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Homework 3

Question 1) Find the recurrence relation $a_n = 2a_{n-1} + 3^n$ with initial condition $a_1 = 5$.

Solution: The associated homogeneous recurrence relation of above recurrence relation is

i) $a_n = 2a_{n-1}$ with characteristic equation $r - 2 = 0 \Rightarrow r = 2$ (degree 1) $\downarrow$ characteristic root.

Then, $\boxed{a_n^{(h)} = \alpha . 2^n}$ for $\alpha \in \mathbb{R}$

ii) To find $a_n^{(p)}$: $F(n) = 3^n$

$a_n^{(p)} = c.3^n$, $c \in \mathbb{R}$

We know that:

$a_n = 2a_{n-1} + 3^n$

$c.3^n = 2(c.3^{n-1}) + 3^n$

for $n=1 \rightarrow c.3 = 2(c.3^0) + 3^1$

$3c = 2c + 3$

$c = 3 \longrightarrow \boxed{a_n^{(p)} = 3.3^n = 3^{n+1}}$

iii) $\boxed{a_n = a_n^{(h)} + a_n^{(p)}}$

$a_n = \alpha . 2^n + 3^{n+1}$

using the initial condition $a_1 = 5$:

for $n=1 \rightarrow a_1 = \alpha.2 + 3^2 = 2\alpha + 9 = 5$

$\alpha = -2$

Answer: $\boxed{a_n = -2^{n+1} + 3^{n+1}}$ //

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Homework 3

Question 2) Find a closed form for the generating function of the sequence

$$a_n = n-1 \text{ for } n=0,1,2,\dots$$

Solution

$$G(x) = \sum_{i=0}^{\infty} (i-1)x^i = \sum_{i=0}^{\infty} i.x^i - \sum_{i=0}^{\infty} x^i$$

$$= \frac{x}{(1-x)^2} - \frac{1}{1-x}$$

$$= \frac{x}{(1-x)^2} - \frac{1-x}{(1-x)^2}$$

$$= \frac{x-(1-x)}{(1-x)^2}$$

$$= \frac{x-1+x}{(1-x)^2}$$

$$= \frac{2x-1}{(1-x)^2}$$

Basic property of formal power series

- $\sum_{i=0}^{\infty} i.x^i = \frac{x}{(1-x)^2}$
- $\sum_{i=0}^{\infty} x^i = \frac{1}{1-x}$ for $|x|<1$

Answer: $\boxed{G(x) = \frac{2x-1}{(1-x)^2}}$