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Recitation Section: A5

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Question 1: In a dormitory, we have 100 students. In a given night, on average % 20 of students are watching TV. Thus, we would always see 20 students at TV room (watching TV) for every night. Is this true?

Solution 1: Statement is false. Because 20 is the expected value, not a deterministic or certain value. It's just an expectation. So, we cannot expect to see 20 students for every night. X : # of students at TV room $E[X] = 20$.

(Also, some nights there can be midterms ☺)

Question 2: X and Y are continuous r.v.'s. Their joint p.d.f. is:

$$f(x,y) = \begin{cases} \frac{1}{5}(y+2), & 0 < y < 1, y-1 < x < y+1 \\ 0 & \text{otherwise} \end{cases}$$

a) Find marginal density of Y . b) Calculate $E[X|Y=0]$.

$$\begin{aligned} \text{a) } f_Y(y) &= \int_{x=y-1}^{y+1} \frac{1}{5}(y+2) dx = \frac{x}{5}(y+2) \Big|_{x=y-1}^{y+1} = \frac{(y+1)(y+2)}{5} - \frac{(y-1)(y+2)}{5} \\ &= \frac{y^2+3y+2 - (y^2+y-2)}{5} = \frac{2y+4}{5}, \quad 0 < y < 1 \end{aligned}$$

b) we need to find $f_{X|Y}(x|0)$.

$$f_{X|Y}(x|0) = \frac{f(x,0)}{f_Y(0)} = \frac{\frac{1}{5}(0+2)}{\frac{0+4}{5}} = \frac{2/5}{4/5} = \frac{1}{2}, \quad -1 < x < 1$$

Because y is given as 0.

$$E[X|Y=0] = \int_{x=-1}^1 x \cdot \frac{1}{2} dx = \frac{x^2}{4} \Big|_{x=-1}^1 = \frac{1}{4} - \frac{(-1)^2}{4} = \frac{1}{4} - \frac{1}{4} = 0 //$$