

1)

a)  $M$  accepts the strings that start with  $a$ .

$$M = \{a^i b^j \mid 0 \leq i \leq j\}$$

b)  $w = aab$ 

$$(q_0, aab, z_0) \vdash (q, ab, z_0) \vdash (q, b, az_0) \vdash (q_1, e, z_0) \quad \checkmark$$

$$(q_0, a, z_0) \rightarrow (q, z_0) \quad (q, a, z_0) \rightarrow (q, az_0) \quad (q, b, a) \rightarrow (q_1, e)$$

 $w = abb$ 

$$(q_0, abb, z_0) \vdash (q, bb, z_0) \vdash$$

$$(q_0, a, z_0) \rightarrow (q, z_0) \quad (q, b, z_0) \rightarrow \text{No such transition} \quad \notin L(M)$$

 $w = aba$ 

$$(q_0, aba, z_0) \vdash (q, ba, z_0) \vdash$$

$$(q_0, a, z_0) \rightarrow (q, z_0) \quad (q, b, z_0) \rightarrow \text{No such transition} \quad \notin L(M)$$

 $w \in L(M)$ 

↑  
Final state //

c)  $w = aaabb$ 

$$(q_0, aaabb, z_0) \vdash (q, aabb, z_0) \vdash (q, abb, az_0) \vdash (q, bb, aa z_0) \vdash (q_1, b, az_0) \vdash (q_1, e, z_0)$$

$$(q_0, a, z_0) \rightarrow (q, z_0) \quad (q, a, z_0) \rightarrow (q, az_0) \quad (q, a, a) \rightarrow (q, aa) \quad (q, b, a) \rightarrow (q_1, e) \quad (q_1, b, a) \rightarrow (q_1, e)$$

 $w = aaab$ 

← SAME UNTIL HERE →

Final state  
↓  
 $w \in L(M)$  //

2)

a)  $Q =$ 

$$\Sigma = \{a, b\}$$

$$T = \{a, b\}$$

$$F = \{q_1\}$$

$$\delta(q_0, a, z_0) \rightarrow (q_0, az_0)$$

$$\delta(q_0, b, z_0) \rightarrow (q_1, z_0)$$

$$\delta(q_1, b, z_0) \rightarrow (q_1, z_0)$$

$$\delta(q_0, a, a) \rightarrow (q_0, aa)$$

$$\delta(q_0, b, a) \rightarrow (q_1, e)$$

$$\delta(q_1, b, a) \rightarrow (q_1, e)$$

$$\delta(q_1, b, b) \rightarrow (q_1, bb)$$

b)  $Q =$ 

$$\Sigma = \{a, c, b\}$$

$$T =$$

$$F = \{q_2\}$$

$$\delta(q_0, a, z_0) \rightarrow (q_0, az_0)$$

$$\delta(q_0, c, x) \rightarrow (q_1, x) \quad (x: \text{generic variable})$$

$$\delta(q_0, a, a) \rightarrow (q_0, aa)$$

$$\delta(q_0, b, a) \rightarrow (q_1, e)$$

$$\delta(q_1, b, a) \rightarrow (q_1, e)$$

$$\delta(q_1, c, z_0) \rightarrow (q_2, e) = \text{Final}$$

c is missing.

$$f) \{a^i b^j c^k \mid i, j > 0\}$$

$Q =$

$$\Sigma = \{a, b, c\}$$

$\Gamma$

$$F = \{q\}$$

$$\delta(q_0, a, z_0) \rightarrow (q_0, az_0)$$

$$\delta(q_0, a, a) \rightarrow (q_0, aa)$$

$$\delta(q_0, b, a) \rightarrow (q_1, e)$$

$$\delta(q_1, b, a) \rightarrow (q, c)$$

$$\delta(q_0, c, a) \rightarrow (q, e)$$

$$\delta(q_1, c, a) \rightarrow (q, e)$$

$$d) \{a^i b^j c^k \mid i+k=j\}$$

$M =$

$$\delta(q_0, a, z_0) \rightarrow (q_0, az_0)$$

$$\delta(q_0, a, a) \rightarrow (q_0, aa)$$

$$\delta(q_0, b, z_0) \rightarrow (q_0, bz_0)$$

$$\delta(q_0, b, b) \rightarrow (q_0, bb)$$

$$\delta(q_0, b, a) \rightarrow (q_1, e)$$

$$\delta(q_1, b, a) \rightarrow (q_1, e)$$

$$\delta(q_1, b, b) \rightarrow (q_1, bb)$$

$$\delta(q_1, c, b) \rightarrow (q_2, e)$$

$$\delta(q_2, c, b) \rightarrow (q_2, e)$$

$$\delta(q_2, e, z_0) \rightarrow (q_3, e) = \text{Final}$$

$$e) \{a^i b^j \mid 0 \leq i \leq j \leq 2i\}$$

$$\text{CFG: } S \rightarrow asb \mid asb^2 \mid \lambda$$

6.3.2)

$$S \rightarrow aAA$$

$$A \rightarrow aS \mid bS \mid a$$

$$\{ \{q\}, \{a, b\}, \{q, b, A, S\}, \delta, q, S \}$$

$$\delta(q, \epsilon, S) \rightarrow \{(q, aAA)\}$$

$$\delta(q, \epsilon, A) \rightarrow \{(q, aS), (q, bS), (q, aA)\}$$

$$\delta(q, a, a) \rightarrow \{(q, \epsilon)\}$$

$$\delta(q, b, b) \rightarrow \{(q, \epsilon)\}$$