TAM 598 Lecture 4 :

CONTINUOUS RANDOM VARIABLES

Announcements:

- HW 1 covers lectures 1-4 ; due on Feb 12
- to be submitted via CANVAS

I. CONTINUOUS RANDOM VARIABLES

Let X be a random variable that can take values in $\mathbb{R}$. If the range of X is uncountable (e.g. if it forms an interval), we say X is a continuous random variable

eg X = mass of a ball bearing, the temperature of a room, ...

1) cumulative distribution function (CDF) of random variable X is denoted by $F_X(x)$ and is the probability that X takes a value less than or equal to x

2) probability density function is a function $f_X(x)$ that gives us the probability that X lies in any Borel subset A of $\mathbb{R}$

3) Expectations of Continuous Random Variables

recall discrete RV:

$$\mathbb{E}[X] = \sum_{x} x\, p(x)$$

4) Variance of continuous random variables is again the average squared distance of X from its expected value. Tells us how "spread out" X is

II. THE UNIFORM DISTRIBUTION - a random variable equally likely to take a value within a given interval.

Between 0 and 1:

- $X \sim \mathcal{U}([0,1])$ has pdf

- $X \sim \mathcal{U}([0,1])$ has cdf

- The probability that X takes values in $[a, b]$ for $a < b$ in $[0, 1]$ is

- The expectation is

- The variance is

Between a and b

- $X \sim \mathcal{U}([a,b])$ has pdf

- $X \sim \mathcal{U}([a,b])$ has cdf

- $\mathbb{E}[X] =$

- $\mathbb{V}[X] =$

Say we know how to obtain samples from $\mathcal{U}([0,1])$, but we want samples from $\mathcal{U}([a,b])$. How do we do it?

III. THE NORMAL DISTRIBUTION a.k.a. THE GAUSSIAN DISTRIBUTION

$$\boxed{X \sim N(\mu, \sigma^2)}$$

- appears over and over again in nature, in measurements, ...
- why?

standard normal distribution

$$Z \sim N(0,1)$$

general normal distribution

$$X \sim N(\mu, \sigma^2)$$

→ consider the pdf of $N(0,1)$:

$$\phi(z) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{z^2}{2}\right)$$

Quantiles of the Normal Distribution

Find the value $Z = z_q$ so that $P(Z \le z_q) = q$, ie

find $\Phi(z_q) = q$

z_q is called the **q-quantile**

want $z_q = \Phi^{-1}(q)$

eg)

eg)

Getting any normal from the standard normal:

let $Z \sim N(0,1)$

How to find a q-quantile x_q of X :

$$P(X \leq x_q) = q$$