TAM 598 Lecture 9:

MONTE CARLO ESTIMATES OF CDF, PDF, EXPECTATION, ETC

Announcements:

- HW 2 covers lectures 4-8; due on Feb 26

I. Cumulative Distribution Function

say Y = RV, $F(Y)$ = CDF.
we draw samples of Y and want to estimate $F(Y)$.

How do we do this?

Alternatively, say we have $X =$ RV, function $g(x)$ and say we want the CDF of $Y = g(x)$.

Then $F(y) =$

II. Probability Density Function

- X = R.V.; $Y = g(X)$
- want to approximate the pdf $p(y)$ of $Y = g(X)$ given samples $X_1, X_2, \dots, X_N$

The constant c_j is the probability that y falls in $[b_{j-1}, b_j]$

III. Estimating Quantiles μ_q

$X = RV, \quad Y = g(x)$

$0 \le q \le 1$

$$F(\mu_q) = p(Y \le \mu_q) = q$$

IV. Example – Propagating Uncertainty Through an ODE

$$\dot{y} = \frac{dy}{dt} = -a\,y(t)$$

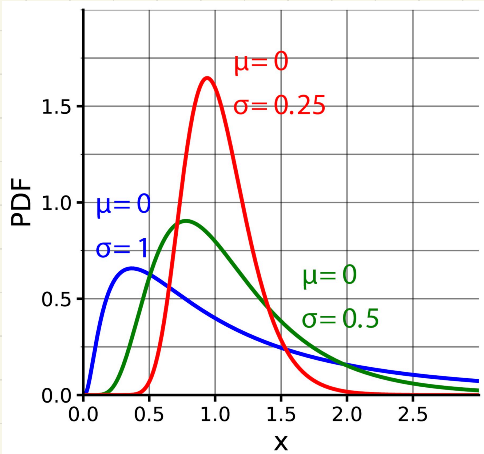
a = exponential decay constant, units t^{-1}

① Decay Rate a

we know $a > 0$

(2) Initial Conditions y_0

we know $y_0 > 0$ so exp or L-N



lognormal (μ, σ^2)

pdf $\quad p(x) = \dfrac{1}{x\sigma\sqrt{2\pi}} \exp\left(-\dfrac{\ln(x-\mu)^2}{2\sigma^2}\right)$

$\mathbb{E}[X] = \exp\left(\mu + \dfrac{\sigma^2}{2}\right)$

$\mathbb{V}[X] = \left| \exp(\sigma^2) - 1 \right| \exp(2\mu + \sigma^2)$

UNCERTAINY PROPAGATION :

Now we are ready to draw samples from a and y_0, and evaluate the statistics associated w/ our ODE