QEC Tutorial – Cheat Sheet (Peer Review Draft)

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Quantum error correction is required to protect quantum information from decoherence and noise. Unlike classical error correction however we can not copy quantum information since observing/ knowing/ copying quantum information will result in that information being lost. So, we instead utilize entangled qubits, which create the basis of quantum error correction algorithms. 1. **Superposition:**
The qubit states of 0 and 1 can exist in a linear combination of both until measured:

$$|0\rangle = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \qquad |1\rangle = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \qquad |\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

2. **Entanglement:**
Bell states are the quantum states of two entangled qubits:
Bell State

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

3. **Quantum Measurement:** When you make a quantum measurement, you collapse the wavefunction and get either 0 or 1 as your qubit measurement. In a mathematical sense, right after the measurement, the state of the system is an eigenstate of the observable, which means that the value of the observable can be exactly known.

$$P_0 = |0\rangle\langle 0| \qquad P_1 = |1\rangle\langle 1| \qquad \langle A\rangle = \langle\psi|A|\psi\rangle$$

4. **Quantum Gates:** Widely used quantum gates (operators).

$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad Z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \qquad H = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \qquad \text{CNOT} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

5. **QEC Methods:** Widely used, fundamental quantum correction methods.
Bit flip code:

$$|0_L\rangle = |0^n\rangle \qquad |1_L\rangle = |1^n\rangle \qquad X|0_L\rangle = |1_L\rangle \qquad X|1_L\rangle = |0_L\rangle$$

Phase flip code:

$$|0_L\rangle = |+\rangle^{\otimes n} \qquad |1_L\rangle = |-\rangle^{\otimes n} \qquad Z|+\rangle = |+\rangle \qquad Z|-\rangle = -|-\rangle$$

6. **QEC Equations:**
Error Correction

$$\text{Corrected State} = X^{e_1} Z^{e_2} |\psi\rangle$$

References

Nielsen, M. A., & Chuang, I. L. (2010). *Quantum Computation and Quantum Information.*
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