This exam is **no book, no gadgets**. You may use **one sheet of notes** (letter size, two sides), which you submit with the exam. There are 28 questions, worth 60 points. Marks will be curved so the median is at least B+ (87).

This exam is my own work. I understand it is governed by the ***Emory Honor Code****.*

Signature: ______________________________

Fill in the Blank: fill each blank appropriately. An answer could be a word, phrase, or formula.

Approximation Algorithms

(2 pts) 1. We saw how to approximately solve the vertex cover problem in a graph, in polynomial time. What approximation ratio did we guarantee? (This is the ratio between the size of our cover, and the best possible.)

1. ______________________________

(2 pts) 2. In Baker's PTAS for MIS (maximum independent set) in a planar graph, we remove every kth layer of vertices. What was our formula for k, in terms of ϵ?

2. ______________________________

(2 pts) 3. In the set cover problem, suppose we know that some k of the given sets will cover $\{1, 2, \ldots, n\}$. The greedy algorithm finds a set cover with (at most) how many sets?

3. ______________________________

(2 pts) 4. Suppose graph G has n vertices and maximum degree Δ (each vertex has at most Δ neighbors). Then by a simple greedy procedure, we can color G with at most how many colors?

4. ______________________________

(2 pts) 5. Suppose graph G has n vertices, and its vertices can be colored by 3 colors. In class, we saw how to color G in polynomial time, but with more colors. How many colors did we need? (big-Oh)

5. ______________________________

Math and Definitions

(2 pts) 6. Solve the recurrence $T(n) = 2 \cdot T(n/2) + n \lg n$. (Big-Oh)

6. ______________________________

(2 pts) 7. In the Master Theorem, we consider the recurrence $T(n) = a \cdot T(n/b) + O(n^d)$, with constants $a > 0$, $b > 1$, and $d \geq 0$. What condition on a, b, d puts us in the "top heavy" case?

7. ______________________________

(2 pts) 8. Suppose G is a graph of n vertices, and it has a tree decomposition of width k. Each tree node defines a "bag" (a subset of vertices of G). What is the maximum size of a bag?

8. ______________________________

(2 pts) 9. Suppose graph G has n vertices, and a min-cut crossed by C edges. What is our lower bound on the probability that Karger's algorithm outputs this particular min-cut?

9. ______________________________

(2 pts) 10. Suppose $(E, \mathcal{I})$ is a matroid. Suppose A and B are in $\mathcal{I}$, and $|A| > |B|$. According to the definition of a matroid, what can we conclude? (existential sentence)

10. ______________________________

(2 pts) 11. What class of graphs did we see, other than trees, with tree decompositions of width $O(1)$?

11. ______________________________

(2 pts) 12. Matrix A is totally unimodular means: every square submatrix S of A has what property?

12. ______________________________

Exact Algorithms

(2 pts) 13. The book's $FFT(a, \omega)$ first computes $(s_0, s_1, \ldots, s_{n/2-1}) = FFT((a_0, a_2, \ldots, a_{n-2}), \omega^2)$ and $(s'_0, s'_1, \ldots, s'_{n/2-1}) = FFT((a_1, a_3, \ldots, a_{n-1}), \omega^2)$. Then it returns $(r_0, r_1, \ldots, r_{n-1})$, where for $j < n/2$, r_j equals what?

13. ______________________________

(2 pts) 14. In the LCS (longest common subsequence) problem, the input is two strings of lengths m and n. In our dynamic programming approach, how many subproblems are there?

14. ______________________________

(2 pts) 15. In the TSP problem, the input is an n by n distance matrix. In our dynamic programming approach, how many subproblems are there? (big-Oh)

15. ______________________________

(2 pts) 16. Suppose graph G has n vertices, and a tree decomposition of width k, with N tree nodes. We want to solve the MIS problem. In our dynamic programming approach, how many subproblems are there? (big-Oh)

16. ______________________________

(2 pts) 17. We exactly solved the MIS problem in a planar graph by divide and conquer, using the Lipton-Tarjan separator theorem. What was the running time of that algorithm? (use big-Oh)

17. ______________________________

(2 pts) 18. Which (named) algorithm did we generalize, in order to find a maximum weight independent set in a matroid?

18. ______________________________

Flows and LP

(2 pts) 19. In our LP examples, we saw that "fractional maximum matching" was dual to what other LP problem?

19. ______________________________

(2 pts) 20. We have a flow network G, a max flow f, and a min cut S (so $s \in S$ and $t \in V - S$). Edge e goes from $V - S$ to S, with capacity $c(e)$. What can we say about $f(e)$?

20. ______________________________

(2 pts) 21. Suppose we run Ford-Fulkerson on a network with integer capacities, V vertices, E edges, and max flow value F. If we always choose a *shortest* augmenting path, what is a bound on the number of paths required? (big-Oh)

21. ______________________________

(2 pts) 22. Continuing the previous question, suppose we always choose a fattest (max capacity) augmenting path. What is an upper bound on the number of paths required? (big-Oh)

22. ______________________

(2 pts) 23. Name a problem, other than min-cut, that we reduced to max-flow.

23. ______________________

(2 pts) 24. Suppose we have dual LP's MAX = $\max\{c \cdot x\colon\ Ax \le b,\, x \ge 0\}$ and MIN = $\min\{b \cdot y\colon\ A^t y \ge c,\, y \ge 0\}$. What does "weak duality" say about the values MAX and MIN?

24. ______________________

(2 pts) 25. Continuing the previous, suppose vectors x^* and y^* are feasible solutions to the LP's, achieving $c \cdot x^* = b \cdot y^*$. Suppose x_1^* (the first component of x^*) is positive. What can we conclude about the constraints on y^*?

25. ______________________

(2 pts) 26. Continuing the previous, suppose MAX' is the maximum value of the first LP, but restricted to vectors x with integer coordinates. What inequality can we state relating MAX and MAX'?

26. ______________________

Short Answer: (please indicate if you write on the back)

(4 pts) 27. Using $\omega = i$ (the square root of -1) and $n = 4$, write down the Fourier transform matrix, and its inverse.

(4 pts) 28. We are given a graph with vertex set $V = \{1, 2, \ldots, n\}$, edge set E, and a positive weight w_i for each vertex $i \in V$. Write the relaxed weighted vertex cover problem as an LP. Also, given a fractional optimal solution to this LP, describe how to "round" it, to get an actual vertex cover.