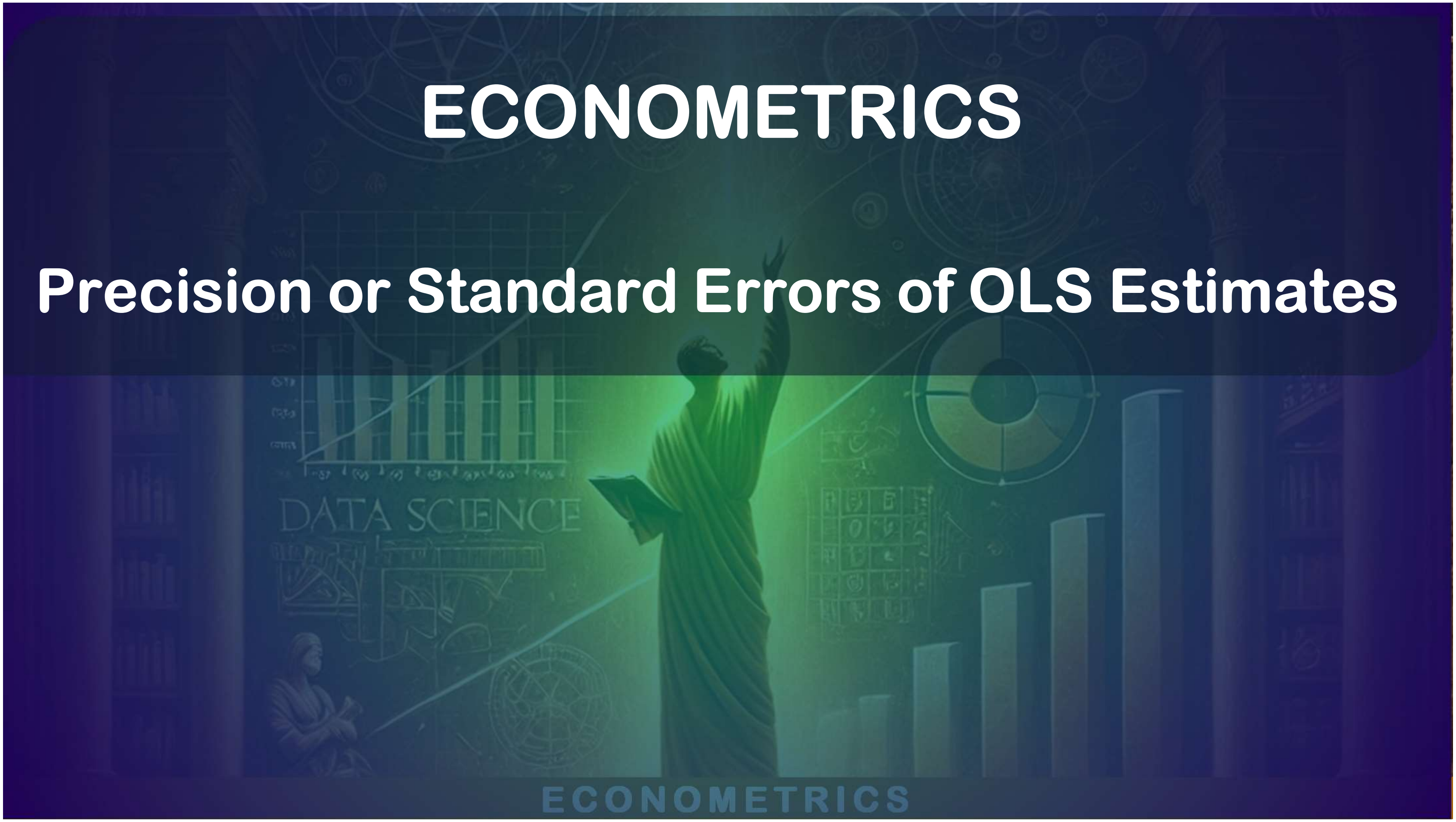
ECONOMETRICS

Precision or Standard Errors of OLS Estimates



Lesson Goal

- Learn to measure precision of OLS estimates via their standard errors.

Precision or Standard Errors of OLS Estimates

- The least squares estimators are obtained from sample data.

- Since data are likely to change from sample to sample, the estimates will change likewise.

- Thus, we need some measure of "**reliability**" or **precision** of the estimates, $\widehat{\beta_1}$ and $\widehat{\beta_2}$.

- In statistics, the precision of an estimate is measured by its **standard error** (se).

Precision or Standard Errors of OLS Estimates

- The **variance** (var) and **standard errors** (se) of the estimates are:

Slope coefficient

$$var(\widehat{\beta_2}) = \frac{\sigma^2}{\sum x_i^2}$$

$$se(\widehat{\beta_2}) = \frac{\sigma}{\sqrt{\sum x_i^2}}$$

Intercept coefficient

$$var(\widehat{\beta_1}) = \frac{\sum \mathrm{X_i^2}}{n \sum x_i^2} \cdot \sigma^2$$

$$se(\widehat{\beta_1}) = \sqrt{\frac{\sum \mathrm{X_i^2}}{n \sum x_i^2}} \cdot \sigma$$

where: σ^2 is the constant or homoscedastic variance of u_i.

Precision or Standard Errors of OLS Estimates

- **The homoscedastic variance $\boldsymbol{\sigma^2}$ can be estimated using:**

$$\hat{\sigma}^2 = \frac{\sum \hat{u}_i^2}{n-2}$$

where: $\hat{\sigma}^2$ is the OLS estimator of the true but unknown variance σ^2;

$n-2$ is the number of degrees of freedom (df);

$\sum \hat{u}_i^2$ is the sum of the residuals squared or residual sum of squares (RSS): $\sum \hat{u}_i^2 = \sum \left(Y_i - \widehat{Y_i}\right)^2$

Precision or Standard Errors of OLS Estimates

- The square root of the true variance σ^2 is obtained:

$$\hat{\sigma} = \sqrt{\frac{\sum \hat{u}_i^2}{n-2}}$$

where: $\hat{\sigma}$ is called the **standard error of estimate**, or the **standard error of the regression** (se).

Features of the Variances of OLS Estimates

- The variance of the slope coefficient $\widehat{\beta_2}$ is directly proportional to σ^2 but inversely proportional to $\sum x_i^2$.

$$var\left(\widehat{\beta_2}\right) = \frac{\sigma^2}{\sum x_i^2}$$

Features of the Variances of OLS Estimates

- The variance of the intercept coefficient $\widehat{\beta_1}$ is directly proportional to σ^2 and $\sum X_i^2$ but inversely proportional to $\sum x_i^2$ and the sample size n.

$$var\left(\widehat{\beta_1}\right) = \frac{\sum \mathrm{X}_\mathrm{i}^2}{n\sum x_i^2} \cdot \sigma^2$$

Features of the Variances of OLS Estimates

- Since $\widehat{\beta_1}$ and $\widehat{\beta_2}$ are estimators, they will not only vary from sample to sample, but in a given sample, they are likely to dependent on each other, and this dependence is measured by the covariance between them.

$$cov\left(\widehat{\beta_1}, \widehat{\beta_2}\right) = -\bar{X} \cdot var\left(\widehat{\beta_2}\right)$$

$$= -\bar{X}\left(\frac{\sigma^2}{\sum x_i^2}\right)$$

THANK YOU!

Next Lesson: Properties of OLS Estimates: Gauss-Markov Theorem (with PROOF)