CS 303 Homework 2

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Q1)

Decimal	Binary	Hexadecimal
546	1000100010	222
128	$(10000000)_2$	$(80)_{16}$
27.375	$(11011.011)_2$	$(1B.6)_{16}$

$546 \mid 2 \rightarrow 0,\ 273 \mid 2 \rightarrow 1,\ 136 \mid 2 \rightarrow 0,\ 68 \mid 2 \rightarrow 0,\ 34 \mid 2 \rightarrow 0,\ 17 \mid 2 \rightarrow 1,\ 8 \mid 2 \rightarrow 0,\ 4 \mid 2 \rightarrow 0,\ 2 \mid 2 \rightarrow 0,\ 1$

$546 \mid 16 \rightarrow 2,\ 34 \mid 16 \rightarrow 2$

$128 \mid 2 \rightarrow 0,\ 64 \mid 2 \rightarrow 0,\ 32 \mid 2 \rightarrow 0,\ 16 \mid 2 \rightarrow 0,\ 8 \mid 2 \rightarrow 0,\ 4 \mid 2 \rightarrow 0,\ 2 \mid 2 \rightarrow 0,\ 1$

$128 \mid 16 \rightarrow 0,\ 8$

$27 \mid 2 \rightarrow 1,\ 13 \mid 2 \rightarrow 1,\ 6 \mid 2 \rightarrow 0,\ 3 \mid 2 \rightarrow 1,\ 1$

$(11011.011)_2 \rightarrow 1\ 11011.011 \rightarrow 1\ \ 11\ \ 6$

$0.375 \times 2 = 0.750 \quad (0)$

$0.750 \times 2 = 1.5 \quad (1)$

$0.5 \times 2 = 1 \quad (1)$

same

-6.375 $\quad 6 \mid 2 \rightarrow 0,\ 3 \mid 2 \rightarrow 1,\ 1$ $\quad 0.375$

$6.375 \Rightarrow 00000100.0110$

$\rightarrow$ complement $\rightarrow 11111011.1001$

$\rightarrow$ add 1 to LSB (0.0001) $\rightarrow 11111011.1010$

- $\underbrace{00000011}_{3}.\underbrace{1101}_{1\cdot\frac{1}{2}+1\cdot\frac{1}{4}+1\cdot\frac{1}{16}=\frac{13}{16}} \Rightarrow 3.8125$ //

- $11111111.1111 \xrightarrow{\text{subtract 1}} 11111111.1110 \xrightarrow{\text{complement}} 00000000.0001 = \frac{1}{16} \Rightarrow -0.0625$ //

- $10100110.0111 \xrightarrow{\text{subtract 1}} 10100110.0110 \xrightarrow{\text{complement}} \underbrace{01011001}_{89}.\underbrace{1001}_{\frac{1}{2}+\frac{1}{16}=\frac{9}{16}} \Rightarrow -89.5625$ //

Q2

$\frac{127}{64} = 1\frac{63}{64}$

$\Rightarrow 1.\underbrace{111111}$

$\frac{63}{64} \cdot 2 = \frac{63}{32}$ (1)

$\frac{31}{32} \cdot 2 = \frac{31}{16}$ (1)

$\frac{15}{16} \cdot 2 = \frac{15}{8}$ (1)

$\frac{7}{8} \cdot 2 = \frac{7}{4}$ (1)

$\frac{3}{4} \cdot 2 = \frac{3}{2}$ (1)

$\frac{1}{2} \cdot 2 = 1$ (1)

6 bits are needed to represent the fraction part in binary.

Q3

a)

```
  111
  00111001
+ 00111001
----------
  01111010
```

⇓ No overflow

b)

```
  11111111
  00000001
+ --------
  100000000
```

⇓ No overflow

c)

00111001

↳ 2's complement: 11000110 + 1

```
  11000110
+        1
----------
  11000111
```

```
  1111111
  00111001
  11000111
+ --------
  100000000
```

⇓ No overflow

d)

11111111

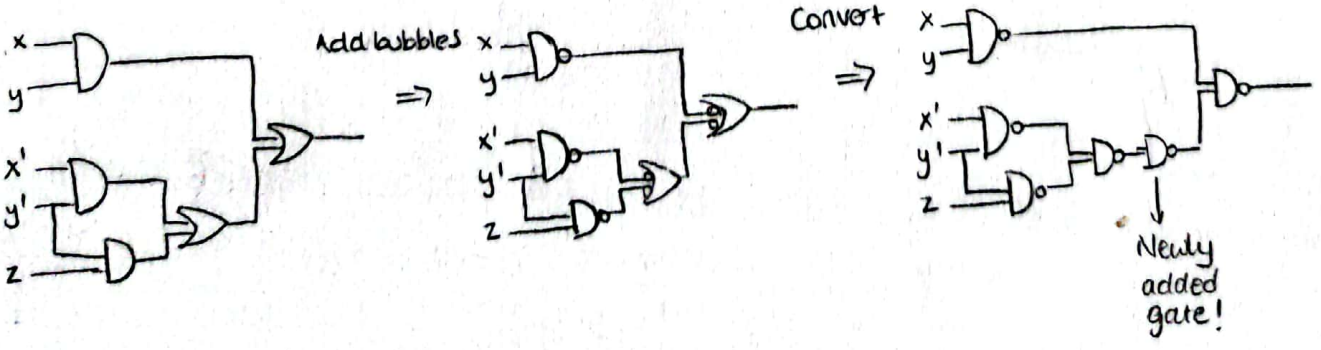
↳ 2's complement: 00000000 + 1

```
  00000000
+        1
----------
  00000001
```

```
  00000000
  00000001
+ --------
  00000001
```

⇓ No overflow

Q4 $F = xy + x'y' + y'z$ (I used only 2-input gates)



Q5

a) $F = (x \oplus y)(z+t)$

$x \oplus y = x'y + xy'$

$F = (x'y + xy')(z+t) = x'yz + x'yt + xy'z + xy't$

$= x'yzt + x'yzt' + x'yzt + x'yz't + xy'zt + xy'zt' + xy'zt + xy'z't$

0111	0110	0111	0101	1011	1010	1011	1001
7	6	7	5	11	10	11	9

$F = \sum(5,6,7,9,10,11)$ //

b) $F = \prod(0,1,2,3,4,8,12,13,14,15)$ //

c)

xy \ zt	00	01	11	10
00				
01		1	1	1
11				
10		1	1	1

$\Rightarrow F = x'yt + x'yz + xy't + xy'z$.

↳ It's still the same. Can not be simplified.

Q6

a)

a_1	a_0	b_1	b_0	d_1	d_0
0	0	0	0	—	—
0	0	0	1	0	0
0	0	1	0	0	0
0	0	1	1	0	0
0	1	0	0	—	—
0	1	0	1	0	0
0	1	1	0	0	1
0	1	1	1	0	1
1	0	0	0	—	—
1	0	0	1	0	0
1	0	1	0	0	0
1	0	1	1	1	0
1	1	0	0	—	—
1	1	0	1	0	0
1	1	1	0	0	1
1	1	1	1	0	0

b) for d1 :

a_1a_0 \ b_1b_0	00	01	11	10
00	x	0	0	0
01	x	0	0	0
11	x	0	0	0
10	x	0	1	0

$$F(d_1) = a_1a_0'b_1b_0$$

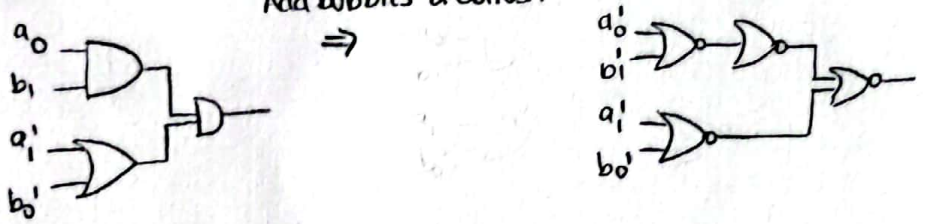
for d0 :

a_1a_0 \ b_1b_0	00	01	11	10
00	x	0	0	0
01	x	0	1	1
11	x	0	0	1
10	x	0	0	0

$$F(d_0) = a_1'a_0b_1 + a_0b_1b_0'$$

c) $F(d_0) = a_1'a_0b_1 + a_0b_1b_0' = a_0b_1(a_1' + b_0')$

Add bubbles & convert ⇒



Q7

$a_2a_1a_0$ \ $b_2b_1b_0$	000	001	011	010	110	111	101	100
000	0	0	0	0	0	0	0	0
001	1	0	0	0	0	0	0	0
011	1	1	0	1	0	0	0	0
010	1	1	0	0	0	0	0	0
110	1	1	1	1	0	0	1	1
111	1	1	1	1	0	0	1	1
101	1	1	1	1	0	0	0	0
100	1	1	1	1	0	0	0	0

$$F(g) = a_2b_2' + a_2'a_1b_2'b_1' + a_2a_1b_2b_1' + a_2'a_0b_2'b_1'b_0' + a_2a_1a_0b_1b_0' + a_2a_0b_2b_1'b_0' + a_2'a_1a_0b_2'b_1b_0'$$