TAM 598 Lecture 20:

State Space Models –

Kalman Filters

Announcements:

- HW 5 covers lectures 17-20; due on Fri Apr 18 last time:

initial conditions $\underset{\sim}{x}_0 = \underline{\mu}_0 + \underline{z}_0$

transitions $\underline{x}_{t+1} = \underline{\underline{A}}\,\underline{x}_t + \underline{\underline{B}}\,\underline{u}_t + \underline{z}_t$

emissions $y_. = C x + w_.$

Kalman Filter - recursively computes the posterior distribution $p(\underline{x}_t \mid \underline{y}_{1:t}, \underline{u}_{1:t})$ using two steps:

prediction step

goal $p(\underline{x}_t \mid \underline{y}_{1:t-1}, \underline{u}_{1:t})$

say we have $p(\underline{x}_{t-1} \mid \underline{y}_{1:t-1}, \underline{u}_{1:t-1}) = N(\underline{x}_{t-1} \mid \underline{\mu}_{t-1}, \underline{\underline{V}}_{t-1})$

we use our update rules and marginalize over $\underline{x}_{t-1}$:

update step

goal: $p(\underline{x}_t \mid \underline{y}_{1:t}, \underline{u}_{1:t})$

we now condition our prior on a new measurement $\underline{y}_t$ to obtain the posterior using Bayes' Rule:

SMOOTHING – now we want the posterior $p(\underline{x}_t \mid \underline{y}_{1:T}, \underline{u}_{1:T})$ of the distribution of the state at t, given all measurements and control inputs up to some future time T.

$$p(\underline{x}_{T-1} \mid \underline{x}_T, \underline{y}_{1:T}, \underline{u}_{1:T})$$

from here:

① our joint distribution $p(\underline{x}_T, \underline{x}_{T-1} \mid \underline{y}_{1:T}, \underline{u}_{1:T})$ is gaussian

② integrate out $\underline{x}_T$ to get our smoothed distribution

③ Result is gaussian with

mean

covariance