ISTANBUL TECHNICAL UNIVERSITY
DEPARTMENT OF MECHANICAL ENGINEERING

NUMERICAL METHODS IN FLUID FLOW AND HEAT TRANSFER

MIA 502E

SPRING 2019

Homework 3

due to April, the 25th

Question1 (50p)

Use the finite difference method to approximate the solution to the given PDE with the prescribed boundary conditions. First use Gauss-Seidel methodology and then GS-SOR taking the relaxation factor 1.25. Compare the speed of convergence and your numerical output with that of the analytical solution u(x,y)=cos(x)cos(y). Take the spatial time steps h = π/10 and k = π/20. Plot the 3D chart of u(x,y) versus x and y.

$$\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=-\left[\cos\left(x+y\right)+\cos\left(x-y\right)\right] \quad ; 0<x<\pi \quad 0<y<\frac{\pi}{2}$$

$$u\left(0,y\right)=\cos y \quad u\left(\pi,y\right)=-\cos y \quad 0\le y\le\frac{\pi}{2}$$

$$u\left(x,0\right)=\cos x \quad u\left(x,\frac{\pi}{2}\right)=0 \qquad 0\le x\le\pi$$

Question2 (50p)

Using standard central difference approximations, obtain solutions of this equation for low values of Re for lid-driven cavity on a rectangular computational domain 1x2 (mesh: 100 x 200). Demonstrate that there are convergence problems for increasing Re and that these can be cured using forward-backward (upwind-downwind) algorithm. Plot contours of u for several Re numbers (Re = 1,5,50,100,1000). The boundary conditions are:

u(0,y) = u(1,y) = u(x,0) = 0 and u(x,2) = U = 1.

$$u\frac{\partial u}{\partial x}=\frac{1}{\mathrm{Re}}\left\{\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right\}$$