Subscripts preceded by a comma indicate partial differentiation with respect to material or spatial coordinates.
In the absence of body forces, the equations of the motion in the reference state are

$$T_{\Delta\beta,\Delta} + T_{3\beta,3} = 0, \quad T_{\Delta 3,\Delta} + T_{33,3} = \rho_3 \tag{1}$$

where T_{Kk} are the components of the first Piola-Kirchhoff stress tensor field accompanying the deformation field (1), a dot over u_3 indicates partial differentiation with respect to $\imath$, and $\rho = \rho(X_\Delta)$ is the density of the layer. The boundary conditions can be written as

$$T_{2k} = 0 \quad on \quad X_2 = h \quad and \quad u_3 = 0 \quad on \quad X_2 = 0 \tag{2}$$

The components of the deformation gradient tensors $x_{k,K}$ and $X_{K,k}$ are as given in the list below

$$x_{\alpha,\Delta=\delta_{\alpha\Delta}}, \quad x_{\alpha,3} = 0, \quad x_{3,\Delta} = u_{3,\Delta}, \quad x_{3,3} = 1, \tag{3}$$

$$X_{\Delta,\alpha} = \delta_{\Delta\alpha}, \quad X_{\Delta,3} = 0, \quad X_{3,\alpha} = -u_{3,\Delta}\delta_{\Delta\alpha}, \quad X_{3,3} = 1 \tag{4}$$

for the deformation field (1) which is isochoric. *i.e.* $j = \det x_{k,K} = 1$. In addition to using the components (4) and (5), if the relations $t_{kl} = j^{-1}x_{k,K}T_{Kl}$ and $T_{Kl} = jX_{K,k^t,kl}$ are taken into consideration, then the components of the Cauchy stress tensor t_{kl} and of the first Piola-Kirchhoff stress tensor T_{Kl} can be written as

$$t_{\alpha\beta} = \delta_{\alpha\Delta}T_{\Delta\beta}, \quad t_{\alpha 3} = \delta_{\alpha\Delta}T_{\Delta 3},$$

$$t_{3\beta} = u_{3,\Delta}T_{\Delta\beta} + T_{3\beta}, \quad t_{33} = u_{3,\Delta}T_{\Delta 3} + T_{33}, \tag{5}$$

$$T_{\Delta\beta} = \delta_{\Delta\alpha}t_{\alpha\beta}, \quad T_{\Delta 3} = \delta_{\Delta\alpha}t_{\alpha 3},$$

$$T_{3\beta} = -u_{3,\Delta}\delta_{\Delta\alpha}t_{\alpha\beta} + t_{3\beta}, \quad T_{33} = -u_{3,\Delta}\delta_{\Delta\alpha}t_{\alpha 3} + t_{33}, \tag{6}$$

respectively. Therefore, the equations of the motion (2) in terms of t_{kl} are expressed as follows:

$$(\delta_{\Delta\alpha}t_{\alpha\beta})_{,\Delta} + (-u_{3,\Delta}\delta_{\Delta\alpha}t_{\alpha\beta} + t_{3\beta})_{,3} = 0,$$

$$(\delta_{\Delta\alpha}t_{\alpha 3})_{,\Delta} + (-u_{3,\Delta}\delta_{\Delta\alpha}t_{\alpha 3} + t_{33})_{,3} = \rho_3. \tag{7}$$

If the layer consists of hyper-elastic materials, there exists a strain energy function $\sum$ which gives the mechanical properties of the constituent materials, and stress constituve equations can be given by

$$T_{Kk} = \frac{\partial\Sigma}{\partial x_{k,K}}. \tag{8}$$

We consider that the constituve materials are isotropic and heterogeneous, so Σ is the function of the principal invariants of the Finger deformation tensor c^{-1} and X_Δ, as

$$I = trc^{-1}, \quad 2II = (trc^{-1})^2 - tr(c^{-2}), \quad III = detc^{-1} \tag{9}$$

and calculated on the deformation field (1) as

$$I = II = 3 + K^2, \quad III = 1 \tag{10}$$

where $K^2 = u_{3,\Delta}u_{3,\Delta}$.
Let us now assume that the heterogeneity is varied only with the depth and uniform in any direction parallel to the boundaries and consider generalized neo-Hookean materials. Hence, the strain energy function Σ has a form

$$\Sigma = \Sigma(I, X_2) \tag{11}$$

Then the stress constituve equations are

$$t_{kl} = 2\frac{d\Sigma}{dI}(-\delta_{kl} + c^{-1}_{kl}) \tag{12}$$

where $c^{-1}_{kl} = x_{k,K^x l,K}$ are the components of Finger deformation tensor, found for the deformation field(1) as follows:

$$c^{-1}_{\alpha\beta} = \delta_{\alpha\Delta}\delta_{\beta\Delta}, \quad c^{-1}_{\alpha 3} = \delta_{\alpha\Delta}u_{3,\Delta}, \quad c^{-1}_{3\beta} = u_{3,\Delta}\delta_{\beta\Delta}, \quad c^{-1}_{33} = 1 + K^2. \tag{13}$$