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Recitation section: B10

Question 10: Show that for any  $n \in \mathbb{Z}^+$ , the numbers of positive divisors of  $n^2$  is odd.

$$\text{Let } n = p_1^{l_1} \cdot p_2^{l_2} \cdot p_3^{l_3} \cdot \dots \cdot p_m^{l_m} \quad (p_i \text{'s are prime numbers})$$

Then,

$$n^2 = p_1^{2l_1} \cdot p_2^{2l_2} \cdot p_3^{2l_3} \cdot \dots \cdot p_m^{2l_m}$$

Let  $b \in \mathbb{Z}^+$  such that

$$b = p_1^{t_1} \cdot p_2^{t_2} \cdot p_3^{t_3} \cdot \dots \cdot p_m^{t_m}$$

$$b | n^2 \Leftrightarrow t_i \leq 2l_i$$

Let  $b$  a divisor of  $n^2$ .

for  $t_1 \rightarrow 2l_1 + 1$  choices

for  $t_2 \rightarrow 2l_2 + 1$  choices

for  $t_3 \rightarrow 2l_3 + 1$  choices

⋮

for  $t_m \rightarrow 2l_m + 1$  choices

$$\begin{array}{l} \text{numbers of} \\ \text{pos divisors} \end{array} = \underbrace{(2l_1+1)}_{\text{odd}} \underbrace{(2l_2+1)}_{\text{odd}} \underbrace{(2l_3+1)}_{\text{odd}} \cdot \dots \cdot \underbrace{(2l_m+1)}_{\text{odd}}$$

multiplication of odd numbers results in an odd number. (Because they do not include factor of 2.)

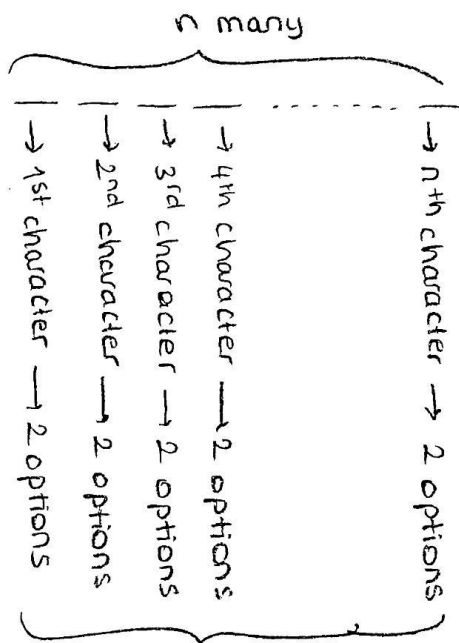
So, we showed that for any  $n \in \mathbb{Z}^+$ , the numbers of positive divisors of  $n^2$  is odd.

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Question 12: How many bit strings are there of length  $n$ ?



For every character in string, there are 2 options as 0 and 1.

$$\underbrace{2 \cdot 2 \cdot 2 \cdot 2 \cdots 2}_{n \text{ many } 2\text{'s.}} \Rightarrow 2^n \text{ bit strings} //$$

$\Rightarrow$  There exist  $2^n$  bit strings of length  $n$ .

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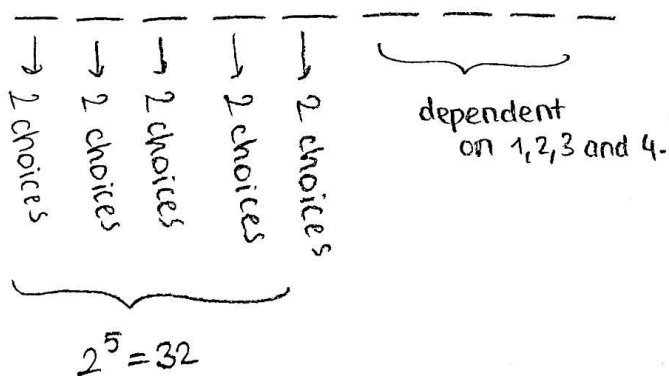
Question 13: How many palindrome binary strings are there of length 9?

1. 2. 3. 4. 5. 6. 7. 8. 9.

To be a palindrome:

- 1. character = 9. character
- 2. = 8.
- 3. = 7.
- 4. = 6.
- 5. is independent.

So, it is enough to calculate first five characters because remaining ones will be dependent on first four characters reverse respectively.



Answer: There are 32 palindrome bit strings of length 9.