

ECONOMETRICS

The background of the slide is a dark blue gradient. It features a central image of a classical statue, possibly representing a deity or scholar, holding a tablet and pointing upwards. To the left of the statue, there is a bar chart and the text 'DATA SCIENCE'. To the right, there is a pie chart and another bar chart. The word 'ECONOMETRICS' is written in a smaller font at the bottom center.

Precision or Standard Errors of OLS Estimates

ECONOMETRICS

Lesson Goal

- Learn to measure precision of OLS estimates via their standard errors.

Precision or Standard Errors of OLS Estimates

- The least squares estimators are obtained from sample data.
- Since data are likely to change from sample to sample, the estimates will change likewise.
- Thus, we need some measure of “**reliability**” or **precision** of the estimates, $\hat{\beta}_1$ and $\hat{\beta}_2$.
- In statistics, the precision of an estimate is measured by its **standard error** (se).

Precision or Standard Errors of OLS Estimates

- The **variance** (var) and **standard errors** (se) of the estimates are:

Slope coefficient

$$var(\widehat{\beta}_2) = \frac{\sigma^2}{\sum x_i^2}$$

$$se(\widehat{\beta}_2) = \frac{\sigma}{\sqrt{\sum x_i^2}}$$

Intercept coefficient

$$var(\widehat{\beta}_1) = \frac{\sum X_i^2}{n \sum x_i^2} \cdot \sigma^2$$

$$se(\widehat{\beta}_1) = \sqrt{\frac{\sum X_i^2}{n \sum x_i^2}} \cdot \sigma$$

where: σ^2 is the constant or homoscedastic variance of u_i .

Precision or Standard Errors of OLS Estimates

- The homoscedastic variance σ^2 can be estimated using:

$$\hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n - 2}$$

where: $\hat{\sigma}^2$ is the OLS estimator of the true but unknown variance σ^2 ;

$n - 2$ is the number of degrees of freedom (df);

$\sum \hat{u}_i^2$ is the sum of the residuals squared or **residual sum of squares (RSS)**: $\sum \hat{u}_i^2 = \sum (Y_i - \hat{Y}_i)^2$

Precision or Standard Errors of OLS Estimates

- The square root of the true variance σ^2 is obtained:

$$\hat{\sigma} = \sqrt{\frac{\sum \hat{u}_i^2}{n - 2}}$$

where: $\hat{\sigma}$ is called the **standard error of estimate**, or the **standard error of the regression** (se).

Features of the Variances of OLS Estimates

- The variance of the slope coefficient $\widehat{\beta}_2$ is directly proportional to σ^2 but inversely proportional to $\sum x_i^2$.

$$\text{var}(\widehat{\beta}_2) = \frac{\sigma^2}{\sum x_i^2}$$

Features of the Variances of OLS Estimates

- The variance of the intercept coefficient $\widehat{\beta}_1$ is directly proportional to σ^2 and $\sum X_i^2$ but inversely proportional to $\sum x_i^2$ and the sample size n .

$$\text{var}(\widehat{\beta}_1) = \frac{\sum X_i^2}{n \sum x_i^2} \cdot \sigma^2$$

Features of the Variances of OLS Estimates

- Since $\widehat{\beta}_1$ and $\widehat{\beta}_2$ are estimators, they will not only vary from sample to sample, but in a given sample, they are likely to be dependent on each other, and this dependence is measured by the covariance between them.

$$\begin{aligned} cov(\widehat{\beta}_1, \widehat{\beta}_2) &= -\bar{X} \cdot var(\widehat{\beta}_2) \\ &= -\bar{X} \left(\frac{\sigma^2}{\sum x_i^2} \right) \end{aligned}$$

THANK YOU!

Next Lesson: Properties of OLS Estimates: Gauss-Markov Theorem (with PROOF)