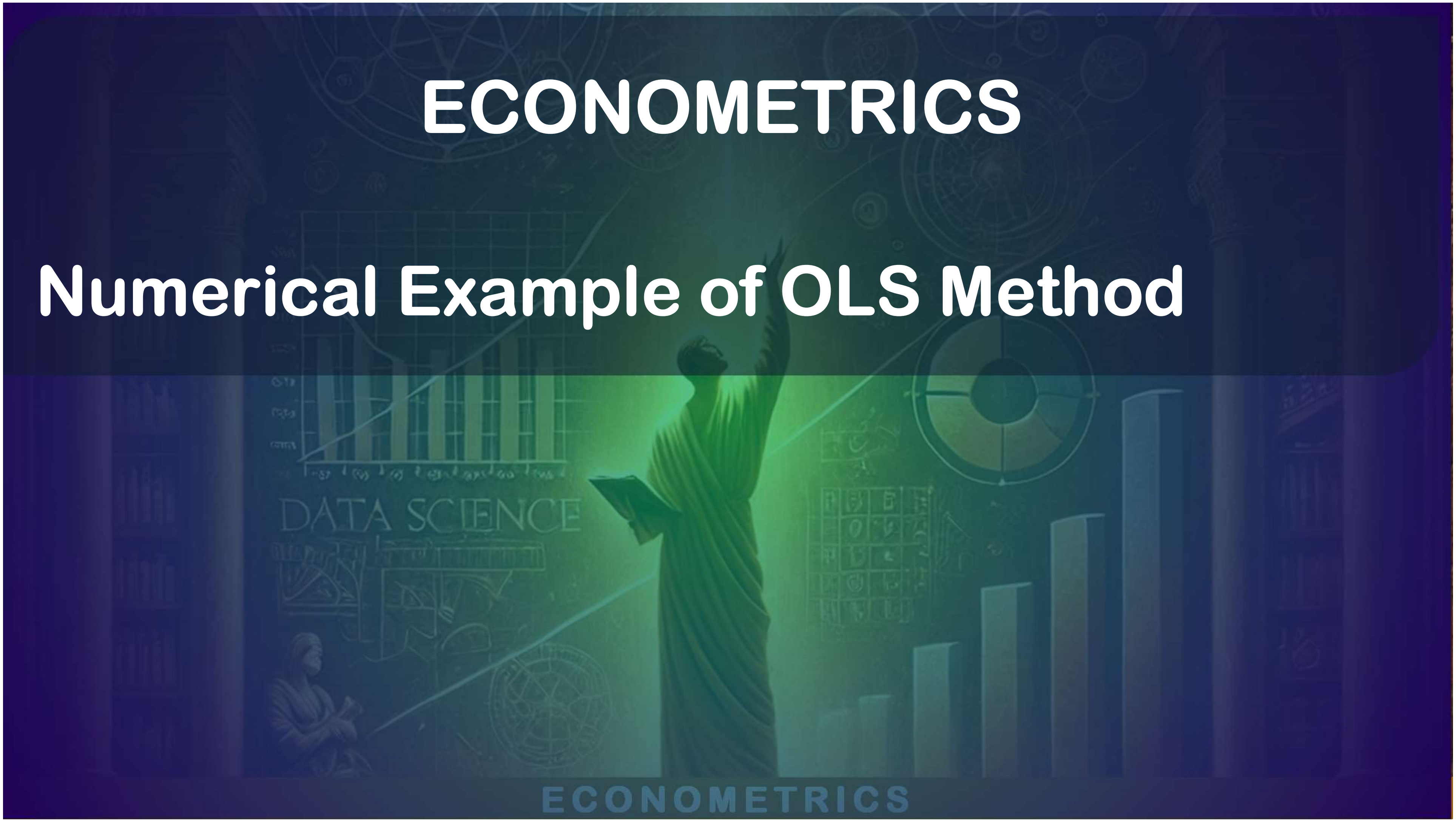
ECONOMETRICS

Numerical Example of OLS Method



Lesson Goal

- Apply the method of ordinary least squares to estimate regression coefficients in a simple linear regression setup.

Numerical Example of OLS Method

Obs.	Y_i	X_i
1	70	80
2	65	100
3	90	120
4	95	140
5	110	160
6	115	180
7	120	200
8	140	220
9	155	240
10	150	260

Y = Dependent Variable
X = Independent Variable

$n = 10$ (sample size)

Population Regression Function:

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

Sample Regression Function:

$$\hat{Y}_i = \widehat{\beta_1} + \widehat{\beta_2} X_i + \widehat{u_i}$$

Regression Coefficients

Slope:

$$\widehat{\beta_2} = \frac{\Sigma(\mathrm{X_i} - \overline{\mathrm{X}})(Y_i - \overline{Y})}{\Sigma(\mathrm{X_i} - \overline{\mathrm{X}})^2}$$

Intercept:

$$\widehat{\beta_1} = \overline{Y} - \widehat{\beta_2}\overline{X}$$

Numerical Example of OLS Method

Obs.	Y_i	X_i
1	70	80
2	65	100
3	90	120
4	95	140
5	110	160
6	115	180
7	120	200
8	140	220
9	155	240
10	150	260
Sum	**1110**	**1700**

Mean of Y:

$$\bar{Y} = \frac{\Sigma Y_\mathrm{i}}{n}$$

Mean of X:

$$\bar{X} = \frac{\Sigma X_\mathrm{i}}{n}$$

Mean of Y:

$$\bar{Y} = \frac{1110}{10} = 111$$

Mean of X:

$$\bar{X} = \frac{1700}{10} = 170$$

Numerical Example of OLS Method

Obs.	Y_i	X_i	$X_i - \bar{X}$
1	70	80	-90
2	65	100	-70
3	90	120	-50
4	95	140	-30
5	110	160	-10
6	115	180	10
7	120	200	30
8	140	220	50
9	155	240	70
10	150	260	90
Sum	**1110**	**1700**	

Mean of Y:

$$\bar{Y} = 111$$

Mean of X:

$$\bar{X} = 170$$

Numerical Example of OLS Method

Obs.	Y_i	X_i	$X_i - \bar{X}$
1	70	80	-90
2	65	100	-70
3	90	120	-50
4	95	140	-30
5	110	160	-10
6	115	180	10
7	120	200	30
8	140	220	50
9	155	240	70
10	150	260	90
Sum	**1110**	**1700**	0

Mean of Y:

$$\bar{Y} = 111$$

Mean of X:

$$\bar{X} = 170$$

Numerical Example of OLS Method

Obs.	Y_i	X_i	$X_i - \bar{X}$	$Y_i - \bar{Y}$
1	70	80	-90	-41
2	65	100	-70	-46
3	90	120	-50	-21
4	95	140	-30	-16
5	110	160	-10	-1
6	115	180	10	4
7	120	200	30	9
8	140	220	50	29
9	155	240	70	44
10	150	260	90	39
Sum	**1110**	**1700**	0	

Mean of Y:
$$\bar{Y} = 111$$
Mean of X:
$$\bar{X} = 170$$

Numerical Example of OLS Method

Obs.	Y_i	X_i	$X_i - \bar{X}$	$Y_i - \bar{Y}$
1	70	80	-90	-41
2	65	100	-70	-46
3	90	120	-50	-21
4	95	140	-30	-16
5	110	160	-10	-1
6	115	180	10	4
7	120	200	30	9
8	140	220	50	29
9	155	240	70	44
10	150	260	90	39
Sum	**1110**	**1700**	**0**	**0**

Mean of Y:

$$\bar{Y} = 111$$

Mean of X:

$$\bar{X} = 170$$

Numerical Example of OLS Method

Obs.	Y_i	X_i	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$
1	70	80	-90	-41	8100
2	65	100	-70	-46	4900
3	90	120	-50	-21	2500
4	95	140	-30	-16	900
5	110	160	-10	-1	100
6	115	180	10	4	100
7	120	200	30	9	900
8	140	220	50	29	2500
9	155	240	70	44	4900
10	150	260	90	39	8100
Sum	1110	1700	0	0	

Numerical Example of OLS Method

Obs.	Y_i	X_i	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$
1	70	80	-90	-41	8100
2	65	100	-70	-46	4900
3	90	120	-50	-21	2500
4	95	140	-30	-16	900
5	110	160	-10	-1	100
6	115	180	10	4	100
7	120	200	30	9	900
8	140	220	50	29	2500
9	155	240	70	44	4900
10	150	260	90	39	8100
Sum	1110	1700	0	0	33000

Numerical Example of OLS Method

Obs.	Y_i	X_i	$X_i - \bar{X}$	$Y_i - \bar{Y}$	$(X_i - \bar{X})^2$	$(X_i - \bar{X})(Y_i - \bar{Y})$
1	70	80	-90	-41	8100	3690
2	65	100	-70	-46	4900	3220
3	90	120	-50	-21	2500	1050
4	95	140	-30	-16	900	480
5	110	160	-10	-1	100	10
6	115	180	10	4	100	40
7	120	200	30	9	900	270
8	140	220	50	29	2500	1450
9	155	240	70	44	4900	3080
10	150	260	90	39	8100	3510
Sum	1110	1700	0	0	33000	

Numerical Example of OLS Method

Obs.	Y_i	X_i	$X_i - \overline{X}$	$Y_i - \overline{Y}$	$(X_i - \overline{X})^2$	$(X_i - \overline{X})(Y_i - \overline{Y})$
1	70	80	-90	-41	8100	3690
2	65	100	-70	-46	4900	3220
3	90	120	-50	-21	2500	1050
4	95	140	-30	-16	900	480
5	110	160	-10	-1	100	10
6	115	180	10	4	100	40
7	120	200	30	9	900	270
8	140	220	50	29	2500	1450
9	155	240	70	44	4900	3080
10	150	260	90	39	8100	3510
Sum	1110	1700	0	0	33000	16800

Numerical Example of OLS Method

$\Sigma \mathrm{X_i} = 1110$ $\bar{X} = 170$ $\Sigma(X_i - \bar{X})^2 = 33000$

$\Sigma Y_{\mathrm{i}} = 1700$ $\bar{Y} = 111$ $\Sigma(X_i - \bar{X})(Y_i - \bar{X}) = 16800$

Slope:

$$\widehat{\beta_2} = \frac{\Sigma(\mathrm{X_i} - \bar{\mathrm{X}})(Y_i - \bar{Y})}{\Sigma(\mathrm{X_i} - \bar{\mathrm{X}})^2} = \frac{16800}{33000} = 0.5091$$

Intercept:

$$\widehat{\beta_1} = \bar{Y} - \widehat{\beta_2}\bar{X} = 111 - (0.5091 \times 170) = 24.453$$

Regression Equation (Estimated Y): $\hat{Y} = 24.453 + 0.5091X$

Ordinary Least Squares (OLS) Method - Interpretation

Regression Equation (Estimated Y):

$$\hat{Y} = 24.453 + 0.5091X$$

Intercept	Slope
When $X = 0$, $\hat{Y} = 24.453 + 0.5091(0) = 24.453$ Thus, the predicted $\hat{Y}$ is estimated to be 24.453 when $X = 0$.	• Shows the change in the estimated $\hat{Y}$ for a unit change in X. • If positive, estimated $\hat{Y}$ increases by $\widehat{\beta_2}$ units (i.e. 0.5091) when X increases by one-unit, and vice-versa.

OLS Method – Visualize using Scatter Diagram

Obs.	Y_i	X_i
1	70	80
2	65	100
3	90	120
4	95	140
5	110	160
6	115	180
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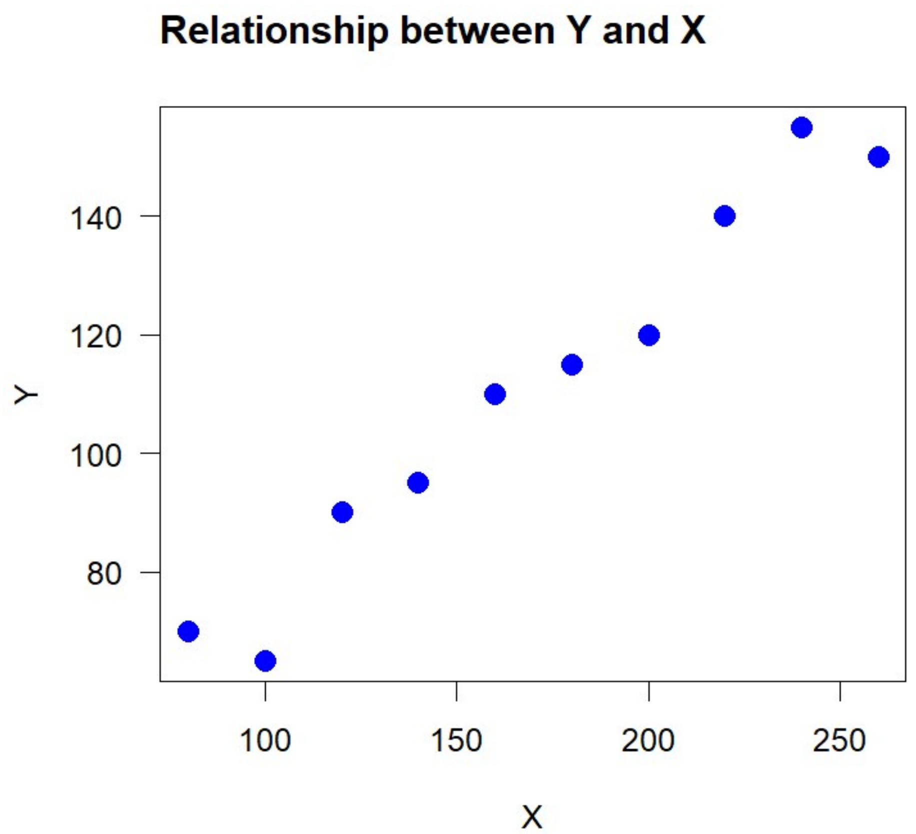
9	155	240
10	150	260



OLS Method – Visualize using Scatter Diagram

Obs.	Y_i	X_i
1	70	80
2	65	100
3	90	120
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6	115	180
7	120	200
8	140	220
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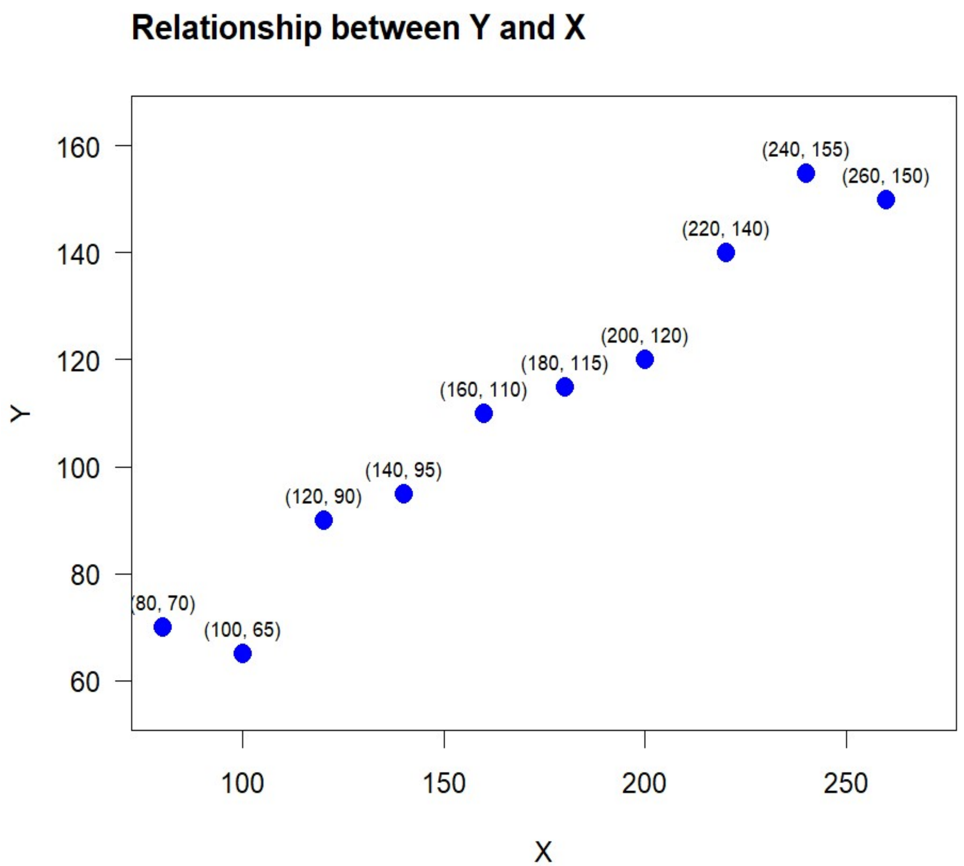

OLS Method – Visualize using Scatter Diagram

Obs.	Y_i	X_i
1	70	80
2	65	100
3	90	120
4	95	140
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8	140	220
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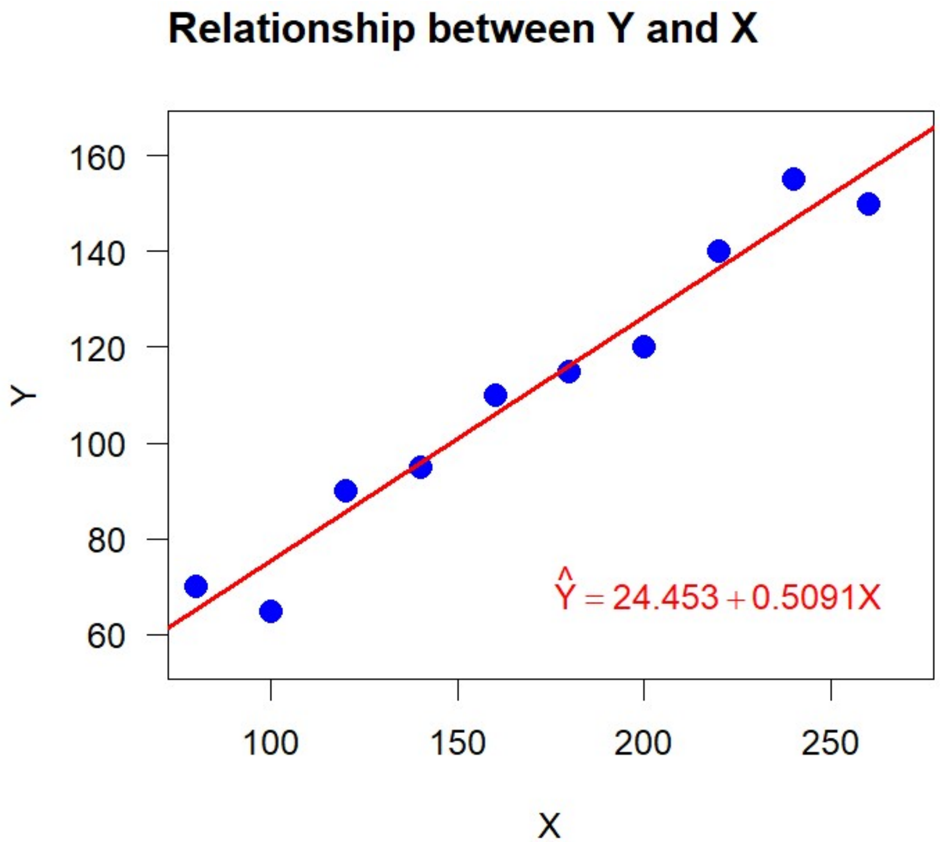

THANK YOU!

Next Lesson: Statistical Properties of OLS Estimates