21}) + 1.(-1)^{3+1}.\det(m_{31})$$

$$= 2.11 + (-1).(-2) + 1.(-7) = 17$$

c) verifying:

$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 2 & 4 \\ 1 & -1 & -1 \end{bmatrix} \begin{bmatrix} 13/2 \\ 23/4 \\ -17/4 \end{bmatrix} = \begin{bmatrix} 26/2 - 23/4 - 17/4 \\ 13/2 + 46/4 - 17 \\ 13/2 - 23/4 + 17/4 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \\ 5 \end{bmatrix} \checkmark$$

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Question 2:

Solution 2:

a) If $A \in M_{n\times n}(\mathbb{R})$ satisfies $A=-A^T$ and n is odd, then A is singular.

$\boxed{\text{True}}$

Some determinant properties
i) $\det(A)=\det(A^T)$ (from lecture)
ii) $\det(kA)=k^n \cdot \det(A)$ (from recitation)

$$A=-A^T \Rightarrow A^T=-A \Rightarrow \underbrace{\det(A^T)}_{\text{by i)}}=\det(-A) \Rightarrow \boxed{\det(A)=\det(-A)}$$

$$\underbrace{\det(-A)=(-1)^n\cdot\det(A)}_{n \text{ is odd}} \Rightarrow \boxed{\det(-A)=-\det(A)} \rightarrow \det(A)=-\det(A)$$

$$\Downarrow$$

$$\det(A)=0$$

A is not invertible (singular) //

b) If $A \in M_{n\times n}(\mathbb{R})$ satisfies $A=-A^T$ and n is even, then A is singular.

$\boxed{\text{False}}$ Counterexample: $A=\begin{bmatrix}0 & 1\\ -1 & 0\end{bmatrix}$

$\det(A)=0.0-(-1).1=1\neq 0 \rightarrow$ So, A is invertible (non-singular). //