

# ECONOMETRICS

## Method of Ordinary Least Squares (OLS)

ECONOMETRICS

The background of the slide is a dark blue gradient with a central green glow. It features a classical statue of a person in a long robe, holding a tablet and pointing upwards. Surrounding the statue are various data visualization elements: a bar chart on the left, a line graph with a rising trend, a pie chart on the right, and a grid of numbers. The words "DATA SCIENCE" are written in a stylized font across the middle. At the bottom, the word "ECONOMETRICS" is repeated in a smaller, blue font.

## Lesson Goal

- Understand the method of ordinary least squares used in estimating regression coefficients.

## Ordinary Least Squares (OLS) Method

- Method of OLS is attributed to Carl Friedrich Gauss.
- Recall the PRF:


$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

- Estimate it from the SRF:

$$Y_i = \underbrace{\widehat{\beta}_1 + \widehat{\beta}_2 X_i}_{\widehat{Y}_i} + \widehat{u}_i$$

$$Y_i = \widehat{Y}_i + \widehat{u}_i$$

## Ordinary Least Squares (OLS) Method

$$Y_i = \widehat{Y}_i + \widehat{u}_i$$


$$\widehat{u}_i = Y_i - \widehat{Y}_i$$


Error or residual term  $\widehat{u}_i$  is difference between the actual  $Y_i$  and estimated  $\widehat{Y}_i$ .

- Determine SRF so that it is closer to Y – sum of residuals criterion.

$$\sum_{i=1}^n \widehat{u}_i = \sum_{i=1}^n (Y_i - \widehat{Y}_i) \Rightarrow \sum \widehat{u}_i = \sum (Y_i - \widehat{Y}_i)$$

## Ordinary Least Squares (OLS) Method

- The sum of residuals is zero

$$\sum \widehat{u}_i = \sum (Y_i - \widehat{Y}_i) = 0$$

- Use least squares criterion – sum of squared residuals (SSR).

$$\sum \widehat{u}_i^2 = \sum (Y_i - \widehat{Y}_i)^2$$

$$\sum \widehat{u}_i^2 = \sum (Y_i - \widehat{\beta}_1 - \widehat{\beta}_2 X_i)^2$$

- Minimize SSR with respect to the estimates  $\widehat{\beta}_1$  and  $\widehat{\beta}_2$ .

## Ordinary Least Squares (OLS) Method

- Minimize SSR with respect to the estimates  $\widehat{\beta}_1$  and  $\widehat{\beta}_2$ .

$$\sum \widehat{u}_i^2 = f(\widehat{\beta}_1, \widehat{\beta}_2)$$

- Apply differential calculus – first order condition.

## Ordinary Least Squares (OLS) Method

$$\sum \hat{u}_i^2 = \sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i)^2$$

$$\frac{\partial \sum \hat{u}_i^2}{\partial \hat{\beta}_1} = 2 \sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i) \times (-1) = 0$$

$$\cancel{-2} \sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i) = \underline{0} \quad \underline{-2}$$

$$\sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i) = 0$$

$$\sum Y_i - n\hat{\beta}_1 - \hat{\beta}_2 \sum X_i = 0 \quad (1)$$

$$\frac{\partial \sum \hat{u}_i^2}{\partial \hat{\beta}_2} = 2 \sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i) \times (-X_i) = 0$$

$$-2 \sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i) X_i = 0$$

$$\sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i) X_i = 0$$

$$\sum (X_i Y_i - \hat{\beta}_1 X_i - \hat{\beta}_2 X_i^2) = 0$$

$$\sum X_i Y_i - \hat{\beta}_1 \sum X_i - \hat{\beta}_2 \sum X_i^2 = 0 \quad (2)$$

## Ordinary Least Squares (OLS) Method

$$\sum \hat{u}_i^2 = \sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i)^2$$

$$\sum Y_i - n\hat{\beta}_1 - \hat{\beta}_2 \sum X_i = 0 \quad (1)$$

$$\sum X_i Y_i - \hat{\beta}_1 \sum X_i - \hat{\beta}_2 \sum X_i^2 = 0 \quad (2)$$

From equation (1)

$$\frac{\sum Y_i}{n} - \hat{\beta}_2 \frac{\sum X_i}{n} = \hat{\beta}_1$$

$$\bar{Y} = \frac{\sum Y_i}{n}, \quad \bar{X} = \frac{\sum X_i}{n}$$

$$\bar{Y} - \hat{\beta}_2 \bar{X} = \hat{\beta}_1$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$$

$$\sum X_i Y_i - (\bar{Y} - \hat{\beta}_2 \bar{X}) \sum X_i - \hat{\beta}_2 \sum X_i^2 = 0$$

$$\sum X_i Y_i - \bar{Y} \sum X_i + \hat{\beta}_2 \bar{X} \sum X_i - \hat{\beta}_2 \sum X_i^2 = 0$$

$$\bar{Y} = \frac{\sum Y_i}{n} \Rightarrow n\bar{Y} = \sum Y_i$$

$$\bar{X} = \frac{\sum X_i}{n} \Rightarrow n\bar{X} = \sum X_i$$

$$\sum X_i Y_i - \bar{Y} (n\bar{X}) + \hat{\beta}_2 \bar{X} (n\bar{X}) - \hat{\beta}_2 \sum X_i^2 = 0$$

$$\sum X_i Y_i - n\bar{X}\bar{Y} + \hat{\beta}_2 n\bar{X}^2 - \hat{\beta}_2 \sum X_i^2 = 0$$

$$\sum X_i Y_i - n\bar{X}\bar{Y} = \hat{\beta}_2 \sum X_i^2 - \hat{\beta}_2 n\bar{X}^2$$

$$\sum X_i Y_i - n\bar{X}\bar{Y} = \hat{\beta}_2 (\sum X_i^2 - n\bar{X}^2)$$



## Ordinary Least Squares (OLS) Method

$$\sum \hat{u}_i^2 = \sum (Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i)^2$$

$$\frac{\sum X_i Y_i - n \bar{X} \bar{Y}}{\sum X_i^2 - n \bar{X}^2} = \hat{\beta}_2 \frac{(\sum X_i^2 - n \bar{X}^2)}{\sum X_i^2 - n \bar{X}^2}$$

$$\frac{\sum X_i Y_i - n \bar{X} \bar{Y}}{\sum X_i^2 - n \bar{X}^2} = \hat{\beta}_2$$

$$\begin{aligned} \sum (X_i - \bar{X})(Y_i - \bar{Y}) &= \sum (X_i Y_i - \bar{Y} X_i - \bar{X} Y_i + \bar{X} \bar{Y}) \\ &= \sum X_i Y_i - \bar{Y} \sum X_i - \bar{X} \sum Y_i + n \bar{X} \bar{Y} \\ &= \sum X_i Y_i - \bar{Y} (n \bar{X}) - \bar{X} (n \bar{Y}) + n \bar{X} \bar{Y} \\ &= \sum X_i Y_i - n \bar{X} \bar{Y} - n \bar{X} \bar{Y} + n \bar{X} \bar{Y} \\ &= \sum X_i Y_i - n \bar{X} \bar{Y} \end{aligned}$$

$$\begin{aligned} \sum (X_i - \bar{X})^2 &= \sum (X_i^2 - 2\bar{X} X_i + \bar{X}^2) \\ &= \sum X_i^2 - 2\bar{X} \sum X_i + n \bar{X}^2 \\ &= \sum X_i^2 - 2\bar{X} (n \bar{X}) + n \bar{X}^2 \\ &= \sum X_i^2 - 2n \bar{X}^2 + n \bar{X}^2 \\ &= \sum X_i^2 - n \bar{X}^2 \end{aligned}$$

$$\Rightarrow \hat{\beta}_2 = \frac{\sum (X_i - \bar{X})(Y_i - \bar{Y})}{\sum (X_i - \bar{X})^2}$$

Let  $x_i = X_i - \bar{X}$  and  $y_i = Y_i - \bar{Y}$

$$\Rightarrow \hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2}$$

## Ordinary Least Squares (OLS) Method

$$\sum \widehat{u}_i^2 = \sum (Y_i - \widehat{\beta}_1 - \widehat{\beta}_2 X_i)^2$$

$$\widehat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2} \quad \text{where } x_i = X_i - \bar{X} \\ y_i = Y_i - \bar{Y}$$

(slope)

$$\widehat{\beta}_1 = \bar{Y} - \widehat{\beta}_2 \bar{X} \quad (\text{intercept})$$

**THANK YOU!**

Next Lesson: **Properties of OLS Estimates**