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Recitation section: B10

Question 10: show that for any $n \in \mathbb{Z}^+$, the numbers of positive divisors of n^2 is odd.

Let $n = p_1^{\ell_1} \cdot p_2^{\ell_2} \cdot p_3^{\ell_3} \cdot \dots \cdot p_m^{\ell_m}$ (p_i 's are prime numbers)

Then,

$$n^2 = p_1^{2\ell_1} \cdot p_2^{2\ell_2} \cdot p_3^{2\ell_3} \cdot \dots \cdot p_m^{2\ell_m}$$

Let $b \in \mathbb{Z}^+$ such that

$$b = p_1^{t_1} \cdot p_2^{t_2} \cdot p_3^{t_3} \cdot \dots \cdot p_m^{t_m}$$

$$b | n^2 \Leftrightarrow t_i \leq 2\ell_i$$

Let b a divisor of n^2 .

for $t_1 \rightarrow 2\ell_1 + 1$ choices

for $t_2 \rightarrow 2\ell_2 + 1$ choices

for $t_3 \rightarrow 2\ell_3 + 1$ choices

\vdots

for $t_m \rightarrow 2\ell_m + 1$ choices

$$\begin{array}{l} \text{numbers of} \\ \text{pos. divisors} \end{array} = \underbrace{(2\ell_1 + 1)}_{\text{odd}} \underbrace{(2\ell_2 + 1)}_{\text{odd}} \underbrace{(2\ell_3 + 1)}_{\text{odd}} \cdot \dots \cdot \underbrace{(2\ell_m + 1)}_{\text{odd}}$$

multiplication of odd numbers results in an odd number. (Because they do not include factor of 2.)

So, we showed that for any $n \in \mathbb{Z}^+$, the numbers of positive divisors of n^2 is odd.

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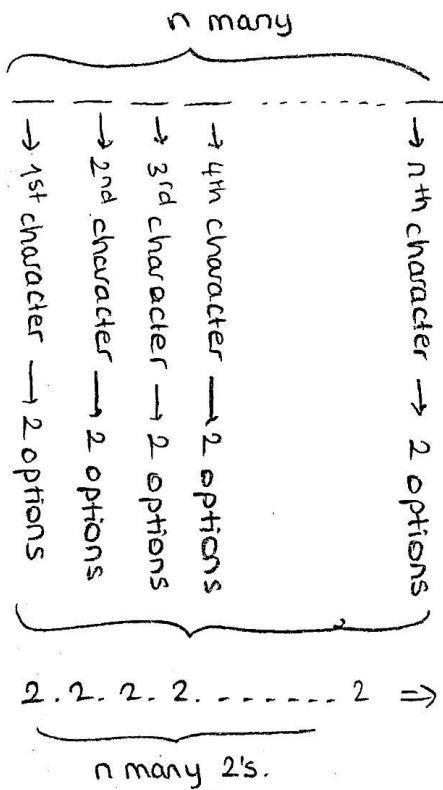
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Question 12: How many bit strings are there of length n ?



For every character in string, there are 2 options as 0 and 1.

\Rightarrow There exist 2^n bit strings of length n .

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Question 13: How many palindrome binary strings are there of length 9?

1. 2. 3. 4. 5. 6. 7. 8. 9.

To be a palindrome:

1. character = 9. character

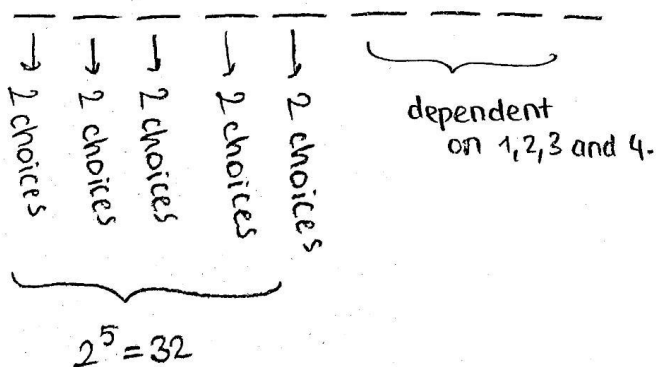
2. = 8.

3. = 7.

4. = 6.

5. is independent.

So, it is enough to calculate first five characters because remaining ones will be dependent on first four characters reverse respectively.



Answer: There are 32 palindrome binary strings of length 9.