Computers & Industrial Engineering 201 (2025) 110883

Contents lists available at ScienceDirect

Computers & Industrial Engineering

journal homepage: www.elsevier.com/locate/caie

Analysis of an all-or-nothing inventory model with price-dependent intermittent demand and supply uncertainty

Elifnur Doğruöz *, Refik Güllü

Department of Industrial Engineering, Boğaziçi University, 34342 Bebek/Istanbul, Turkey

ARTICLE INFO

Keywords:
Pricing
Inventory
Uncertain demand
Uncertain yield

ABSTRACT

In this article, we consider an inventory model under intermittent demand where the demand occurrence depends on the price and the yield uncertainty of the item. Once the demand occurs, the size of the demand is a random variable. We restrict ourselves to an all-or-nothing replenishment model and primarily focus on the effects of pricing and supply uncertainty. We present analytical (for the single period) and numerical findings (for the multi-periods) for the optimality of a threshold replenishment policy and compare responsive (after observing hold replenishment policy yield) versus unresponsive (before observing yield) pricing schemes. In particular, we show that the profit difference between responsive and unresponsive pricing cases can be substantial. We present numerical results on the sensitivity of the responsive optimal price on the yield level and other system parameters. We also discuss our findings for different demand size distributions.

1. Introduction and related literature

Supply chains are the backbones that ensure the seamless flow of goods and services from manufacturers to end consumers. The dynamic relationship between supply and demand lies at the core of the supply chains. Over the recent decades, the focus of supply chain planning shifted from being cost driven to providing a competitive advantage through improving customer service (Snyder & Shen, 2019). Synchronizing supply with demand is fundamental to supply chain management and serves multiple purposes, with customer satisfaction being the most significant. Aligning production with market needs ensures product availability, fostering customer loyalty and trust. Beyond customer satisfaction, matching the supply with the demand is crucial in managing costs. While overproduction leads to high storage costs, underproduction results in missed sales opportunities and the loss of customers. Thus, balancing supply and demand is essential for economic viability and profitability.

Pricing is a powerful tool for aligning supply and demand in supply chain management. As the demand is mostly price-sensitive, dynamic pricing offers a remarkable advantage to retailers. In the last decades, in addition to airlines and hotels, industries like retail have adopted dynamic pricing solutions to gain a competitive edge (Elmaghraby & Keskinocak, 2003). Advances in technology have led to the use of AI-driven pricing tools, further enhancing competitiveness (Bharadiya, 2023).

Adopting dynamic pricing in the business process requires an understanding of the demand. Intermittent demand is commonly observed in industries such as the spare parts industry, the automotive sector, telecommunication systems, etc. In this paper we consider pricing and replenishment problems of an inventory system facing price-dependent intermittent demand and yield uncertainty. Price-dependent intermittent demand primarily occurs in environments where the rate of customer arrivals is low, and the customers are price sensitive due to competition. Price-dependent intermittent demand can be observed in industry, for instance, when quoting a lower price in a tender process increases the chance of winning the tender, resulting in demand. The demand size is decided by the tender organizer and is random to the retailer. Additionally, this demand model can be used to represent price-sensitive customers who buy in bulk when the price is right for them. To our knowledge, this demand model is novel in supply chain literature. Huang et al. (2013) present an extensive survey of price-dependent demand models.

Major research questions that we seek to answer are as follows:

a. Can we obtain a plausible intermittent demand model where the time between demand arrivals depends on the price level and the supply uncertainty?
b. How to price an item if the demand occurrence probability depends on the price?
c. How different is the responsive (after observing the yield) pricing from the unresponsive (before ordering the yield) pricing in an intermittent demand model?

* Correspondence to: Ahmediye Mah. Pirnal Sokak Sanliayaz Apt. 12/5 Uskudar/Istanbul, Turkey.
E-mail address: elifnurdogruoz@gmail.com (E. Doğruöz).

https://doi.org/10.1016/j.cie.2025.110883
Received 26 June 2024; Received in revised form 7 January 2025; Accepted 11 January 2025
Available online 21 January 2025

In what follows we review the relevant literature: joint inventory and pricing models and the models with yield uncertainty.

Joint pricing and inventory optimization:

A substantial amount of literature explores the combined analysis of inventory and pricing. One of the early works in this stream is the study of Federgruen and Heching (1999). They investigated simultaneous pricing and inventory decisions of a single item under demand uncertainty in periodic review. Chen and Simchi-Levi (2004) worked on a very similar model but included a fixed cost alongside the linear ordering cost and assumed price-dependent demand. They aimed to find a policy maximizing the total expected profit and showed that (s, S, p) and a modified (s, S, p) policies are optimal for additive and generalized demand functions, respectively.

Alfares and Ghaithan (2016) developed an economic order quantity inventory model that incorporates price-dependent demand, time-varying inventory holding costs, and order size-dependent purchasing costs. They also proposed a solution algorithm to determine the selling price and order quantity. More recently, Bardhan et al. (2024) examined a multi-period inventory model where demand depends on price, time, and service levels, demonstrating the existence of an optimal pricing strategy.

Supply and yield uncertainty models with a focus on pricing:

Many of the studies in the literature address demand uncertainty, considering supply as certain. Supply chain resilience has been a point of interest, especially after the COVID-19 pandemic. Comprehending and managing supply uncertainty holds significant importance for businesses in safeguarding the resilience and continuity of their operations. During and even after the pandemic, the demand for some products has risen extremely, surpassing supply (Ivanov, 2022). For some other products, both demand and supply have dramatically decreased. After the global pandemic, geopolitical conflicts, natural disasters, and other unexpected events, such as supply fluctuations and economic crises, have raised even more questions about the resilience of supply chains. Consequently, our research addresses supply chain disruption.

In yield uncertainty models, the retailer gives an order of size Q and receives rQ, where r is the realization of a random variable δ representing the supply yield. We examine two pricing methods: responsive (postponed) pricing, where the retailer sets the price after receiving rQ, and unresponsive (simultaneous) pricing, where the price is set before knowing the received amount. We will compare these methods, showing that responsive pricing results in higher profits, and investigate through a numerical study if there is a threshold policy.

We have seen many examples of scarcity increasing the desirability of goods during the pandemic. As Islam et al. (2021) said, scarcity increased the consumers' eagerness to buy stuff such as toiletries, frozen food, etc., obsessively in panic and resulted in empty store shelves. Taking this into account, in addition to the price, we relate the yield uncertainty with the demand occurrence probability in our intermittent demand pattern. While building our demand function involving yield uncertainty, we use attraction models based on market share and derive our demand models through attraction models.

Tang (2006) provided a broad review of quantitative methods of supply chain risk management. Snyder et al. (2016) also comprehensively reviewed operations research/management science methods in supply chain disruption literature and mitigation methods. Golmohammadi and Hassini (2020) reviewed lot-sizing problem literature with random demand and supply. Ben-Ammar et al. (2022) provided a literature review for supply planning and inventory control under lead time uncertainty. Our study focuses on yield uncertainty, assuming a supplier delivers a random part of the order quantity.

Tang and Yin (2007) aimed to find the order amount and price, maximizing a retailer's profit under supply uncertainty with price-dependent demand. They compared *responsive* and *no responsive* pricing policies and proved that the retailer would obtain a higher expected profit in the responsive pricing policy, especially when the uncertainty or unit cost is higher. Surti et al. (2013) extended this work with stochastic demand with additive and multiplicative error and stochastic yield. They could not prove the joint concavity of the expected profit function in quantity and price; however, they showed that separately it was concave in quantity and price. They proposed a search algorithm for decision variables to find the optimal order quantity and optimal price. They also proved that the postponed pricing profit dominates the simultaneous pricing profit. Tang et al. (2012) and He (2013) studied similar problems, finding that dynamic pricing policies are advantageous under certain conditions. Chen and Dong (2018) examined sourcing and responsive pricing decisions for two correlated products with price-dependent demand under supply and demand uncertainty. They showed that the 'order-up-to' inventory policy is not always valid; it is valid only under specific conditions.

Xu and Lu (2013) studied a newsvendor problem with random yield. The demand was assumed to be the product of a deterministic and price-dependent function and a random noise. They compared in-house production and procurement cost structures and observed that as variability in yield rate decreases, expected profit increases and optimal price decreases in both procurement and in-house production cases, which is aligned with the papers mentioned above. Moon et al. (2012) investigated an EOQ model with variable supply capacity and random yield. Serel (2017) focused on emergency suppliers' capacity uncertainty, identifying optimal pricing strategies under different demand uncertainties.

Lu et al. (2018) explored production and pricing decisions for a company producing two substitutable products, where one product has yield uncertainty, and demand is modeled as a linear function of price. Golmohammadi and Hassini (2019) focused on joint lot-sizing and pricing problems in agriculture with uncertain supply and demand, identifying conditions for optimal threshold policies for joint and sequential decisions on price and production. Another recent work, Dong et al. (2023), investigated supply diversification and pricing under yield uncertainty, finding that responsive pricing positively affects decision-making. Geng et al. (2023) examined postponement methodologies wherein the retailer can choose to pay for procurement at the time of order placement or upon delivery. Additionally, the study explored the choice of pricing as either responsive or unresponsive with a game theoretic approach. Both Dong et al. (2023) and Geng et al. (2023) used a deterministic price-dependent demand function. Shah et al. (2023) and De et al. (2024) investigated an inventory model for deteriorating goods whose demand depends on the selling price and stock level. Considering uncertainties in the capacity process, Abdali et al. (2024) proposed a machine learning-driven approach to prioritizing customers and capacity allocation.

The replenishment policy:

In industrial applications, the order amounts are either restricted by physical constraints (such as process batch sizes and container capacities), or there are high enough fixed costs that prohibit placing frequent orders with small order quantities. In our paper, we consider an all-or-nothing type of ordering policy. Therefore, in any given period, our ordering decision only involves whether to place an order of a certain size or not.

When production quantity is limited, and a fixed cost is incurred for each order, the form of the optimal policy is not exactly known. Gallego and Scheller-Wolf (2000) showed that the optimal policy follows an (s, S)-like structure. Building on this, Chao et al. (2012) investigated a periodic review inventory system with a fixed cost and finite capacity in the finite horizon, using a random additive price-dependent demand model. They found that the same ordering policy as in the work of Gallego and Scheller-Wolf (2000) was optimal. Additionally, they defined an optimal pricing policy based on the inventory level after replenishment using the inverse function of the demand.

Hu et al. (2019) studied a periodic review, joint pricing and inventory control problem with a fixed cost and piecewise linear variable

Table 1
Notation table.

Parameters	
x	On-hand inventory of the retailer
c	Unit variable cost of ordering
h	Holding cost per unit time
Q	Order quantity
$\bar{K}$	Fixed cost paid if the order is made ($Q > 0$)
b_1, b_2	Coefficients utilized in demand occurrence probability $g(p,r)$
s	Threshold inventory level below which an order must be placed
Z	Random variable denoting the demand size
δ	Random variable denoting the supply yield, $0 \leq \delta \leq 1$
r	A realization of δ
$A(r)$	A function of r
$g(p,r)$	Demand occurrence probability as a function of p and r
$D(p,r)$	Demand realization as a function of p and r
$G(x,p)$	Expected profit as a function of x and p
Decision Variables	
p	Selling price
y	Takes value of x or $x+Q$

cost, using stochastic price-dependent demand. Gallego and Toktay (2004) focused on a single item, periodic-review inventory problem with both fixed and variable costs. They developed a special case of the model of Gallego and Scheller-Wolf (2000) by introducing an all-or-nothing concept, where the orders are given only in full capacity. They aimed to minimize the total cost and investigated the conditions, enabling an optimal threshold policy with respect to the inventory position. To achieve this, they showed that there is a single sign change (from negative to positive) between the cost-to-go functions for the options of ordering and not ordering.

We next list our contributions to the aforementioned literature.

Contributions of our paper:

The major contributions of our paper are summarized as follows.

1. We propose a price-dependent intermittent demand model. Our demand function differs from the demand functions used in the above-mentioned studies. We also incorporate supply uncertainty in the demand function.
2. We develop optimal pricing policies under responsive (after the yield is observed) and unresponsive (before the yield is observed) cases.
3. We present managerial insights and observations based on our numerical computations.

The rest of the paper is structured as follows: A detailed explanation of our demand function is presented in Section 2. Section 3 explores simultaneous inventory and pricing decisions under price-dependent intermittent demand and yield uncertainty. Our numerical study is presented in Section 4. Finally, our conclusions and future research directions are provided in Section 5.

2. Modeling the customer demand

In this section, we present the notations and assumptions used in the model, followed by a detailed explanation of customer demand.

2.1. Notation and assumptions

This article uses the notation in Table 1. Major assumptions of our model are as follows:

1. All-or-nothing policy is utilized. Orders can only be full capacity Q or zero, meaning no order is placed.
2. Unsatisfied demand is lost, there is no backlogging.
3. It is assumed that the demand is intermittent. Demand size follows a random distribution, and the demand occurrence probability is associated with the product's price and yield uncertainty.
4. We assume an uncertainty in the yield, that is, whenever the retailer orders an amount of Q it receives only an amount of rQ, where r is a realization of the random variable δ denoting the supply yield.
5. Initial inventory is zero.
6. A holding cost h is incurred if a unit of product is carried to the next period.

2.2. Modeling customer demand

Customer demand is one of the main components of supply chains. Representing the demand correctly is crucial for supply chain management as it affects the inventory, supply policy, revenue, and profitability. Various demand models have been utilized in supply chain literature and other economy or business-related research areas to reflect consumer behavior correctly.

Huang et al. (2013) comprehensively reviewed widely used demand models and characterized single firm price-dependent demand models in four categories concerning the elements affecting the demand in addition to price: deterministic models, stochastic models, willingness-to-pay models, and Poisson flow models. In deterministic price-dependent demand models, demand is a function of price. While deterministic models are widely used in the literature for their simplicity and ease of extracting solutions, stochastic models are also preferred as they represent randomness and variability in reality.

Our research uses a price and yield-dependent stochastic demand function. We assume that our demand is intermittent (sporadic), meaning several periods may have zero demand between the demand occurrences. When demand occurs, the demand size can be small or large. In addition to the stochasticity of the demand size, the duration of the intervals between non-zero demand occurrences is another source of uncertainty. Despite many application areas mentioned in Section 1, forecasting intermittent demand is not easy. Exponential smoothing is one of the traditional methods for forecasting intermittent demand. However, since exponential smoothing underperforms, Croston (1972) proposed a method that uses exponential smoothing on demand size and intervals separately. Syntetos and Boylan (2005) put forward Syntetos–Boylan approximation method by adjusting Croston's method to eliminate bias. Recently, neural networks have also been used in intermittent demand forecasting and have proven more successful (Turkmen et al., 2020).

In our research, we adjust the intermittent demand model to associate the demand occurrence probability with the product's price and uncertainty in the yield. In yield uncertainty, the firm orders an amount of product, Q, and receives an amount $rQ \leq Q$ due to a shortfall in yield. Therefore, demand realization $D(p,r)$ is represented by following distribution:

$$D(p,r) = \begin{cases} Z, & \text{with probability } g(p,r) \\ 0, & \text{with probability } 1 - g(p,r) \end{cases} \tag{1}$$

where $0 \leq g(p,r) \leq 1$. This demand model is particularly useful for price-sensitive consumers who strategically purchase days of supply when prices are favorable, effectively optimizing their purchasing decisions to maximize savings. Besides, uncertainty in supply increases their appetite to purchase.

In our research, we utilize two distinct demand functions, denoted as $g(p,r)$. The first function, referred to as D1, is given by

$$g(p,r) = \frac{1}{1 + (b_1 p + b_2 p^2)A(r)}, \tag{2}$$

where b_1 and b_2 are coefficients, and $A(r)$ is a function of r that incorporates supply uncertainty into the model. If $A(r)$ increases as r

increases (*i.e.*, uncertainty decreases), it indicates that as the retailer receives a larger portion of the order, the probability of demand occurrence (*i.e.*, purchasing) decreases as customers anticipate future stock. Conversely, if $A(r)$ decreases with r, as uncertainty decreases, the probability of demand occurrence increases as customers are unsure if the stock will be available in the future. In this study, we use $A(r)$ functions that are increasing in r, such as $A(r) = r$.

Attraction models represent the products' market share and are widely used to estimate consumer behavior, subsequently, the demand in economics, marketing, and operations management (Huang et al., 2013). Logit is one of the most well-known discrete choice models. In binary logit models, the decision-maker is able to select from two options. We will use this as the second $g(p,r)$ function, which will be referred to as D2 since it is a valid model and convenient for interpreting customer behavior based on the utility approach. In our research, demand occurrence is a binary process. The utility of the decision-maker, or the customer in our case, is a function of price p and yield uncertainty r. Hence, the probability of the demand occurrence, in other words, the probability that the customer buys the product becomes

$$g(p,r) = \frac{1}{1+exp(b_1 p + b_2 A(r))}. \tag{3}$$

Please see Appendix for the derivation of D2.

3. The optimal inventory and pricing model

We consider an all-or-nothing ordering policy for single-item periodic-review pricing and inventory control problem for a retailer. When an order is placed by the retailer, a variable cost c and also a fixed cost $\tilde{K}$ are incurred. Unsatisfied demand is lost. We assume that there is an initial inventory of x. A holding cost is incurred if a unit of product is carried to the next period. We also assume an uncertainty in the yield, *i.e.*, the firm orders Q; however, it only receives rQ, where r is a realization of the random variable δ denoting the supply yield.

In the all-or-nothing policy, orders can only be full capacity Q or zero, meaning no order is placed. As mentioned in Section 2, we use various demand functions and observe their effect on the profit. Throughout this research, we assume that the demand is intermittent, and the demand occurrence depends only on the price and yield uncertainty as described in Eq. (1). We will use two different demand functions through two different demand occurrence probability functions, $g(p,r)$: D1 using Eq. (2) and D2 using Eq. (3).

In our models, $G(x,p)$ denotes the expected profit function, where the on-hand inventory is x and the price is p. We will explore two alternative pricing methods which differ in the order of events: The first one is responsive (dynamic) pricing, in which the firm decides on pricing after they observe the delivered amount, and the second is an unresponsive (static) pricing model in which the price is decided beforehand. We want to show if the optimal policy has a simple form threshold policy such that 'order if $x < s$, do not order otherwise'.

In Fig. 1, we illustrate a sample path for the system described. At the beginning of period 1, the on-hand inventory is equal to x, which is below the threshold s. As a result, an order of size Q is placed. The yield realization for period 1 is r_1, so $r_1 Q$ is received, and the demand $D_1(r_1)$ is observed. No demand occurs in the second and third periods, as the demand is intermittent and depends on price. The next order occurs in the sixth period as the inventory level falls below the order threshold.

3.1. Responsive pricing

The main difference between the responsive and unresponsive pricing schemes is in the order of events. In the responsive pricing scheme, the order of events is as below and Fig. 2:

Step 1: The retailer decides if the order is placed or not,

Step 2: The retailer receives the order and observes the supply yield,

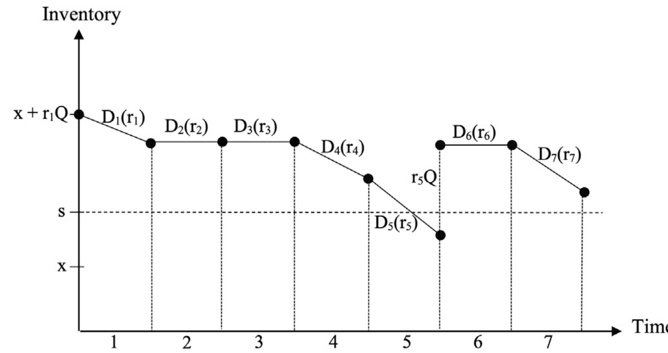

Fig. 1. Inventory diagram of the system.

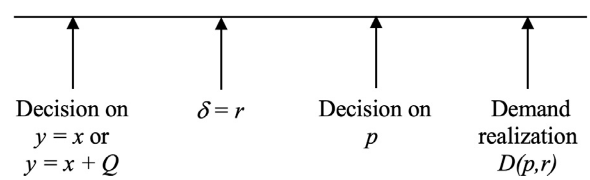

Fig. 2. Order of events in responsive pricing scheme.

Step 3: The retailer decides on the price,

Step 4: Demand occurs.

We compose the dynamic programming equations for the profit-to-go functions. Denote $p_n(r)$ as the price in period n when the realization of supply yield is $\delta = r$. Define $J_n(x,r)$ as the expected profit for the current and subsequent periods when the starting inventory is x, the yield is observed as r and there are n periods to go until the end of horizon. Then, the optimal expected profit $f_n(x)$ is given as

$$f_n(x) = max\{-K + E[J_n(x+Q,\delta)], E[J_n(x,\delta)]\}, \tag{4}$$

where the expectations in Eq. (4) are taken over δ. Without loss of generality we let $f_0(x) = 0 \quad \forall x \geq 0$ as the end of horizon condition. $J_n(x,r)$ is written as:

$$J_n(x,r) = \max_{p_n(r)\geq 0}\{G(x,p_n(r)) + E[f_{n-1}((x - D(p_n(r)))^+)]\}, \tag{5}$$

with $G(x,p_n(r))$ is the profit of the current period and $E[f_{n-1}((x - D(p_n(r)))^+)]$ is the expected optimal profit for the subsequent periods. Eqs. (4) and (5) describe the standard dynamic programming recursion of the problem (Bertsekas, 1987).

We can further write $G(x,p_n(r))$ as

$$\begin{aligned}
G(x,p_n(r)) &= p\, E[min(x, D(p_n(r)))] - h\, E[(x - D(p_n(r)))^+] \\
&= p_n(r)x - (p_n(r)+h)\, E[(x - D(p_n(r)))^+] \\
&= p_n(r)x - (p_n(r)+h)\, g(p_n(r),r)\, L(x) \\
&\quad - (p_n(r)+h)\,(1 - g(p_n(r),r))\, x \\
&= p_n(r)x + (p_n(r)+h)\, g(p_n(r),r)\,[x - L(x)] - (p_n(r)+h)\, x \\
&= (p_n(r)+h)\, g(p_n(r),r)\,[x - L(x)] - hx,
\end{aligned} \tag{6}$$

where

$$L(x) = \int_0^x (x - w)\, q_z(w)\, dw, \tag{7}$$

being the expected remaining inventory with $q_z(w)$ representing the probability density function of the demand size. $f_n(x)$ can further be clarified as

$$\begin{aligned}
f_n(x) &= max\{-K + E[J_n(x+Q,\delta)], E[J_n(x,\delta)]\} \\
&= -min\{K - E[J_n(x+Q,\delta)], -E[J_n(x),\delta]\} \\
&= -[-E[J_n(x,\delta)] + min\{E[J_n(x,\delta)] + K - E[J_n(x+Q,\delta)], 0\}] \\
&= E[J_n(x,\delta)] + max\{E[H_n(x,\delta)], 0\}.
\end{aligned}$$

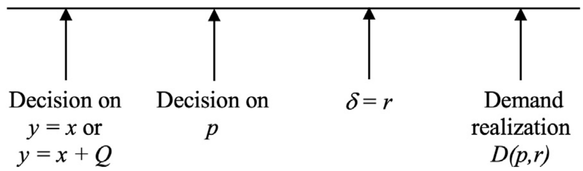

Fig. 3. Order of events in unresponsive pricing scheme.

$H_n(x,r)$ is the difference in expected profits between ordering and not ordering in period n with the on-hand inventory x and the yield realization is r. $H_n(x,r)$ is defined as follows:

$$H_n(x,r) = J_n(x+Q,r) - K - J_n(x,r). \tag{8}$$

It is optimal to order if $H_n(x,r) > 0$ and not to order if $H_n(x,r) < 0$.

We conjecture that the optimal policy has a simple threshold form: 'order if $x < s$, do not order otherwise'. To demonstrate this, we need to show $H_n(x)$ is non-increasing in x, hence, has a unique sign change from positive to negative. We will validate this conjecture for the single period problem in Section 3.3 and present numerical results on its validity in Section 4.

3.2. Unresponsive pricing

Unlike responsive pricing, in unresponsive pricing, the price is decided before observing the yield rate. The order of events is as follows and presented in Fig. 3:

Step 1: The retailer decides if the order is placed or not,
Step 2: The retailer decides on the price,
Step 3: The retailer receives the order and observes the supply yield,
Step 4: Demand occurs.

In the unresponsive pricing case, we decide on price and if we order or not based on the expectation of yield uncertainty due to the order of events. In this case, the expected profit, $J_n(x)$, does not depend on the yield realization and is given as

$$J_n(x) = \max_{p_n \ge 0} \{E[G(x,p_n)] + E[f_{n-1}((x - D(p_n(\delta)))^+)]\},$$

where the expectations above are taken on both demand and yield. $G(x,p_n)$ is defined as:

$$G(x,p_n) = (p_n + h)\, E_\delta[g(p_n,\delta)]\,[x - L(x)] - hx. \tag{9}$$

The remainder is the same as in the responsive case.

3.3. The single period problem

We next analyze the single-period problem by deriving $H(x)$, which represents the profit difference between ordering and not ordering for the single period. This analysis helps us better understand the problem under D1 and D2, the demand occurrence probability functions defined in Eqs. (2) and (3), respectively.

3.3.1. Responsive pricing

For the responsive pricing scheme, under D1, the profit function for the single period is stated as

$$G(y,p) = (p+h)\, g(p,r)\,[y - L(y)] - hy. \tag{10}$$

As the next step, we will take the derivative of this function with respect to p to obtain the optimal price for the current on-hand inventory level y as

$$\frac{\partial G(y,p)}{\partial p} = [y - L(y)]\,[g(p,r) + \frac{\partial g(p,r)}{\partial p}\,(p+h)]. \tag{11}$$

$\frac{\partial g(p,r)}{\partial p}$ is derived as

$$\frac{\partial g(p,r)}{\partial p} = \frac{-A(r)(b_1 + 2b_2 p)}{(1 + (b_1 p + b_2 p^2)A(r))^2}. \tag{12}$$

By inserting this into Eq. (11) we obtain p^* as

$$p^* = \frac{2hA(r)b_2 + \sqrt{4h^2(A(r))^2 b_2^2 + 4A(r)b_2(1 - hA(r)b_1)}}{-2A(r)b_2}, \tag{13}$$

provided that $b_2 \neq 0$, $A(r) \neq 0$. We are using the positive root since the price cannot be negative. For a given x and r, $G(x,p^*(r))$ and $G(x+Q,p^*(r))$ become

$$G(x,p^*(r)) = (p^*(r) + h)\, g(p^*(r),r)\,[x - L(x)] - hx, \tag{14}$$

and

$$G(x+Q,p^*(r)) = (p^*(r)+h)\, g(p^*(r),r)\,[x+rQ - L(x+rQ)] - h(x+rQ). \tag{15}$$

The difference in the profit between ordering and not ordering is calculated as

$$\begin{aligned} H(x) &= G(x+Q,p^*(r)) - K - G(x,p^*(r)) \\ &= (p^*(r)+h)\, g(p^*(r),r)\,[rQ - L(x+rQ) + L(x)] - h\,r\,Q - K, \end{aligned} \tag{16}$$

where $K = \tilde{K} + crQ$.

Proposition 1. *Under D1, the optimal policy for the responsive pricing scheme for a single period has a simple threshold form: 'order if $x < s$, do not order otherwise'.*

Proof. To demonstrate this, we will show $H_n(x)$ is non-increasing in x, and therefore, has a unique sign change from positive to negative. $H(x)$ is non-increasing in x.

$$\frac{\partial H(x)}{\partial x} = (p^*(r)+h)\, g(p^*(r),r)[-\frac{\partial L(x+Q)}{\partial x} + \frac{\partial L(x)}{\partial x}] < 0. \quad \square \tag{17}$$

For D2, we follow a similar procedure as D1 and obtain p^* as

$$p^* = \frac{W_0(exp(-b_2\, A(r)) * exp(b_1\, h - 1)) - b_1\, h + 1}{b_1}, \tag{18}$$

where W_0 is the principal branch of Lambert W function, provided that $b_1 \neq 0$. Lambert W function is the multi-valued inverse function of $f(w) = w\,exp(w)$ where w is any complex number, and $exp(w)$ is the exponential function (Corless et al., 1996). The solution of $a = w\,exp(w)$ is $w = W(a)$, which is used to solve exponential equations by converting them into this form. $W_{(k)}$ represents the branches of Lambert W function. Due to analytical difficulty, we conduct a numeric study in the following section.

3.3.2. Unresponsive pricing

For the unresponsive pricing scheme, the profit function becomes

$$G(y,p) = (p+h)\, E_\delta\,[g(p,\delta)]\,[y - L(y)] - hy, \tag{19}$$

and

$$p^*(y) := \arg\max_p G(y,p). \tag{20}$$

As seen in Eq. (19), we need to find $E_\delta[g(p,\delta)]$ to find the price maximizing $G(y,p)$. Under D1, stated in Eq. (2), $E_\delta[g(p,\delta)]$ becomes

$$E_\delta[g(p,\delta)] = \int_0^1 \frac{1}{1 + (b_1\, p + b_2\, p^2)\, A(r)} f_\delta(r)\, dr. \tag{21}$$

Let us assume δ is uniformly distributed on $[0,1]$ and $A(r) = r$ then

$$\begin{aligned} E_\delta[g(p,\delta)] &= \int_0^1 \frac{1}{1 + (b_1\, p + b_2\, p^2)\, r} f_\delta(r) dr \\ &= \frac{\ln(1 + (b_1\, p + b_2\, p^2))}{(b_1\, p + b_2\, p^2)} \end{aligned} \tag{22}$$

for $(b_1\, p + b_2\, p^2) \ge -1$.

Even with the uniformly distributed δ, it is very complex to derive p^* maximizing $G(y,p)$ for the unresponsive pricing scheme. Hence, we examine the results numerically in Section 4.

3.4. Comparison of responsive and unresponsive pricing

To compare the effect of responsive and unresponsive pricing on profit, we work on a simple, pure revenue-based profit model in which the revenue is denoted as $G(p,r)$ for illustration purposes. We utilize a simple demand function $D(p,r) = a(r) - bp + \epsilon$, where ϵ is a random variable with mean 0 and variance σ^2. To ensure the non-negativity of demand, we assume that $a(r) - bp + \epsilon \geq 0$ with probability 1. This function differs from the typical demand functions used in this study, serving only for illustration purposes in this section. As in previous sections, r is the realization of the random variable δ denoting the supply yield.

For the responsive case, below is our profit function:

$$\begin{aligned} G(p,r) &= E[D(p,r)]p \\ &= (a(r) - bp)p, \end{aligned} \tag{23}$$

therefore, p^* maximizing Eq. (23) is

$$p^*_{res} = \frac{a(r)}{2b}, \tag{24}$$

and so the average profit as

$$\begin{aligned} \Pi_{res} &= E[G(p^*_{res}(\delta),\delta)] \\ &= \frac{1}{4b}E[a(\delta)^2]. \end{aligned} \tag{25}$$

For the unresponsive case, our expected profit function is

$$E[G(p,\delta)] = (E[a(\delta)] - bp)p, \tag{26}$$

giving

$$p^*_{unres} = \frac{E[a(\delta)]}{2b}. \tag{27}$$

Consequently, the average profit for the unresponsive case is

$$\begin{aligned} \Pi_{unres} &= E[G(p^*_{unres}(\delta),\delta)] \\ &= \frac{1}{4b}E[a(\delta)]^2. \end{aligned} \tag{28}$$

Then,

$$\begin{aligned} \Pi_{res} &= \frac{1}{4b}(Var[a(\delta)] + E[a(\delta)]^2) \\ &= \frac{1}{4b}Var[a(\delta)] + \Pi_{unres}. \end{aligned} \tag{29}$$

Although it is difficult to show that responsive pricing always performs better than unresponsive pricing, we could show that the expected average profit is higher in responsive pricing under this simple stylized condition.

Let us look at our version with $\delta \sim Uniform(0,1)$ and $A(r) = r$ under D1. In responsive case, we insert p^*_{res}, Eq. (13), into (10) and obtain

$$\begin{aligned} \Pi_{res} &= E[G(p^*_{res}(\delta),\delta)] \\ &= \int_0^1 \{(p^*_{res}(r) + h)\, g(p^*_{res}(r),r)\,[x - L(x)] - hx\}\, dr \end{aligned} \tag{30}$$

and in unresponsive case we find p^* maximizing Eq. (19) and value of the profit for $x = 0,1,\ldots,10$. As an illustrative example, we assume that the demand size is exponentially distributed with $\lambda = 1, b_1 = b_2 = 1$ and observe that p^* satisfies the bounds (*i.e.* $0 \leq E[G(p^*_{res}(\delta),\delta)] \leq 1$). Table 2 demonstrates that, as expected, responsive pricing yields higher profits compared to unresponsive pricing. Additionally, the price under responsive pricing is lower than that of unresponsive pricing. It is also observed that the expected profit difference between responsive and unresponsive pricing is non-increasing in x.

4. Numerical results

We conduct a numerical study to observe the policy for single and two-period models for responsive and unresponsive pricing schemes.

4.1. Numerical results for the single period problem

First, we start with the single-period study for simplicity. For both responsive and unresponsive pricing schemes, we carry out the numerical study for demand model D1 by using R and MATLAB software. We use $A(r) = r$, $b_1 \in \{1,2\}$, $b_2 \in \{1,2\}$. For the demand size distribution we use exponentially distributed demand size with a mean of 1. The replenishment batch size is used as $Q = 2$. The cost parameters are chosen as $c = 0$, $\tilde{K} = 0.1$, $h \in \{0, 0.02, 0.04, 0.06, 0.08, 0.1\}$. As state variables, we choose $x \in \{0,1,2,\ldots,10\}$, and $r \in \{0.1, 0.2, \ldots, 0.9, 1\}$. Additionally, for the responsive pricing scheme, we conducted a similar numerical study for the demand model D2; however, due to the complexity of the Lambert W function, we could not pursue this analysis for the unresponsive pricing scheme.

4.1.1. Responsive pricing

First, for the single-period study, we conduct the numerical study under the responsive pricing scheme. Our observations based on the expected profit value at the optimum price are as follows:

- We observe that $H(x)$ is non-increasing in x while r (and therefore and $p(r)$) are constant; consequently, there is a threshold x level after which ordering does not make sense.
- At each x value, $H(x)$ increases up to an r level and then starts decreasing as r increases when h is small. However, at higher levels of h, $H(x)$ decreases as r increases.
- p^*, $g(p^*,r)$, and $G(x,p^*)$ decreases as r increases at each x value.

We also examined the behavior of $H(x)$ and other functions as we change the price p. As expected, $g(p,r)$ is non-increasing in p when r and x are held as constant due to the nature of the function. Based on the results, we observed that both $H(x)$ and $G(x,p)$ increase up to a certain value of p, after which they begin to decrease, while p increases and r and x remain constant.

Using MATLAB to handle the Lambert W function, we carried out a numerical analysis using D2 and identified similar outcomes. Here, again, we observed a non-increasing $H(x)$ leading to the existence of threshold policy in our numerical results. Additionally, we explored the use of $A(r) = \log(r)$ and noticed a similar non-increasing trend in the behavior of $H(x)$. Notably, there was an observed increase in p^*, $g(p^*,r)$, $G(x,p^*(r))$, and $H(x)$ when $A(r) = \log(r)$. We investigated various combinations of $b_1 \in \{1,2\}$ and $b_2 \in \{1,2\}$ as well. The results demonstrate that as the value of b_1 increases there is a corresponding decrease in $G(x,p)$ at the same y level. Similar trends were observed for b_2; however, the impact of b_2 on $G(x,p)$ is more pronounced than that of b_1, particularly when r is small. This effect increases with the growth of r, surpassing the influence of b_1 as r increases. This observation implies that for the demand model D2, the level of yield is more important than the price level.

4.1.2. Unresponsive pricing

For the unresponsive pricing scheme, firstly, we assume uniformly distributed yield uncertainty. Even with the uniformly distributed yield uncertainty, deriving p^* maximizing $G(x,p)$ is very complex for the single period. Hence, we examine the results numerically only under D1. Fig. 4 shows $G(x,p)$ calculated for initial on-hand inventory $x \in \{1,2,\ldots,10\}$, prices $p \in \{0.1, 0.2, \ldots, 4.9, 5\}$, and parameters $\lambda = 1$, $b_1 = b_2 = 1$, $h = 0.02$ with $\delta \sim Uniform[0,1]$, satisfying $0 \leq E_\delta[g(p,\delta)] \leq 1$. We observe that $G(x,p)$ increases up to a certain price level before starting to decrease, with δ following a Uniform[0,1] distribution. As illustrated in Fig. 5, as x increases, $G(x,p^*)$ initially rises to a peak value before subsequently declining.

We compared the single-period optimal prices for responsive and unresponsive pricing under uniformly distributed yield uncertainty, using different holding costs in scenario D1. The optimal price for varying r values is denoted as p^*_{res}, while p^*_{unres} remains constant regardless of r,

Table 2
Comparison of responsive and unresponsive pricing under D1.

h	λ	Q	x	p_{res}	p_{unres}	Π_{res}	Π_{unres}	Diff[a]
0	1	2	1	2.00	2.30	0.5126	0.412	24.44%
0	1	2	2	2.00	2.30	0.7012	0.5635	24.43%
0	1	2	3	2.00	2.30	0.7706	0.6193	24.43%
0	1	2	4	2.00	2.30	0.7961	0.6398	24.43%
0	1	2	5	2.00	2.30	0.8055	0.6473	24.43%
0	1	2	6	2.00	2.30	0.8089	0.6501	24.43%
0	1	2	7	2.00	2.30	0.8102	0.6511	24.43%
0	1	2	8	2.00	2.30	0.8107	0.6515	24.43%
0	1	2	9	2.00	2.30	0.8108	0.6516	24.43%
0	1	2	10	2.00	2.30	0.8109	0.6517	24.43%
0.01	1	2	1	1.99	2.28	0.505	0.4038	25.08%
0.01	1	2	2	1.99	2.28	0.6845	0.546	25.37%
0.01	1	2	3	1.99	2.28	0.7442	0.592	25.72%
0.01	1	2	4	1.99	2.28	0.7598	0.6026	26.10%
0.01	1	2	5	1.99	2.28	0.7592	0.6001	26.51%
0.01	1	2	6	1.99	2.28	0.7527	0.5929	26.95%
0.01	1	2	7	1.99	2.28	0.744	0.5839	27.41%
0.01	1	2	8	1.99	2.28	0.7345	0.5743	27.88%
0.01	1	2	9	1.99	2.28	0.7246	0.5645	28.38%
0.01	1	2	10	1.99	2.28	0.7147	0.5545	28.89%

[a] Diff represents the percentage difference in Π, calculated as $(\Pi_{res} - \Pi_{unres})/\Pi_{unres}$.

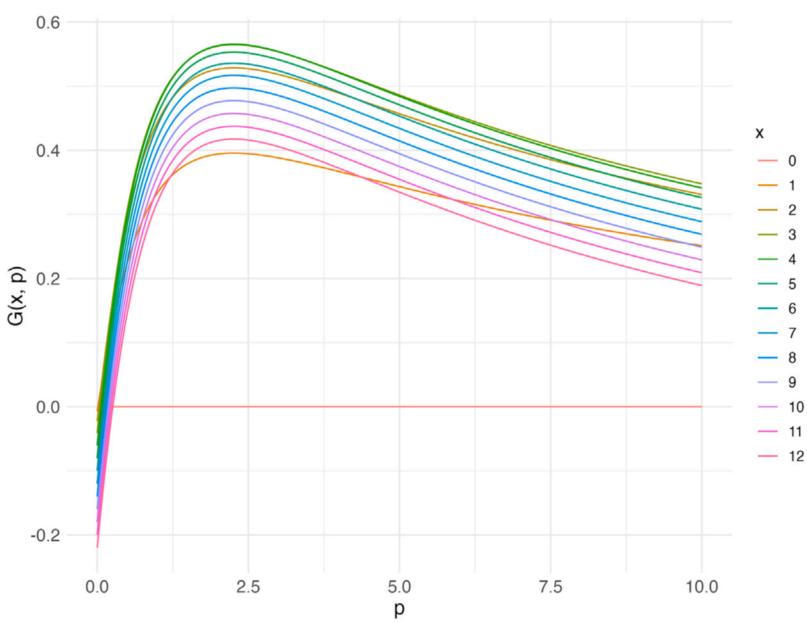

Fig. 4. Profit function in unresponsive pricing with uniformly distributed δ.

as it is determined before observing the realization of r. Fig. 6 illustrates an r value at which p^*_{res} and p^*_{unres} are equal for each holding cost. As the holding cost increases, this intersection r value also increases, while the price value $p^*_{res} = p^*_{unres}$ at this intersection decreases.

In addition to the uniform distribution, the beta distribution is widely used in modeling probabilities due to its flexibility and bounded domain [0,1]. Calculating $E_\delta[g(p,\delta)]$ with the assumption that δ belongs to beta distribution is very difficult analytically; therefore, we apply numerical integration. We conducted another numerical study to examine various shape parameters α and β, resulting in different mean and variance. Fig. 7 shows the shape in the first demand function with $A(r) = r$, shape parameters $\alpha \in \{1, 2, \ldots, 6\}$, $\beta \in \{1, 2, \ldots, 6\}$ for $x \in \{1, 2, \ldots, 10\}$, and prices $p \in \{0.1, 0.2, \ldots, 5\}$. Other parameters are $h = 0.02$, $\lambda = 1$, and $b_1 = b_2 = 1$. As seen in Fig. 7, we observe a similar shape for $G(x,p)$ when considering a beta-distributed δ compared to that of a uniformly distributed δ. $G(x,p)$ decreases as $\alpha = \beta$ increases for the same x and p values. It can be interpreted that as $\alpha = \beta$ increases, variance increases, negatively impacting the profit.

4.2. Numerical results for the two-period problem

We also conducted a numeric study for responsive and unresponsive pricing for two-period problems by using R for D1 as we did for the single period problem. Our parameter set is $h = 0.02$, $b_1 = b_2 = 1$, $c = 0$, $\tilde{K} = 0.1$, $Q = 2$ as in the single period problem.

Since the true distribution of $E[f_1((x - D(p_2(\delta)))^+)]$ is unknown, we used below discrete random variables (DRV) for demand to calculate $E[f_1((x - D(p_2(\delta)))^+)]$ numerically:

DRV1: values 3, 4, 5 with probability 0.1, 0.8, 0.1, respectively,

DRV2: values 2, 3, 4, 5, 6 with probability 0.1, 0.1, 0.6, 0.1, 0.1, respectively,

DRV3: values 1, 2, 3, 4, 5, 6, 7 with probability 0.1, 0.1, 0.1, 0.4, 0.1, 0.1, 0.1, all having a mean of 4 and a variance of 0.2, 1, and 2.8, respectively.

4.2.1. Responsive pricing

For $r \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$, and for $x \in \{1, 2, \ldots, 11, 12\}$ firstly, we calculate $J_2(x)$ by searching for p^* that maximizes $\max_{p\geq 0}\{G(x, p_2(\delta)) +$

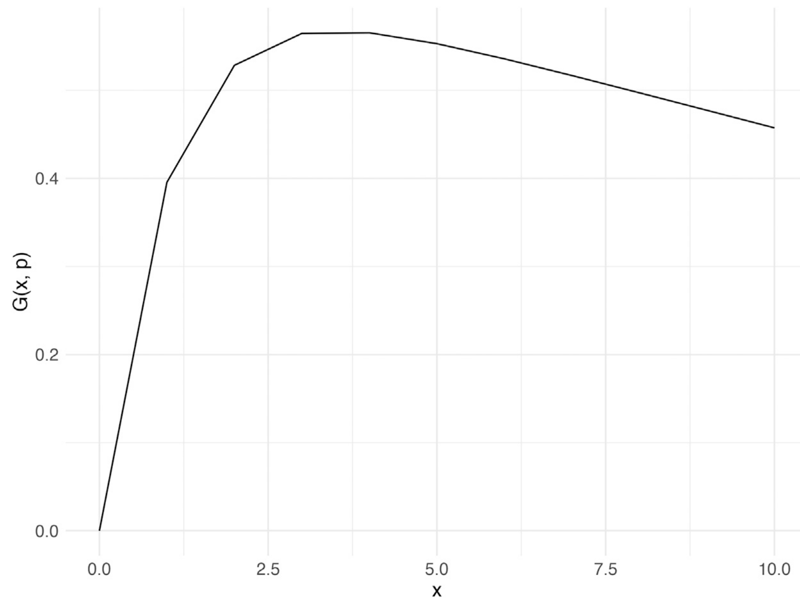

Fig. 5. Profit function in unresponsive pricing with uniformly distributed δ with optimum prices.

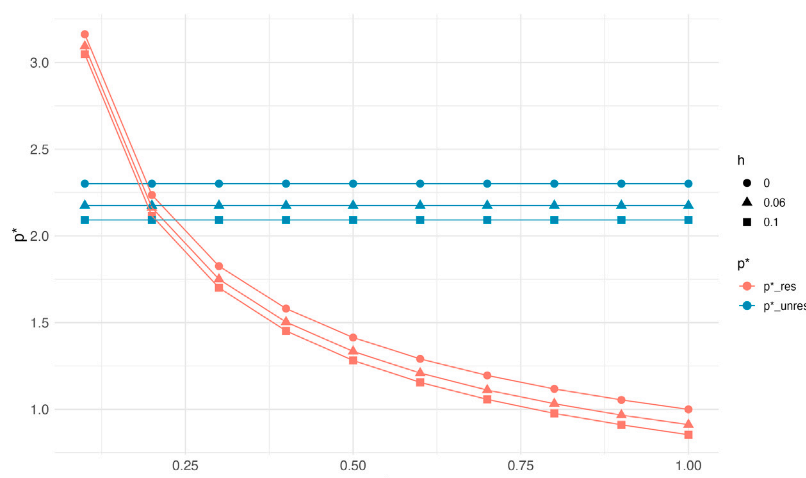

Fig. 6. Comparison of p^*_{res} and p^*_{unres}.

$E[f_1((x - D(p_2)(\delta))^+)]\}$ to observe how the expected profit changes as x increases, as shown in Fig. 8. As depicted in Fig. 8, we observe that

- $H_2(x)$ for $\forall x$ with $r \in \{0.1, 0.3, 0.5, 0.7, 0.9\}$ exhibits a non-increasing trend and only one sign change. As such, we observe a threshold beyond which ordering is anticipated to be less profitable; hence, the findings substantiate our claim.
- $J_2(x)$ increases up to a certain level of x and starts to decrease thereafter when r is smaller. However, as r gets bigger, *e.g.*, when $r = 0.9$, we observe that this pattern is broken; however, it does not affect the non-increasing trend in $H_2(x)$.
- p^* is non-increasing (decreasing or staying constant) until reaching that level of x and remains constant.
- As variance increases, the expected profit decreases.

4.2.2. *Unresponsive pricing*

We used the same parameter set for unresponsive pricing with the same discrete random variables for the demand, namely, DRV1, DRV2, and DRV3. Fig. 9 illustrates how the expected profit, $J_2(x)$, changes as x increases under the unresponsive pricing scheme for three different discrete random variables. $J_2(x)$ is calculated by searching for p^* that maximizes $\max_{p\geq 0}\{G(x, p_2(\delta)) + E[f_1((x - D(p_2)(\delta))^+)]\}$.

As seen from Fig. 9, with discrete random demand,

- $J_2(x)$ increases up to a certain level of x and starts to decrease beyond that level. We can conclude that $J_2(x)$ is unimodal but, not necessarily concave.
- As variance increases, the expected profit decreases or stays the same.

4.2.3. *Observations and insights*

We can infer the following observations and insights, based on our numerical computations.

1. The profit difference between responsive and unresponsive pricing cases can be substantial. If the pricing decision should precede the yield realization, then one should at least obtain a reliable yield forecast before deciding on the price.
2. Supporting our conjecture, in all the cases that we numerically considered, the threshold policy turned out to be optimal: there

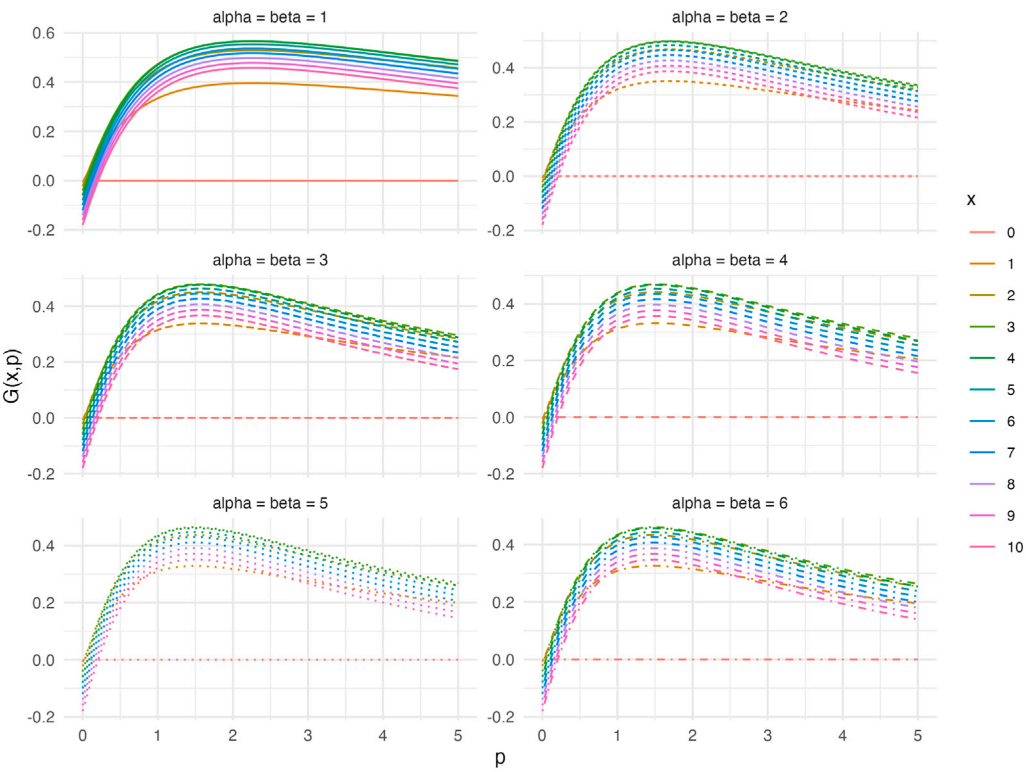

Fig. 7. Profit function in unresponsive pricing with beta distributed δ.

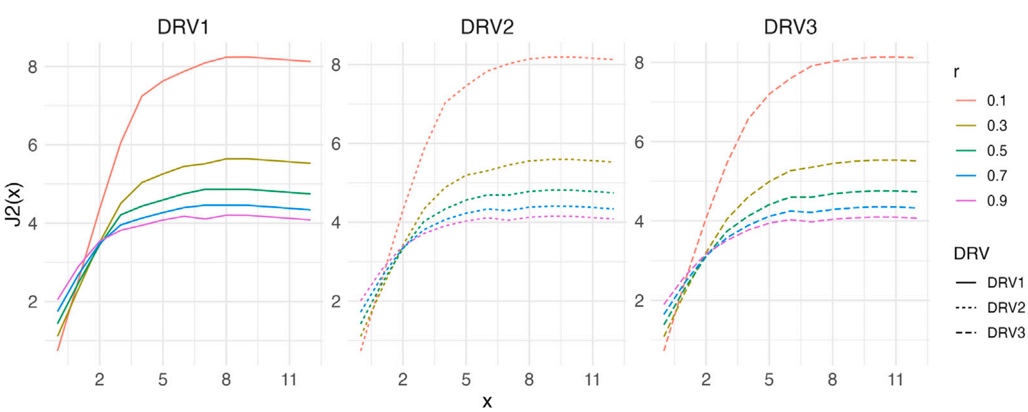

Fig. 8. Expected profit for two-period problem under responsive pricing scheme with discrete random demand.

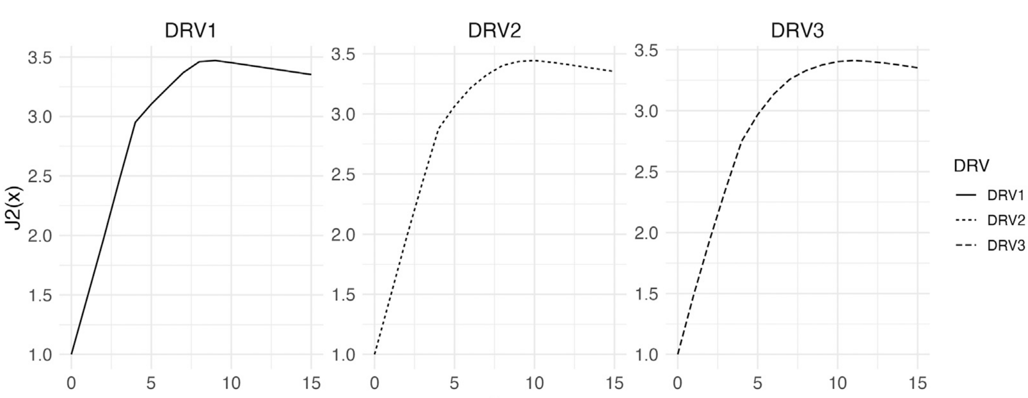

Fig. 9. Expected profit for two-period problem under unresponsive pricing scheme with discrete random demand.

is a threshold inventory level for which it is optimal to place an order if the inventory is less than or equal to this level; no order is placed otherwise.

3. If the customers are less likely to place an order at higher yield levels (with the anticipation that the product is expected to be available in the future), then the optimal yield dependent price decreases as observed yield increases.
4. There is a critical yield level r^0 so that if the yield is smaller (higher) than r^0, responsive pricing yields a larger (smaller) price compared to the unresponsive pricing. This critical yield level increases as the holding cost increases. Therefore, the tracking of the yield level becomes more important for more expensive items.
5. As the demand variability increases, the uncertainty rises, leading to a decrease in expected profit. Conversely, a more stable and predictable demand pattern contributes to higher profitability.

5. Conclusion

In this research, we investigated an all-or-nothing inventory and pricing problem with yield and price-dependent intermittent demand under supply uncertainty faced by a retailer in a single and multi-period setting. In yield and price-dependent intermittent demand, the demand occurs with a probability based on a function of the price p and yield realization r, called $g(p,r)$. We worked on two different forms of $g(p,r)$ as given in Eqs. (2) and (3), called D1 and D2, respectively. When demand occurs, its size follows a probability distribution, modeled using exponential and discrete random variables.

In addition to ordering or not ordering, the retailer has to decide on the price that will be applied to the products. For yield uncertainty, we investigated two pricing schemes: responsive (after observing hold replenishment policy yield) and unresponsive pricing (before observing yield). Our aim was to determine if there is a threshold policy where the retailer orders up to a level of x and does not order after that level, and we presented analytical (for the single period) and numerical findings (for the multi-period) for the optimality of a threshold replenishment policy and compared responsive versus unresponsive pricing schemes.

To the best of our knowledge, our demand model is novel in the supply chain literature. Our main contribution lies in developing a pricing and inventory policy specifically tailored for intermittent demand scenarios, where the occurrence of demand is influenced by pricing. Furthermore, our study introduces a novel dimension by incorporating yield uncertainty into the context of price-dependent intermittent demand, thereby extending the understanding of real-world supply chain dynamics. Additionally, we developed optimal pricing policies under responsive and unresponsive pricing schemes.

5.1. Discussion and managerial insights

In both our analytical and numerical analyses, we observed that a threshold policy, where orders are placed only when inventory falls below or reaches a specific level, is optimal. Furthermore, we identified significant profit differences between responsive and unresponsive pricing, emphasizing the importance of obtaining reliable yield forecasts when pricing decisions must precede yield realization.

Another key finding is that, when customers anticipate future availability, the optimal price decreases as yield increases. Additionally, we discovered a critical yield level, r^0, such that if the yield is lower (higher) than r^0, responsive pricing results in a higher (lower) price compared to unresponsive pricing. This critical yield level rises as holding costs increase, highlighting the growing importance of yield tracking for more expensive items. Lastly, as expected, we observed that increased demand variability reduces expected profit, whereas stable demand leads to greater profitability.

5.2. Future directions

There are several future research topics. Considering the problem complexity in our single-period and two-period settings, providing analytical results for the problem in a multi-period situation can be significantly challenging. Nonetheless, developing a heuristic for the multi-period problem is still a significant direction for future research. Another direction could be that instead of an all-or-nothing policy, the decision of the order amount could be incorporated into the problem.

Our research assumed that the demand size follows a random distribution. We examined exponential and discrete distributions for the demand size. Other distributions could be explored to determine if this threshold policy also holds for them. Additionally, we assumed that the demand size is unrelated to the price, while demand occurrence depends on the price. Moreover, our study of two-period problems assumed that only inventory connects the periods. The demand or yield uncertainty in the first period does not influence those in the second period. Another extension could involve introducing a link between the uncertainty of the first period and its impact on the demand in the subsequent period.

Another important future topic could be integrating demand learning for intermittent demand. In reality, we hardly know the true distribution of the demand; hence, information on the demand is obtained by observing the historical data. Also, intermittent demand forecasting has not been addressed in the literature thoroughly (Boylan & Syntetos, 2021; Nikolopoulos, 2021). As a follow-up research, a data-driven intermittent demand learning algorithm can be developed for dynamic pricing and inventory decisions.

CRediT authorship contribution statement

Elifnur Doğruöz: Writing – review & editing, Writing – original draft, Visualization, Validation, Methodology, Investigation, Formal analysis, Data curation, Conceptualization. **Refik Güllü:** Writing – review & editing, Writing – original draft, Validation, Supervision, Project administration, Methodology, Investigation, Formal analysis, Data curation, Conceptualization.

Appendix. Derivation of the demand models

We will use Train's notation to derive the random utility models (Train, 2009) and then lead the way to our demand models. In the random utility model, a decision-maker, n, has to choose among J options. Utility obtained from this selection of option j is shown as U_{nj}, $j = 1,\dots,J$. The decision-maker knows about the utility of every option j and chooses the one with the maximum utility. Hence, the decision-maker n chooses an option i if and only if $U_{ni} > U_{nj}, \forall j \neq i$. In this utility, there is an observed part by the researcher called V_{nj} and an unobserved part called ε_{nj}. Observed, in other words representative, utility V_{nj} is a function attributes of the choices, x_{nj}, and the attributes of the decision-maker, s_n, *i.e.* $V_{nj} = V(x_{nj}, s_n)$. Unobserved part ε_{nj} is simply the difference between the real and representative utility. Hence, the real utility can be written as

$$U_{nj} = V_{nj} + \varepsilon_{nj}. \tag{A.1}$$

ε_{nj} is unknown to the researcher and seen as random. $f(\varepsilon_n)$ is defined as the joint density of the random vector $\varepsilon_n = \langle \varepsilon_{n1},\dots,\varepsilon_{nj}\rangle$. This distribution helps us make inferences about the choices of the decision-maker. For the decision-maker n to choose option i is $P_{ni} = Prob\{U_{ni} > U_{nj}\}$.

Logit is one of the most well-known discrete choice models. It assumes that ε_{nj} is distributed independently, identically extreme value. Density and cumulative distribution of iid extreme valued ε_{nj} is

$$f(\varepsilon_{nj}) = exp(-\varepsilon_{nj})\,exp(-exp(-\varepsilon_{nj})), \tag{A.2}$$

and

$$F(\varepsilon_{nj}) = exp(-exp(-\varepsilon_{nj})), \tag{A.3}$$

respectively. Then,

$$P_{ni} = \frac{exp(V_{ni})}{\sum_j exp(V_{nj})}. \tag{A.4}$$

We can assume that the observed (representative) utility is linear in covariates, *i.e.* $V_{nj} = \beta' x_{nj}$, where x_{nj} is a vector of the attributes (variables) related to option j and β is the vector of coefficients. Therefore, we can rewrite P_{ni} as

$$P_{ni} = \frac{exp(\beta' x_{ni})}{\sum_j exp(\beta' x_{nj})}. \tag{A.5}$$

When there is a binary choice, the probability becomes

$$\begin{aligned} P_{ni} &= \frac{exp(\beta' x_{ni})}{exp(\beta' x_{ni}) + exp(\beta' x_{nj})} \\ &= \frac{1}{1 + exp(\beta' x_{nj} - \beta' x_{ni})} \end{aligned} \tag{A.6}$$

In our research, demand occurrence is a binary process. Our second $g(p,r)$ in Eq. (3) reflects the probability of demand occurrence under price p and yield uncertainty r. The utility of the decision-maker, or the customer in our case, is a function of price p and yield uncertainty r. Hence, the probability of the demand occurrence, in other words, the probability that the customer buys the product becomes

$$g(p,r) = \frac{1}{1 + exp(b_1 p + b_2 A(r))}$$

as in Eq. (3).

Data availability

Data will be made available on request.

References

Abdali, N., Heidari, S., Alipour-Vaezi, M., Jolai, F., & Aghsami, A. (2024). A priority queueing-inventory approach for inventory management in multi-channel service retailing using machine learning algorithms. *Kybernetes*, http://dx.doi.org/10.1108/K-07-2023-1281.

Alfares, H. K., & Ghaithan, A. M. (2016). Inventory and pricing model with price-dependent demand, time-varying holding cost, and quantity discounts. *Computers & Industrial Engineering*, *94*, 170–177. http://dx.doi.org/10.1016/j.cie.2016.02.009.

Bardhan, S., Modak, I., & Giri, B. C. (2024). A multi-period inventory model with price, time and service level dependent demand under preservation technology investment. *European Journal of Industrial Engineering*, http://dx.doi.org/10.1504/EJIE.2025.143085.

Ben-Ammar, O., Dolgui, A., Hnaien, F., & Louly, M.-A. O. (2022). Supply planning and inventory control under lead time uncertainty: A literature review and future directions. *IFAC-PapersOnLine*, *55*(10), 2749–2754. http://dx.doi.org/10.1016/j.ifacol.2022.10.138.

Bertsekas, D. P. (1987). *Dynamic Programming: Deterministic and stochastic models*. Prentice-Hall, Inc.

Bharadiya, J. P. (2023). Machine learning and AI in business intelligence: Trends and opportunities. *International Journal of Computer (IJC)*, *48*(1), 123–134.

Boylan, J. E., & Syntetos, A. A. (2021). *Intermittent demand forecasting: Context, methods and applications*. John Wiley & Sons.

Chao, X., Yang, B., & Xu, Y. (2012). Dynamic inventory and pricing policy in a capacitated stochastic inventory system with fixed ordering cost. *Operations Research Letters*, *40*(2), 99–107. http://dx.doi.org/10.1016/j.orl.2011.12.002.

Chen, W., & Dong, M. (2018). Joint inventory control across products with supply uncertainty and responsive pricing. *Journal of the Operational Research Society*, *69*(8), 1215–1226. http://dx.doi.org/10.1080/01605682.2017.1390527.

Chen, X., & Simchi-Levi, D. (2004). Coordinating inventory control and pricing strategies with random demand and fixed ordering cost: The finite horizon case. *Operations Research*, *52*(3), 887–896. http://dx.doi.org/10.1287/opre.1040.0127.

Corless, R. M., Gonnet, G. H., Hare, D. E. G., Jeffrey, D. J., & Knuth, D. E. (1996). On the LambertW function. *Advances in Computational Mathematics*, *5*(1), 329–359. http://dx.doi.org/10.1007/bf02124750.

Croston, J. D. (1972). Forecasting and stock control for intermittent demands. *Journal of the Operational Research Society*, *23*(3), 289–303. http://dx.doi.org/10.1057/jors.1972.50.

De, P. K., Devi, S. P., & Narang, P. (2024). Inventory model for deteriorating goods with stock and price-dependent demand under inflation and partial backlogging to address post Covid-19 supply chain challenges. *Results in Control and Optimization*, *14*, Article 100369. http://dx.doi.org/10.1016/j.rico.2023.100369.

Dong, L., Xiao, G., & Yang, N. (2023). Supply diversification under random yield: The impact of price postponement. *Production and Operations Management*, *32*(4), 1112–1132. http://dx.doi.org/10.1111/poms.13915.

Elmaghraby, W., & Keskinocak, P. (2003). Dynamic pricing in the presence of inventory considerations: Research overview, current practices, and future directions. *Management Science*, *49*(10), 1287–1309. http://dx.doi.org/10.1287/mnsc.49.10.1287.17315.

Federgruen, A., & Heching, A. (1999). Combined pricing and inventory control under uncertainty. *Operations Research*, *47*(3), 454–475. http://dx.doi.org/10.1287/opre.47.3.454.

Gallego, G., & Scheller-Wolf, A. (2000). Capacitated inventory problems with fixed order costs: Some optimal policy structure. *European Journal of Operational Research*, *126*(3), 603–613. http://dx.doi.org/10.1016/s0377-2217(99)00314-8.

Gallego, G., & Toktay, L. B. (2004). All-or-nothing ordering under a capacity constraint. *Operations Research*, *52*(6), 1001–1002. http://dx.doi.org/10.1287/opre.1040.0153.

Geng, X., Guo, X., Xiao, G., & Yang, N. (2023). Supply risk mitigation in a decentralized supply chain: Pricing postponement or payment postponement? *Manufacturing & Service Operations Management*, http://dx.doi.org/10.1287/msom.2022.0198.

Golmohammadi, A., & Hassini, E. (2019). Capacity, pricing and production under supply and demand uncertainties with an application in agriculture. *European Journal of Operational Research*, *275*(3), 1037–1049. http://dx.doi.org/10.1016/j.ejor.2018.12.027.

Golmohammadi, A., & Hassini, E. (2020). Review of supplier diversification and pricing strategies under random supply and demand. *International Journal of Production Research*, *58*(11), 3455–3487.

He, Y. (2013). Sequential price and quantity decisions under supply and demand risks. *International Journal of Production Economics*, *141*(2), 541–551. http://dx.doi.org/10.1016/j.ijpe.2012.09.010.

Hu, P., Lu, Y., & Song, M. (2019). Joint pricing and inventory control with fixed and convex/concave variable production costs. *Production and Operations Management*, *28*(4), 847–877. http://dx.doi.org/10.1111/poms.12950.

Huang, J., Leng, M., & Parlar, M. (2013). Demand functions in decision modeling: A comprehensive survey and research directions. *Decision Sciences*, *44*(3), 557–609. http://dx.doi.org/10.1111/deci.12021.

Islam, T., Pitafi, A. H., V. Arya, V., Wang, Y., Akhtar, N., Mubarik, S., & Xiaobei, L. (2021). Panic buying in the COVID-19 pandemic: A multi-country examination. *Journal of Retailing and Consumer Services*, *59*, Article 102357. http://dx.doi.org/10.1016/j.jretconser.2020.102357.

Ivanov, D. (2022). Viable supply chain model: Integrating agility, resilience and sustainability perspectives—Lessons from and thinking beyond the COVID-19 pandemic. *Annals of Operations Research*, *319*(1), 1411–1431. http://dx.doi.org/10.1007/s10479-020-03640-6.

Lu, F., Xu, H., Chen, P., & Zhu, S. X. (2018). Joint pricing and production decisions with yield uncertainty and downconversion. *International Journal of Production Economics*, *197*, 52–62. http://dx.doi.org/10.1016/j.ijpe.2017.12.020.

Moon, I., Ha, B., & Kim, J. (2012). Inventory systems with variable capacity. *European Journal of Industrial Engineering*, *6*(1), 68–86. http://dx.doi.org/10.1504/EJIE.2012.044811.

Nikolopoulos, K. (2021). We need to talk about intermittent demand forecasting. *European Journal of Operational Research*, *291*(2), 549–559. http://dx.doi.org/10.1016/j.ejor.2019.12.046.

Serel, D. A. (2017). A single-period stocking and pricing problem involving stochastic emergency supply. *International Journal of Production Economics*, *185*, 180–195. http://dx.doi.org/10.1016/j.ijpe.2016.12.016.

Shah, N. H., Rabari, K., & Patel, E. (2023). Greening efforts and deteriorating inventory policies for price-sensitive stock-dependent demand. *International Journal of Systems Science: Operations & Logistics*, *10*(1), Article 2022808. http://dx.doi.org/10.1080/23302674.2021.2022808.

Snyder, L. V., Lawrence, V., Atan, Z., Peng, P., Rong, Y., Schmitt, A. J., & Sinsoysal, B. (2016). OR/MS models for supply chain disruptions: A review. *IIE Transactions*, *48*(2), 89–109. http://dx.doi.org/10.1080/0740817x.2015.1067735.

Snyder, L., & Shen, Z. (2019). *Fundamentals of supply chain theory* (2nd ed.) Wiley.

Surti, C., Hassini, E., & Abad, P. (2013). Pricing and inventory decisions with uncertain supply and stochastic demand. *Asia-Pacific Journal of Operational Research*, *30*(06), Article 1350030. http://dx.doi.org/10.1142/s0217595913500309.

Syntetos, A. A., & Boylan, J. E. (2005). The accuracy of intermittent demand estimates. *International Journal of Forecasting*, *21*(2), 303–314. http://dx.doi.org/10.1016/j.ijforecast.2004.10.001.

Tang, C. S. (2006). Perspectives in supply chain risk management. *International Journal of Production Economics*, *103*(2), 451–488. http://dx.doi.org/10.1016/j.ijpe.2005.12.006.

Tang, O., Musa, S. N., & Li, J. (2012). Dynamic pricing in the newsvendor problem with yield risks. *International Journal of Production Economics*, *139*(1), 127–134. http://dx.doi.org/10.1016/j.ijpe.2011.01.01.

Tang, C. S., & Yin, R. (2007). Responsive pricing under supply uncertainty. *European Journal of Operational Research*, *182*(1), 239–255. http://dx.doi.org/10.1016/j.ejor.2006.07.011.

Train, K. E. (2009). *Discrete choice methods with simulation*. Cambridge University Press.

Turkmen, A. C., Januschowski, T., Wang, Y., & Cemgil, A. T. (2020). Intermittent demand forecasting with renewal processes. http://dx.doi.org/10.48550/ARXIV.2010.01550, arXiv:2010.01550.

Xu, M., & Lu, Y. (2013). The effect of supply uncertainty in price-setting newsvendor models. *European Journal of Operational Research*, *227*(3), 423–433. http://dx.doi.org/10.1016/j.ejor.2012.12.008.