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Fuzzy Magic Labeling of Simple Graphs

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Abstract

The study of labeling graphs exposed to various distance constraints is motivated by the problem of minimizing the span of non-interfering frequencies assigned to radio transmitters. However, fuzzy labeling models yield more precision, flexibility and compatibility to the system compared to the classical models. In this paper we show that whether any simple graph is fuzzy magic labelizing, by considering the concept of fuzzy magic labeling of graphs. In fact, we prove that every connected graph is a fuzzy magic labelizing graph. Finally, we give some applications for fuzzy magic labeling graphs.

Keywords: Fuzzy graph, fuzzy magic labeling.

2010 MSC: 03E72.

1 Introduction

Zadeh [21] was first introduced the concept of fuzzy relations which describes the phenomena of uncertainty in real life situation and has a widespread application in pattern recognition. Replacing the classical sets by Zadeh's fuzzy sets has the advantage of giving more accuracy and precision in theory, as well as more efficiency and system compatibility in applications.

A graph is a convenient way of representing information involving the relationship between objects. The objects are represented by vertices and the relations by edges. Graph theory is a very important tool to represent many real world problems. Nowadays, graphs do not represent all the systems properly due to the uncertainty or haziness of the system parameters. In general, designing a fuzzy graph model is necessary when there is vagueness in describing the objects, their relationships, or both. In addition, fuzziness should be added to the representation if the relations among accounts are to be measured as good or bad according to the frequency of contacts among the accounts, which motivated to define fuzzy graphs, along with many other problems. Rosenfeld first introduced the concept of fuzzy graphs and obtained the fuzzy analogues of several basic graph theoretic concepts like bridges, paths, cycles, trees and connectedness and established some of their properties [14]. Then, fuzzy graph theory has attracted a lot of attention. Fuzzy graph has some applications such as data mining, image segmentation, clustering, image capturing, networking, communication, planning, scheduling, and the like [1, 13, 15, 16]. Crisp graph and fuzzy graph are structurally similar(regarding the generalization of the crisp graph). However, fuzzy graph is emphasized more when there is an uncertainty on vertices and edges. Further, the fuzzy graph occurs in many real life situations due to the existence of uncertainty in the world. Fuzzy graph theory is developed with large number of branches.

2 Preliminaries

In this section, we review some definitions and results from [5, 8], which we need in what follows.

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A (simple)graph G is a finite nonempty set V of objects called *vertices* (the singular is called *vertex*) together with a set E of 2-element subsets of V called *edges* and is shown with $G = (V, E)$. Two graphs G and H are isomorphic (they have the same structure) if there exists a bijective function $\varphi : V(G) \longrightarrow V(H)$ such that two vertices u and v are adjacent in G if and only if vertices $\varphi(u)$ and $\varphi(v)$ are adjacent in H. Then the function φ is called an isomorphism and we write $G \cong H$.

Let $G^* = (V, E)$ be a simple graph. Then $G = (\sigma, \mu)$ is called a *fuzzy graph* on G^*, if $\sigma : V \to [0, 1]$ and $\mu : E \to [0, 1]$ and for all $xy \in E$

$$\mu(xy) \leq \sigma(x) \wedge \sigma(y)(= \min\{\sigma(x), \sigma(y)\})$$

A fuzzy graph $G = (\sigma, \mu)$ on G^* is called a *fuzzy labeling graph*, if σ and μ are one to one maps and for all $xy \in E$

$$\mu(xy) < \sigma(x) \wedge \sigma(y)$$

A fuzzy labeling graph $G = (\sigma, \mu)$ on G^* is called a *fuzzy magic labeling graph*, if there exists $m \in (0, 3)$ (which is called a *magic value*) such that for all $xy \in E$

$$\sigma(x) + \sigma(y) + \mu(xy) = m$$

From now on, in this paper $G^* = (V, E)$ is a simple graph.

3 Fuzzy magic labelizing graphs

In this section, we investigate how to construct the fuzzy magic labeling on a simple graph . For this, first we show that any path graphs, cyclic graphs, star graphs and complete graphs are fuzzy magic labelizing. Then by using that results, we prove that any connected graph is a fuzzy magic labelizing graph.

Definition 3.1. *A simple graph G^* is called a fuzzy magic labelizing graph, if there exists a fuzzy magic labeling graph $G = (\sigma, \mu)$ on G^*.*

Example 3.2. *Consider the Petersen graph G^*. We define a fuzzy magic labeling $G = (\sigma, \mu)$ on G^* in Figure 1, as follows:*

$$\begin{aligned}
&\mu(0.35, 0.48) = 0.15, \mu(0.35, 0.47) = 0.16, \mu(0.35, 0.3) = 0.33, \mu(0.47, 0.44) = 0.07,\\
&\mu(0.47, 0.49) = 0.02, \mu(0.49, 0.43) = 0.06, \mu(0.49, 0.46) = 0.03, \mu(0.46, 0.42) = 0.1,\\
&\mu(0.46, 0.48) = 0.04, \mu(0.41, 0.48) = 0.09, \mu(0.41, 0.44) = 0.13, \mu(0.41, 0.43) = 0.14,\\
&\mu(0.3, 0.43) = 0.25, \mu(0.44, 0.42) = 0.12, \text{ and } \mu(0.3, 0.42) = 0.26.
\end{aligned}$$

Hence the Petersen graph G is a fuzzy magic labelizing graph.

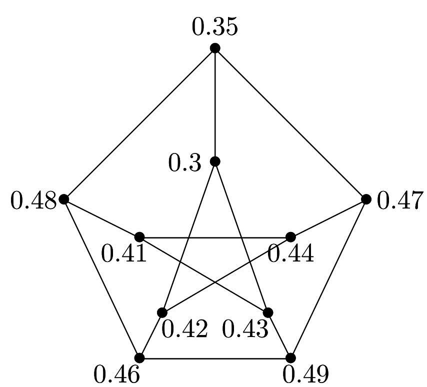

Figure 1: A fuzzy magic labeling of Petersen graph

Now, first we show that any path graph is a fuzzy magic labelizing graph.

Theorem 3.3. *Any path graph P_n, for $2 \leq n \in \mathbb{N}$, is a fuzzy magic labelizing graph.*

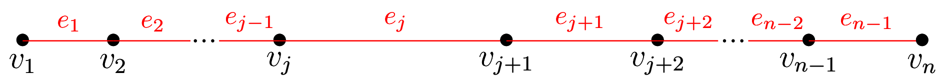

Figure 2: Path graph P_n.

Proof. Let $2 \leq n \in \mathbb{N}$, $V = \{v_1, v_2, v_3, ..., v_n\}$, $E = \{e_i = v_i v_{i+1} \mid 1 \leq i \leq n-1\}$ and $P_n = (V, E)$ be a path graph as Figure 2.

For all $1 \leq i \leq n$, we define the map $\sigma : V \to [0, 1]$ as follows:

$$\sigma(v_i) = \begin{cases} \dfrac{4n-(i+1)}{2(2n-1)}, & \text{if } i \text{ is an odd,} \\ \dfrac{3n-(i+1)}{2(2n-1)}, & \text{if } i \text{ is an even and } n \text{ is an odd,} \\ \dfrac{3n-i}{2(2n-1)}, & \text{if } i \text{ and } n \text{ are evens.} \end{cases}$$

and for all $1 \leq i \leq n-1$, we define the map $\mu : E \to [0, 1]$ by $\mu(e_i) = \dfrac{i}{2n-1}$.

Therefore, for all $2 \leq n \in \mathbb{N}$, the path graph P_n is a fuzzy magic labelizing graph. □

Example 3.4. (i) *Consider the path graph P_6 in Figure 3.*

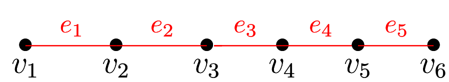

Figure 3: Path graph P_6.

For all $1 \leq i \leq 5$, we define $\mu(e_i) = \dfrac{i}{11}$ and

$$\sigma(v_1) = 1, \sigma(v_2) = \frac{8}{11}, \sigma(v_3) = \frac{10}{11}, \sigma(v_4) = \frac{7}{11}, \sigma(v_5) = \frac{9}{11}, \sigma(v_6) = \frac{6}{11}$$

Then by Theorem 3.3, $G = (\sigma, \mu)$ is a fuzzy magic labeling on P_6 with magic value $\dfrac{20}{11}$.

(ii) *Consider the path graph P_7 in Figure 4.*

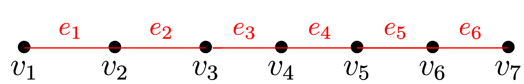

Figure 4: Path graph P_7.

For all $1 \leq i \leq 6$, we define $\mu(e_i) = \frac{i}{13}$ and

$$\sigma(v_1) = 1, \sigma(v_2) = \frac{9}{13}, \sigma(v_3) = \frac{12}{13}, \sigma(v_4) = \frac{8}{13}, \sigma(v_5) = \frac{11}{13}, \sigma(v_6) = \frac{7}{13}, \sigma(v_7) = \frac{10}{13}$$

Let $e_i = v_i v_{i+1} \in E$. We show that $\mu(e_i) < \sigma(v_i) \wedge \sigma(v_{i+1})$, for all $1 \leq i \leq n-1$.

Then by Theorem 3.3, $G = (\sigma, \mu)$ is a fuzzy magic labeling graph on P_7 with magic value $\dfrac{23}{13}$.

4 Applications of fuzzy magic labelizing graphs

In this section, we apply the concept of fuzzy magic labeling of graphs for modeling the real time applications.

First, the graph coloring is emphasized as one of the most important concepts in graph theory, which is used in many real time applications like job scheduling, aircraft scheduling, computer network security, map coloring, GSM mobile phone networks, and automatic channel allocation for small wireless local area networks. The proper coloring of a graph is the coloring of the vertices with minimal number of colors so that two adjacent vertices should not have the same color. The minimum number of colors is called "the chromatic number" and the graph is called "properly colored graph".

Two examples of the applications of fuzzy magic labeling of graphs in heating systems and traffic networks are as follows:

Floor heating system for apartments:

Consider the floor heating of apartment in Figure 5. We can assume the floorś pipeline array as some kinds of

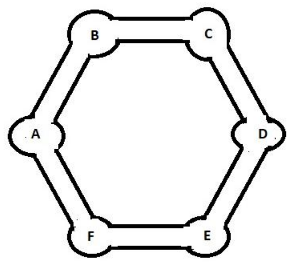

Figure 5: Model of an apartment floor heating system.

graph such as, path graph(P_n), cycle graph(C_n), star graph(S_n) or complete graph(K_n). Thus, the pipe nodes and inter–vertex pipes are considered vertex and the edge, respectively. In a floor heating system of apartments, the warm water flows through the pipes, and the heat is conveyed to the floor. Therefore, the graph should be fuzzy, because the amount of water of each pipe should be less than node's (i.e. pipe's have two vertices) a minimum capacity which is necessary for water to flow. In addition, fuzzy magic graph should be preserved to equalize heat and reach uniform distribution across the apartment floor. In other words, the sum of intra–pipe water plus vertex nodes should be a fixed value.

5 Conclusions

In the present paper, we investigated whether any simple graph is fuzzy magic labelizing, by considering the concept of fuzzy magic labeling of graphs. In fact, we proved that every path graph, cycle graph, star graph and finally any connected graph is a fuzzy magic labelizing graph. Then we introduced good applications about "Floor heating system for apartments" and "Model of a road network" by using fuzzy magic labeling graphs.

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