ECONOMETRICS

Numerical Properties of OLS Estimates

Lesson Goal

- Understand the numerical properties of OLS estimates.

Numerical Properties of OLS Estimates

- The regression coefficients obtained by the OLS method are called "least-squares estimators".

- The least squares estimators have numerical properties.

  "Numerical properties are those that hold as a consequence of the use of ordinary least squares, regardless of how the data were generated."

  (Russell Davidson and James G. MacKinnon, Estimation and Inference in Econometrics, Oxford University Press, New York, 1993, p. 3.)

Numerical Properties of OLS Estimates

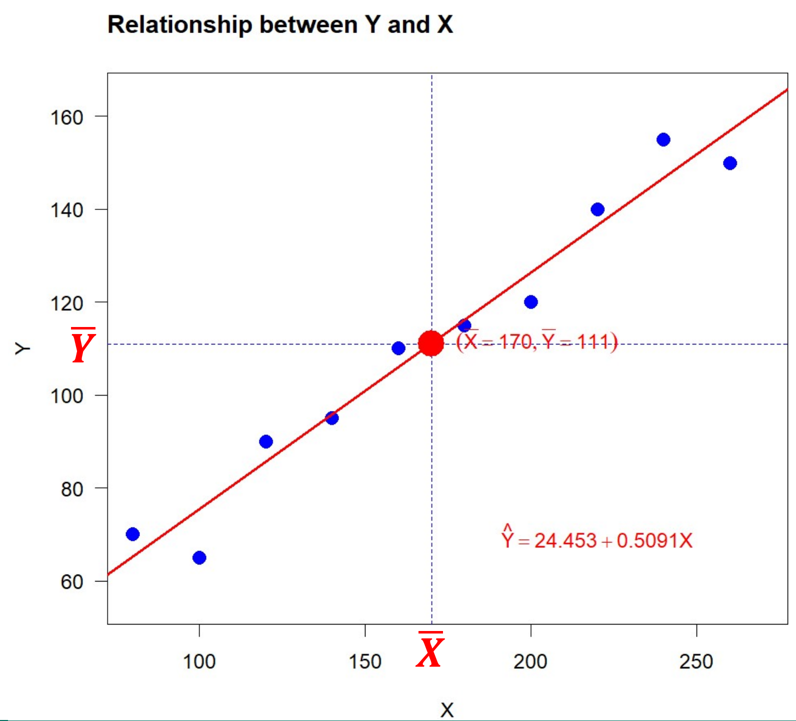
1. The OLS estimators are expressed solely in terms of observable sample quantities of X and Y.

2. They are point estimators. Given the sample, each estimator will yield a single value of the relevant population parameter.

3. The sample regression line can be easily obtained once OLS estimates are obtained from the sample.

Numerical Properties of OLS Estimates – Regression Line

- The regression line has the following properties:
  - It passes through the sample means of X and Y.



Numerical Properties of OLS Estimates – Regression Line

- The regression line has the following properties:
  - The mean value of the estimated Y ($\bar{\hat{Y}}$) equals the mean of the actual Y ($\bar{Y}$).

Recall, $\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2\bar{X}$

Sample Regression Function (SRF)

$$\hat{Y}_i = \hat{\beta}_1 + \hat{\beta}_2 X_i$$

$$\hat{Y}_i = \bar{Y} - \hat{\beta}_2\bar{X} + \hat{\beta}_2 X_i$$

$$\hat{Y}_i = \bar{Y} + \hat{\beta}_2(X_i - \bar{X})$$

Sum both sides, and divide by n

$$\frac{\Sigma\hat{Y}_i}{n} = \frac{\Sigma\left[\bar{Y} + \hat{\beta}_2(X_i - \bar{X})\right]}{n}$$

$$\bar{\hat{Y}} = \frac{\cancel{n}\bar{Y}}{\cancel{n}} + \frac{\hat{\beta}_2 \cancel{\Sigma(X_i - \bar{X})} \rightarrow 0}{n}$$

$$\Sigma(X_i - \bar{X}) = 0$$

$$\bar{\hat{Y}} = \bar{Y}$$

Numerical Properties of OLS Estimates – Regression Line

- The regression line has the following properties:
  - The mean value of the residuals, $\hat{u}_i$ is zero.

$$\frac{\partial \sum \hat{u}_i^2}{\partial \hat{\beta}_1} = 2 \sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i)^{2-1} \times (-1) = 0$$

$$\frac{\cancel{-2} \sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i)}{\cancel{-2}} = \frac{0}{-2}$$

$$\left\{ \begin{array}{l} \sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i) = 0 \\ \sum Y_i - n\hat{\beta}_1 - \hat{\beta}_2 \sum X_i = 0 \quad (1) \end{array} \right.$$

$$\sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i) = 0$$

$$\sum [Y_i - \underbrace{(\hat{\beta}_1 + \hat{\beta}_2 X_i)}_{\hat{Y}_i}] = 0$$

$$\sum \underbrace{(Y_i - \hat{Y}_i)}_{\hat{u}_i} = 0 \quad , \quad \hat{u}_i = Y_i - \hat{Y}_i$$

$$\frac{\sum \hat{u}_i}{n} = \frac{0}{n} \Rightarrow \bar{\hat{u}}_i = 0$$

Numerical Properties of OLS Estimates – Regression Line

- The regression line has the following properties:
  - The residuals $\hat{u}_i$ are uncorrelated with the predicted Y_i.

Sample Regression:

$$Y_i = \hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i \quad \text{—— (1)}$$

Mean Sample Regression

$$\frac{\sum Y_i}{n} = \frac{\sum(\hat{\beta}_1 + \hat{\beta}_2 X_i + \hat{u}_i)}{n}$$

$$\bar{Y} = \frac{n\hat{\beta}_1}{n} + \frac{\hat{\beta}_2 \sum X_i}{n} + \cancel{\frac{\sum \hat{u}_i}{n}} \rightarrow 0$$

$$\bar{Y} = \hat{\beta}_1 + \hat{\beta}_2 \bar{X} \quad \text{—— (2)}$$

Deviation from the mean. (1) − (2)

$$Y_i - \bar{Y} = \hat{\beta}_1 - \hat{\beta}_1 + \hat{\beta}_2 X_i - \hat{\beta}_2 \bar{X} + \hat{u}_i - 0$$

$$Y_i - \bar{Y} = \hat{\beta}_2 (X_i - \bar{X}) + \hat{u}_i - 0$$

$$y_i = Y_i - \bar{Y}, \quad x_i = X_i - \bar{X}, \quad \hat{u}_i = \hat{u}_i - 0$$

$$y_i = \underbrace{\hat{\beta}_2 x_i}_{\hat{y}_i} + \hat{u}_i \quad \text{(deviation form)} \quad \text{—— (3)}$$

$$\searrow \hat{u}_i = y_i - \hat{\beta}_2 x_i$$

where $\hat{y}_i = \hat{\beta}_2 x_i$

Numerical Properties of OLS Estimates – Regression Line

- The residuals $\widehat{u_i}$ are uncorrelated with the predicted Y_i. (continued)

$$\hat{y}_i = \hat{\beta}_2 x_i$$

$$\sum \hat{y}_i \hat{u}_i = \sum \left[\hat{\beta}_2 x_i \hat{u}_i\right]$$

$$\sum \hat{y}_i \hat{u}_i = \hat{\beta}_2 \sum x_i \hat{u}_i$$

from (3)

$$\sum \hat{y}_i \hat{u}_i = \hat{\beta}_2 \sum x_i (y_i - \hat{\beta}_2 x_i)$$

$$\sum \hat{y}_i \hat{u}_i = \hat{\beta}_2 \sum x_i y_i - \hat{\beta}_2^2 \sum x_i^2$$

$$\sum \hat{y}_i \hat{u}_i = \hat{\beta}_2^2 \sum x_i^2 - \hat{\beta}_2^2 \sum x_i^2$$

$$\sum \hat{y}_i \hat{u}_i = 0$$

$\hat{\beta}_2 \sum x_i y_i$,

but $\hat{\beta}_2 = \dfrac{\sum x_i y_i}{\sum x_i^2}$

$$\frac{\sum x_i y_i}{\sum x_i^2} \cdot \frac{\sum x_i y_i}{\sum x_i^2} \cdot \sum x_i^2$$

$$\hat{\beta}_2 \cdot \hat{\beta}_2 \cdot \sum x_i^2$$

$$\hat{\beta}_2^2 \sum x_i^2$$

$$\hat{\beta}_2 \sum x_i y_i \equiv \hat{\beta}_2^2 \sum x_i^2$$

Numerical Properties of OLS Estimates – Regression Line

- The regression line has the following properties:
  - The residuals $\widehat{u_i}$ are uncorrelated with X_i.

$$\frac{\partial \sum \hat{u}_i^2}{\partial \hat{\beta}_2} = 2\sum(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i)^{2-1} \times (-X_i) = 0$$

$$-2\sum(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i) X_i = 0$$

$$\left\{ \sum(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i) X_i = 0 \right.$$

$$\sum(X_i Y_i - \hat{\beta}_1 X_i - \hat{\beta}_2 X_i^2) = 0$$

$$\sum X_i Y_i - \hat{\beta}_1 \sum X_i - \hat{\beta}_2 \sum X_i^2 = 0 \quad (2)$$

$$\sum(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i) X_i = 0$$

$$\sum[Y_i - (\hat{\beta}_1 + \hat{\beta}_2 X_i)] X_i = 0$$

$$\sum(\underbrace{Y_i - \hat{Y}_i}_{\hat{u}_i}) X_i = 0$$

$$\sum \hat{u}_i X_i = 0$$

THANK YOU!

Next Lesson: Statistical Properties of OLS Estimates (Assumptions of Classical Linear Regression Model)