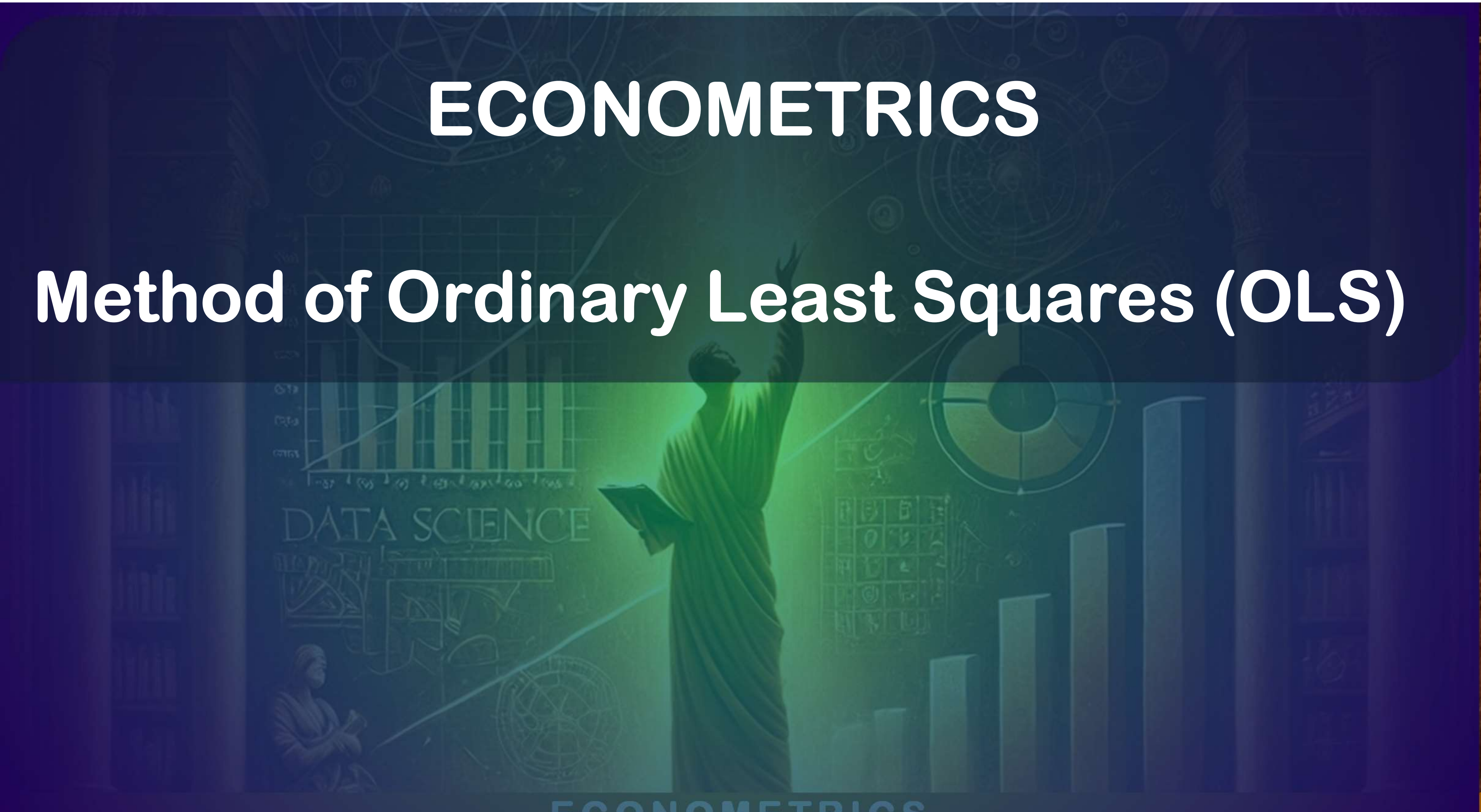
ECONOMETRICS

Method of Ordinary Least Squares (OLS)



Lesson Goal

- Understand the method of ordinary least squares used in estimating regression coefficients.

Ordinary Least Squares (OLS) Method

- Method of OLS is attributed to Carl Friedrich Gauss.
- Recall the PRF:

$$Y_i = \beta_1 + \beta_2 X_i + u_i$$

- Estimate it from the SRF:

$$Y_i = \underbrace{\widehat{\beta_1} + \widehat{\beta_2} X_i}_{\widehat{Y_i}} + \widehat{u_i}$$

$$Y_i = \widehat{Y_i} + \widehat{u_i}$$

Ordinary Least Squares (OLS) Method

$$Y_i = \widehat{Y_i} + \widehat{u_i}$$

$$\widehat{u_i} = Y_i - \widehat{Y_i}$$

Error or residual term $\widehat{u_i}$ is difference between the actual Y_i and estimated $\widehat{Y_i}$.

- Determine SRF so that it is closer to Y – sum of residuals criterion.

$$\sum_{i=1}^{n} \widehat{u_i} = \sum_{i=1}^{n} (Y_i - \widehat{Y_i}) \quad \Rightarrow \quad \sum \widehat{u_i} = \sum (Y_i - \widehat{Y_i})$$

Ordinary Least Squares (OLS) Method

- The sum of residuals is zero

$$\sum \widehat{u_i} = \sum \left(Y_i - \widehat{Y_i}\right) = 0$$

- Use least squares criterion – sum of squared residuals (SSR).

$$\sum \widehat{u_i}^2 = \sum \left(Y_i - \widehat{Y_i}\right)^2$$

$$\sum \widehat{u_i}^2 = \sum \left(Y_i - \widehat{\beta_1} - \widehat{\beta_2} X_i\right)^2$$

- Minimize SSR with respect to the estimates $\widehat{\beta_1}$ and $\widehat{\beta_2}$.

Ordinary Least Squares (OLS) Method

- Minimize SSR with respect to the estimates $\widehat{\beta_1}$ and $\widehat{\beta_2}$.

$$\sum \widehat{u_i}^2 = f(\widehat{\beta_1}, \widehat{\beta_2})$$

- Apply differential calculus – first order condition.

Ordinary Least Squares (OLS) Method

$$\sum \widehat{u_i}^2 = \sum \left(Y_i - \widehat{\beta_1} - \widehat{\beta_2} X_i\right)^2$$

$$\frac{\partial \sum \hat{u}_i^2}{\partial \hat{\beta}_1} = 2\sum\left(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i\right)^{2-1} \times (-1) = 0$$

$$\frac{\cancel{-2}\sum\left(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i\right)}{\cancel{-2}} = \frac{0}{-2}$$

$$\sum\left(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i\right) = 0$$

$$\sum Y_i - n\hat{\beta}_1 - \hat{\beta}_2 \sum X_i = 0 \quad (1)$$

$$\frac{\partial \sum \hat{u}_i^2}{\partial \hat{\beta}_2} = 2\sum\left(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i\right)^{2-1} \times (-X_i) = 0$$

$$-2\sum\left(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i\right) X_i = 0$$

$$\sum\left(Y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i\right) X_i = 0$$

$$\sum\left(X_i Y_i - \hat{\beta}_1 X_i - \hat{\beta}_2 X_i^2\right) = 0$$

$$\sum X_i Y_i - \hat{\beta}_1 \sum X_i - \hat{\beta}_2 \sum X_i^2 = 0 \quad (2)$$

Ordinary Least Squares (OLS) Method

$$\sum \widehat{u_i}^2 = \sum \left(Y_i - \widehat{\beta_1} - \widehat{\beta_2} X_i\right)^2$$

$$\Sigma Y_i - n\hat{\beta}_1 - \hat{\beta}_2 \Sigma X_i = 0 \qquad (1)$$

$$\Sigma X_i Y_i - \hat{\beta}_1 \Sigma X_i - \hat{\beta}_2 \Sigma X_i^2 = 0 \qquad (2)$$

From equation (1)

$$\frac{\Sigma Y_i}{n} - \frac{\hat{\beta}_2 \Sigma X_i}{n} = \frac{n\hat{\beta}_1}{n}$$

$$\bar{Y} = \frac{\Sigma Y_i}{n}, \quad \bar{X} = \frac{\Sigma X_i}{n}$$

$$\bar{Y} - \hat{\beta}_2 \bar{X} = \hat{\beta}_1$$

$$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$$

$$\Sigma X_i Y_i - (\bar{Y} - \hat{\beta}_2 \bar{X}) \Sigma X_i - \hat{\beta}_2 \Sigma X_i^2 = 0$$

$$\Sigma X_i Y_i - \bar{Y} \Sigma X_i + \hat{\beta}_2 \bar{X} \Sigma X_i - \hat{\beta}_2 \Sigma X_i^2 = 0$$

$$\bar{Y} = \frac{\Sigma Y_i}{n} \Rightarrow n\bar{Y} = \Sigma Y_i$$

$$\bar{X} = \frac{\Sigma X_i}{n} \Rightarrow n\bar{X} = \Sigma X_i$$

$$\Sigma X_i Y_i - \bar{Y}(n\bar{X}) + \hat{\beta}_2 \bar{X}(n\bar{X}) - \hat{\beta}_2 \Sigma X_i^2 = 0$$

$$\Sigma X_i Y_i - n\bar{X}\bar{Y} + \hat{\beta}_2 n\bar{X}^2 - \hat{\beta}_2 \Sigma X_i^2 = 0$$

$$\Sigma X_i Y_i - n\bar{X}\bar{Y} = \hat{\beta}_2 \Sigma X_i^2 - \hat{\beta}_2 n\bar{X}^2$$

$$\Sigma X_i Y_i - n\bar{X}\bar{Y} = \hat{\beta}_2 (\Sigma X_i^2 - n\bar{X}^2)$$

Ordinary Least Squares (OLS) Method

$$\sum \widehat{u_i}^2 = \sum (Y_i - \widehat{\beta_1} - \widehat{\beta_2} X_i)^2$$

$$\frac{\sum X_i Y_i - n\bar{X}\bar{Y}}{\sum X_i^2 - n\bar{X}^2} = \frac{\hat{\beta}_2(\sum X_i^2 - n\bar{X}^2)}{\sum X_i^2 - n\bar{X}^2}$$

$$\frac{\sum X_i Y_i - n\bar{X}\bar{Y}}{\sum X_i^2 - n\bar{X}^2} = \hat{\beta}_2$$

$$\begin{aligned}
\sum(X_i - \bar{X})(Y_i - \bar{Y}) &= \sum(X_i Y_i - \bar{Y}X_i - \bar{X}Y_i + \bar{X}\bar{Y}) \\
&= \sum X_i Y_i - \bar{Y}\sum X_i - \bar{X}\sum Y_i + n\bar{X}\bar{Y} \\
&= \sum X_i Y_i - \bar{Y}(n\bar{X}) - \bar{X}(n\bar{Y}) + n\bar{X}\bar{Y} \\
&= \sum X_i Y_i - n\bar{X}\bar{Y} - \cancel{n\bar{X}\bar{Y}} + \cancel{n\bar{X}\bar{Y}} \\
&= \sum X_i Y_i - n\bar{X}\bar{Y}
\end{aligned}$$

$$\begin{aligned}
\sum(X_i - \bar{X})^2 &= \sum(X_i^2 - 2\bar{X}X_i + \bar{X}^2) \\
&= \sum X_i^2 - 2\bar{X}\sum X_i + n\bar{X}^2 \\
&= \sum X_i^2 - 2\bar{X}(n\bar{X}) + n\bar{X}^2 \\
&= \sum X_i^2 - 2n\bar{X}^2 + n\bar{X}^2 \\
&= \sum X_i^2 - n\bar{X}^2
\end{aligned}$$

$$\Rightarrow \hat{\beta}_2 = \frac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{\sum(X_i - \bar{X})^2}$$

Let $x_i = X_i - \bar{X}$ and $y_i = Y_i - \bar{Y}$

$$\Rightarrow \hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2}$$

Ordinary Least Squares (OLS) Method

$$\sum \widehat{u_i}^2 = \sum \left(Y_i - \widehat{\beta_1} - \widehat{\beta_2} X_i\right)^2$$

$\hat{\beta}_2 = \dfrac{\sum x_i y_i}{\sum x_i^2}$ where $x_i = X_i - \bar{X}$

$y_i = Y_i - \bar{Y}$

(slope)

$\hat{\beta}_1 = \bar{Y} - \hat{\beta}_2 \bar{X}$ (intercept)

THANK YOU!

Next Lesson: Properties of OLS Estimates