TAM 598 Lecture 12:

Bayesian Inference - Analytical Examples

Announcements:

- HW 3 covers lectures 8-12; due on Mar 12

Bayesian Inference - a way to fit a model to data.

Say our model predicts the result of a random experiment.
we describe the model using pdf

our problem: how do we use the data to learn about θ?

Maximum Likelihood Principle

likelihood of a single data point x_n is $p(x_n \mid \theta)$

likelihood of the entire dataset $x_{1:N}$ is the joint pdf of all observations. Since all observations are independent, conditioned on θ

Max. Likelihood: find the θ that maximizes $p(x_{1:N} \mid \theta)$

This gives us a point estimate of parameters θ. But there should be some uncertainty in our selected θ, given limited data $x_{1:N}$. How do we quantify this? Bayes rule.

so: $p(\theta \mid x_{1:N}) =$

and $p(\theta \mid x_{1:N})$

Example: inferring the probability of a coin toss from data.

- toss a coin N times, $x_{1:N} = (x_1, \dots, x_N)$ are the results
  tails $= 1$, heads $= 0$. θ = prob. of heads

- say we know nothing initially:

- likelihood of data is

- each measurement

① each measurement $X_n \mid \theta \sim \text{Bernoulli}(\theta)$ so $p(x_n|\theta) = \begin{cases} \theta & x_n = 1 \\ 1-\theta & x_n = 0 \end{cases}$

① so then likelihood

① and our posterior $\propto$ (likelihood)(prior)

Credible Intervals: an interval inside which the parameter θ lies with high probability. Eg, a 95% credible interval (l, u) for θ is

Decision Making: what if you are asked to report a single value for θ? You need to make a decision.

approach: minimize the expected loss

posterior predictive checking

- say you've built a model using data $x_{1:n}$, and you rerun the same experiment. What do you expect to observe?

"posterior predictive checking"