ISTANBUL TECHNICAL UNIVERSITY
DEPARTMENT OF MECHANICAL ENGINEERING

NUMERICAL METHODS IN FLUID FLOW AND HEAT TRANSFER

MIA 502E

SPRING 2018-19

Homework 2

due to April, the 11th

Question1 (50p)

A semi-finite marble slab that is L = 1 m thick is initially at a temperature distribution $T(x,0) = \cos\left[\pi(x-0.5)\right]$. Suddenly both surfaces are lowered to 0ºC and are maintained at that temperature. The problem is governed by:

$$\frac{\partial T}{\partial t} = \frac{1}{\pi^2}\frac{\partial^2 T}{\partial x^2} \quad \text{for } 0<x<1 \text{ [m] and } t>0$$

(a) Develop an explicit finite-difference scheme (FTCS) for the determination of the temperature distribution in the slab as a function of position and time as well as the heat flux at the boundary surface (k = 100 W/mK). Take Δt = 0.04 seconds.

(b) Develop Crank-Nicolson scheme for the determination of the temperature distribution in the slab as a function of position and time as well as the heat flux at the boundary surface (k = 100 W/mK). Take Δt = 0.04 seconds.

(c) For N=10 nodes and given time step size plot T(x,t=0;0.1;0.2;0.3;0.4;4;40 s) compare your numerical results with the analytical solution. The exact analytical solution of this problem for temperature distribution is:

$$T(x,t) = e^{-t}\cos\left[\pi(x-0.5)\right]$$

Question2 (50p)

Consider the problem of the time-dependent 2D heat conduction in the 3x1 rectangular plate shown below. Assume that initially the plate is at a room temperature of 18℃. Suppose that at t=0, the temperature on the lower face is suddenly raised to 150℃. Estimate the time required for the temperature of the center of the block to reach 30℃. Plot the temperature distribution (isotherms) for t → ∞. Temperature distribution over the plate should be indicated in colors. Use computational domains such as 30x10, 90x30 and 180x60 and compare your results for each mesh where b = 10 cm.

$$\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \quad 0 \le x \le a \quad 0 \le y \le b$$