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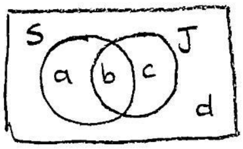
Question 1:

S: Buying second-hand car

J: Buying Japanese car



$d = 0.55$

$a+b = 0.25$ $\Rightarrow$

$b+c = 0.30$

$a+b+c = 0.45$ → Because total must be 1.

$\boxed{\begin{array}{l} a = 0.15 \\ b = 0.10 \\ c = 0.20 \\ d = 0.55 \end{array}}$

a) $P(S \cap J) = P(b) = 0.10 = \%10$ //

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b) $P(J/S) = ?$ $\quad P(J/S) = \dfrac{P(J \cap S)}{P(S)} = \dfrac{0.10}{0.25} = \dfrac{10}{25} = \dfrac{2}{5}$ //

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c) Independency check → $P(S \cap J) = P(S) \times P(J)$ this equation should satisfy.

$P(S) = a+b = 0.25$, $\quad P(S \cap J) = 0.10$ (from part a)

$P(J) = b+c = 0.30$

→ $P(S \cap J) = P(S).\ P(J)$

$0.10 \neq 0.25 \times 0.30$ → They are not equal.

So, S and J events are not independent. (They are dependent.)

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Question 2:

a) Experiments

Inventory → A : inventory is 1, A' : inventory is 2

Demand → B : inventory ≥ demand, B' : inventory < demand

Question gives us:

X : the number demands

- $P(A) = P(A') = 1/2$
- $P(B' \backslash A) = f(2) + f(3) = (P(X>1))$
  $= 0.4 + 0.2 = 0.6$
- $P(B' \backslash A') = f(3) = (P(X>2))$
  $= 0.2$

Question → $P(B') = ?$

$$P(B') = P(B' \cap A) + P(B' \cap A')$$
$$= P(B' \backslash A).P(A) + P(B' \backslash A').P(A')$$
$$= 0.6 \times \frac{1}{2} + 0.2 \times \frac{1}{2} = 0.3 + 0.1 = 0.4$$ //

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b) X : # of remaining cameras . Find p.d.f. → $X \in \{0, 1, 2\}$

Experiments

Inventory:
A : inventory is 1
A' : inventory is 2

Demand:
B = demand is 0
C = " " 1
D = " " 2
E = " " 3

$f(2) = P(B \backslash A') = P(B) = 0.1$ ($P(B \backslash A') = P(B)$: because question says that they are independent)

$f(1) = P(C \backslash A') + P(B \backslash A) = P(C) + P(B) = 0.1 + 0.3 = 0.4$

$f(0) = 1 - (f(2) + f(1)) = 1 - 0.1 - 0.4 = 0.5$

x	0	1	2
f(x)	$\frac{5}{10}$	$\frac{4}{10}$	$\frac{1}{10}$

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Question 3

Experiments

Type of exam — First question

E: Hard — F: Difficult marked

E': Easy — F': Not difficult

Question gives us:

$P(E) = 0.80$

$P(E') = 0.20$

$P(F \backslash E) = 0.90$

$P(F \backslash E') = 0.15$

a) $P(F) = ?$

$$P(F) = P(F \cap E) + P(F \cap E')$$
$$= P(F \backslash E).P(E) + P(F \backslash E').P(E')$$
$$= 0.90 \times 0.80 + 0.15 \times 0.20 = 0.72 + 0.03$$
$$= 0.75 \text{ //}$$

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b) $P(E \backslash F) = ?$

$$P(E \backslash F) = \frac{P(E \cap F)}{P(F)} = \frac{P(F \backslash E).P(E)}{P(F)} = \frac{0.72}{0.75} = \frac{24}{25} \text{ //}$$

we found it in part a.

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Question 4:

A: total winning. (winning - loss)

a) $P(A \geq 500) = ?$

	A:	
10 times winning	$\rightarrow$ 1000 dollars	So, we need to win at least 8 games.
9 " "	$\rightarrow$ 900-150 = 750 dollars	
8 " "	$\rightarrow$ 800-300 = 500 dollars	

We should use binomial distribution because there are two outcomes, success probability is constant and number of game is given.

$b(x;n,\theta) = \binom{n}{x} \theta^x (1-\theta)^{n-x}$ X: the number of games I won

$$P(X \geq 8) = P(X=8) + P(X=9) + P(X=10)$$

$$= b(8;10,0.6) + b(9;10,0.6) + b(10;10,0.6)$$

$$= \binom{10}{8} \cdot (0.6)^8 \cdot (0.4)^2 + \binom{10}{9} (0.6)^9 \cdot (0.4)^1 + \binom{10}{10} (0.6)^{10} \cdot (0.4)^0 //$$

b) X: the number of games I won

$$Gain(x) = 100x - 150(10-x) = 100x - 1500 + 150x = 250x - 1500$$

Q: $E(Gain(x)) = ?$

$$E(Gain(x)) = E(250x - 1500) = 250 \cdot E[x] - 1500 = 250 \cdot 6 - 1500$$

$$= 0 //$$

$E[x] = n \cdot \theta = 10 \times 0.6 = 6$

from binomial distribution

In the long run, I expect to earn 0 $ dollars.

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Question 5:

Poisson distribution $\rightarrow$ $p(x;\lambda) = \dfrac{\lambda^x \cdot e^{-\lambda}}{x!}$

X: # of policies sold by salesman

a) $P(X \geq 1) = ?$

$P(X \geq 1) = 1 - P(X=0)$

$= 1 - p(0;4) = 1 - \dfrac{4^0 \cdot e^{-4}}{0!} = 1 - e^{-4}$ //

b) So, salesman should sell 0 policies during 2 weeks.

Question: $P(X=0) = ?$

$P(X=0) = p(0;8) = \dfrac{8^0 \cdot e^{-8}}{0!} = e^{-8}$ //

(This time $\lambda=8$, because if the average is 4 for one week, then for two weeks, it makes 8.)

1 week $\rightarrow$ $\lambda=4$
2 week $\rightarrow$ $\lambda=8$

c) 1 week $\rightarrow$ 5 days $\rightarrow$ $\lambda=4$
1 day $\rightarrow$ $\lambda = 4/5$

Question: $P(X=1) = ?$

$P(X=1) = p(1;\tfrac{4}{5}) = \dfrac{\left(\frac{4}{5}\right)^1 \cdot e^{-\left(\frac{4}{5}\right)}}{1!} = \dfrac{4}{e} \cdot e^{-4/5}$ //

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Question 6:

$$f(x) = \begin{cases} \frac{2}{5}(x+2) & \text{if } 0<x<1 \\ 0 & \text{otherwise} \end{cases}$$

a) $$P(X<\tfrac{1}{2}) = \int_0^{1/2} \frac{2}{5}(x+2)\,dx = \frac{2}{5}\int_0^{1/2}(x+2)\,dx = \frac{2}{5}\cdot\left(x^2+2x\Big|_0^{1/2}\right)$$

$$= \frac{2}{5}\left(\frac{1}{4}+1-(0+0)\right)$$

$$= \frac{2}{5}\cdot\frac{5}{4} = \frac{1}{2}$$

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b) $E[X] = ?$ $Var[X] = ?$

$$E[x] = \int_x x f(x)$$

$$E[x] = \int_0^1 x\cdot\frac{2}{5}(x+2)\,dx = \int_0^1 \frac{2x^2}{5}+\frac{4x}{5}\,dx = \left(\frac{2x^3}{15}+\frac{2x^2}{5}\right)\Bigg|_0^1 = \frac{2}{15}+\frac{2}{5}-0-0 = \frac{8}{15}$$

$$E[x^2] = \int_0^1 x^2\cdot\frac{2}{5}(x+2)\,dx = \int_0^1 \frac{2}{5}x^3+\frac{4}{5}x^2\,dx = \left(\frac{2x^4}{20}+\frac{4x^3}{15}\right)\Bigg|_0^1 = \frac{1}{10}+\frac{4}{15}-0-0 = \frac{11}{30}$$

$$Var[x] = E[x^2]-(E[x])^2$$

$$= \frac{11}{30}-\left(\frac{8}{15}\right)^2 = \underset{(15)}{\frac{11}{30}} - \underset{(2)}{\frac{64}{15.15}} = \frac{165-128}{450} = \frac{37}{450}$$