2.2. Temos que resolver o problema

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} \\ u(x,0) = f(x) = 20, \; 0 < x < 40 \\ u(0,t) = 0, \; u(40,t) = 60 \end{cases}$$

A solução é então

$$u(x,t) = \frac{3x}{2} + \sum_{n=1}^{\infty} c_n \operatorname{sen} \frac{n\pi x}{40} e^{-\frac{n^2\pi^2}{1600}\,t}$$

em que c_n são os coeficientes da série de senos de

$$f(x) - \frac{3x}{2} = 20 - \frac{3x}{2}$$

ou seja,

$$\begin{aligned} c_n &= \frac{1}{20}\int_0^{40} f(x)\operatorname{sen}(\frac{n\pi x}{40})dx \\ &= 20 c_n(f_{0,1}^{(0)}) - \frac{3}{2} c_n(f_{0,1}^{(1)}) \\ &= -\frac{40}{n\pi}\cos s\Big|_0^{n\pi} - \frac{120}{n^2\pi^2}\left(-s\cos s + \operatorname{sen} s\right)\Big|_0^{n\pi} \\ &= -\frac{40}{n\pi}(\cos(n\pi) - 1) - \frac{120}{n^2\pi^2}\left(-n\pi\cos(n\pi)\right) \\ &= \frac{40(1+2(-1)^n)}{n\pi}, \; n = 1,2,3\ldots \end{aligned}$$

Portanto a solução é dada por

$$u(x,t) = \frac{3x}{2} + \frac{40}{\pi}\sum_{n=1}^{\infty} \frac{1+2(-1)^n}{n} \operatorname{sen}\frac{n\pi x}{40} e^{-\frac{n^2\pi^2}{1600}\,t}$$

Quando t tende a mais infinito a solução tende a solução estacionária $v(x,t) = \dfrac{3x}{2}$.

2.3. (a) Temos que resolver o problema

$$\begin{cases} \dfrac{\partial u}{\partial t} = \dfrac{\partial^2 u}{\partial x^2} \\ u(x,0) = f(x) = \frac{3x}{2}, \; 0 < x < 40 \\ \dfrac{\partial u}{\partial t}(0,t) = 0, \; \dfrac{\partial u}{\partial t}(40,t) = 0 \end{cases}$$

A solução é então

$$u(x,t) = \sum_{n=0}^{\infty} c_n \cos\frac{n\pi x}{40} e^{-\frac{n^2\pi^2}{1600}\,t}$$

em que c_n são os coeficientes da série de cossenos de $f(x)$, ou seja,

$$\begin{aligned} c_0 &= \frac{1}{40}\int_0^{40} f(x)dx = 30, \\ c_n &= \frac{1}{20}\int_0^{40} f(x)\cos\frac{n\pi x}{40}\,dx \\ &= \frac{3}{2}c_n(f_{0,1}^{(1)}) = \frac{120}{n^2\pi^2}\left(s\operatorname{sen} s + \cos s\right)\Big|_0^{n\pi} \\ &= 120\frac{(-1)^n - 1}{n^2\pi^2}, \; n = 1,2,3\ldots \end{aligned}$$

Portanto a solução é dada por

$$\begin{aligned} u(x,t) &= 30 + \frac{120}{\pi^2}\sum_{n=1}^{\infty}\frac{(-1)^n - 1}{n^2}\cos\frac{n\pi x}{40} e^{-\frac{n^2\pi^2}{1600}\,t} \\ &= 30 - \frac{240}{\pi^2}\sum_{n=0}^{\infty}\frac{1}{(2n+1)^2}\cos\frac{(2n+1)\pi}{40}x\, e^{-\frac{(2n+1)^2\pi^2}{1600}\,t} \end{aligned}$$

(b) $\lim\limits_{t\to\infty} u(x,t) = 30.$