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A multi-modal approach to the location of a rapid transit line

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Abstract

The location of a rapid transit line (RTL) represents a very complex decision problem because of the large number of decision makers, unquantifiable criteria and uncertain data. In this context Operational Research can help in the design process by providing tools to generate and assess alternative solutions. For this purpose two bicriterion mathematical programming models – the Maximum Coverage Shortest Path model and the Median Shortest Path model – have been developed in the past. In this paper a new bicriterion model, which can evaluate in a more realistic way the attractivity of an RTL is introduced. To calculate an estimation of the non-inferior solution set of the problem, a procedure based on a k-shortest path algorithm was developed. This approach was applied to a well-known sample problem and the results are discussed and compared with those obtained using a Median Shortest Path model. © 1998 Elsevier Science B.V.

Keywords: Location; Location-routing; Network design; Transportation

1. Introduction

Rapid transit lines (RTL), such as traditional underground metro systems, surface rail rapid transit, light rail transit and monorails, are becoming increasingly common to improve mobility and reduce road congestion (Sullivan, 1985; Gendreau et al., 1995). The planning of such systems is a very complex problem which involves considerable effort and huge financial resources. Decisions have to be made about the topological configuration, the type of system to use, the number and location of lines and stations, line frequencies, location of interchange parking, etc. In this context, both quantifiable and non-quantifiable criteria have to be considered: construction and operating costs, travel times, demand satisfaction, utilization and accessibility of open space and historic sites, visual impact, air and noise pollution and so forth (see, e.g., Steenbrink, 1974; Current, 1993). Furthermore, some data are not completely known in advance and decisions are the consequence of the interaction of several actors: engineers, central and local administrations, environmentalists and lobbies (see, e.g., Colcord, 1971).

In this context, the core issue – the location of a single line and the corresponding stations – is a classical multi-objective network design problem (Magnanti and Wong, 1984; Current, 1993) whose solution is rather complicated because of the large

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number of decision makers, unquantifiable criteria and uncertain data. Within this framework Operational Research can help in the design process by providing tools to generate and assess valid alternative solutions. As it is very difficult to take into account all relevant criteria, a two step procedure is generally employed (Laporte, 1995):

1. generation of a (small) number of efficient solutions considering two main criteria (a *cost* and a *benefit*);
2. choice of the "best" line location among these solutions, considering both quantifiable and non-quantifiable criteria by means of multi-attribute decision aids such as the Promethee method (Brans et al., 1984).

To tackle the first step two bicriterion mathematical programming models have been developed: the Maximum Covering Shortest Path model (Current et al., 1985) and the Median Shortest Path model (Current et al., 1987). Both models evaluate the alternative solutions trading off the construction cost and a measure of the social benefit (Sugden and Williams, 1978).

The criterion of *coverage*, which supposes that a nodal demand d_k is satisfied or "covered" if a facility (an RTL station, in this case) is located within a given fixed distance from node k, is the basis of the formulation of the *Maximum Covering Shortest Path Problem* (MCSPP). The MCSPP objectives consist of the minimization of the total path construction cost and the maximization of the total demand satisfied (defined as the sum of the nodal demands covered by the path).

The objectives of the *Median Shortest Path Problem* (MSPP) are the minimization of the total path construction cost (like MCSPP) and the maximization of the *accessibility* of the path (i.e. the total weighted travel distance that mobility demand, supposed to be concentrated in the nodes, must cover to reach the closest node belonging to the path).

Neither model provides a satisfactory description of real-world situations mainly for two reasons:

- the absence of alternative (concurrent) transportation systems;
- the unrealistic assumptions about the users' behavior.

As far as the latter reason is concerned, the concept of accessibility used by MSPP supposes that users gain access to the line independently of their destination; in the same way, the MCSPP formulation assumes that the only factor affecting the path choice is the distance from it.

In the following a more realistic bicriterion path location model is described. It assumes that each user chooses (among some alternatives) the transportation system corresponding to the *least travel cost*. Travel costs represent a composite measure of different factors such as travel time, monetary cost and comfort. The specification and calibration of such costs are interesting research topics which are beyond the aims of this paper.

If, for the sake of simplicity, we assume that the private mode is the only alternative to hybrid pedestrian–public system to be designed, each user chooses the route of least perceived travel cost between:

a. the shortest path, covered by car or motorbike, on the road network (the *private network*, below);
b. the shortest path on a hybrid pedestrian–public network (the *bi-modal network*, below) which includes the RTL to be located.

This is basically a location-routing problem (for a review, see Laporte, 1988). In fact, the location of the RTL and the determination of the users' routes must be affected at the same time because they depend on one another. Table 1 compares the assumptions on which MCSPP, MSPP and the new model are based.

2. Problem formulation

To formulate the above problem four networks are introduced:

- *the pedestrian network* (G_{PED});
- *the public network* (G_{PUB});
- *the private network* (G_{PRI});
- *a bi-modal (pedestrian–public) network* (G_{BIM}).

The *pedestrian network* G_{PED} includes possible links between origin–destination nodes (zone centroids) which can be covered on foot. $G_{\mathrm{PED}}(V^{\mathrm{PED}}, A^{\mathrm{PED}}, c_{ij}^{\mathrm{PED}})$ is an undirected graph described by a node set V^{PED}, an arc set A^{PED} and a travel cost

Table 1
Comparison between models

Model/characteristics	MCSPP	MSPP	New model
Objective functions	1. Construction cost 2. Covered demand	1. Construction cost 2. Accessibility	1. Construction cost 2. User's travel cost
Demand is associated to...	Single nodes	Single nodes	Origin–destination pairs of nodes
Demand is satisfied by a path Γ...	If node k is at a distance $\leqslant S$ from Γ	Always	If some arc $\in \Gamma$ is on the minimum cost route from node h to node k
Demand and costs are...	Deterministic	Deterministic	Deterministic

${c_{ij}}^{\mathrm{PED}}$ (called *pedestrian cost*) associated with each arc $(i,j) \in A^{\mathrm{PED}}$.

As the starting and arrival points of any trip coincide with a node of V^{PED}, an *origin–destination demand* d_{hk} is also associated with each pair of nodes $h, k \in V^{\mathrm{PED}}$.

The *private network* G_{PRI} is formed by possible links between nodes which can be covered by private vehicles like cars or motorbikes. $G_{\mathrm{PRI}}(V^{\mathrm{PRI}}, A^{\mathrm{PRI}}, {c_{ij}}^{\mathrm{PRI}})$ is a directed graph described by a node set $V^{\mathrm{PRI}} \subseteq V^{\mathrm{PED}}$, an arc set A^{PRI} and a travel cost ${c_{ij}}^{\mathrm{PRI}}$ (called *private cost*) associated with each arc $(i,j) \in A^{\mathrm{PRI}}$. In this formulation we assume that V^{PRI} and V^{PED} coincide ($V^{\mathrm{PRI}} \equiv V^{\mathrm{PED}}$) and private costs are independent of flows on the links. Furthermore ${c_{hk}}^{\mathrm{PRI}}$ denotes the travel cost corresponding to a *"private" minimum cost path* from node h to node k on the private network.

The *public network* G_{PUB} describes possible links between nodes which can be included in the RTL to be designed. $G_{\mathrm{PUB}}(V^{\mathrm{PUB}}, A^{\mathrm{PUB}}, {c_{ij}}^{\mathrm{C}}, {c_{ij}}^{\mathrm{PUB}})$ is an undirected graph with a node set V^{PUB} (the set of potential stations), an arc set A^{PUB}, a *construction cost* ${c_{ij}}^{\mathrm{C}}$ (the amount of money needed to build the arc) and a travel time ${c_{ij}}^{\mathrm{PUB}}$ (called *public cost*) associated with each arc $(i,j) \in A^{\mathrm{PUB}}$ (the travel time from i to j along the arc using the transit line). It is also supposed that each node $j \in V^{\mathrm{PUB}}$ is associated with a unique node $i \in V^{\mathrm{PED}}$ (which represents the pedestrian node corresponding to station j). Furthermore, all construction and utilization costs are scaled over the same planning horizon (e.g. a year).

To model the hybrid pedestrian–public transportation system, a *bi-modal network* G_{BIM} is also defined. Let $i \in V^{\mathrm{PED}}$ be the pedestrian node corresponding to node $j \in V^{\mathrm{PUB}}$, A^{BOARD} the set of the links (i,j) connecting a centroid i to a transit stop j (*boarding arcs*, with associated *boarding costs* ${c_{ij}}^{\mathrm{BOARD}}$) and A^{ALIGH} the set of the links (j,i) connecting a transit stop j to a centroid i (*alighting arcs*, with associated *alighting costs* ${c_{ji}}^{\mathrm{ALIGH}}$). Boarding cost ${c_{ij}}^{\mathrm{BOARD}}$ represents a composite measure of fare, pedestrian travel time from centroid i to the corresponding station j and waiting time at station j. Alighting cost ${c_{ji}}^{\mathrm{ALIGH}}$ represents the pedestrian travel time from station j to the corresponding centroid i. $G_{\mathrm{BIM}}(V^{\mathrm{BIM}}, A^{\mathrm{BIM}}, {c_{ij}}^{\mathrm{BIM}})$ (Fig. 1) is the network with node set

$$V^{\mathrm{BIM}} = V^{\mathrm{PED}} \cup V^{\mathrm{PUB}},$$

arc set

$$A^{\mathrm{BIM}} = A^{\mathrm{PED}} \cup A^{\mathrm{PUB}} \cup A^{\mathrm{BOARD}} \cup A^{\mathrm{ALIGH}},$$

and arc costs

$$c_{ij}^{\mathrm{BIM}} = \begin{cases} c_{ij}^{\mathrm{PED}} & \text{if } (i,j) \in A^{\mathrm{PED}}, \\ c_{ij}^{\mathrm{PUB}} & \text{if } (i,j) \in A^{\mathrm{PUB}}, \\ c_{ij}^{\mathrm{BOARD}} & \text{if } (i,j) \in A^{\mathrm{BOARD}}, \\ c_{ij}^{\mathrm{ALIGH}} & \text{if } (i,j) \in A^{\mathrm{ALIGH}}. \end{cases}$$

In this context, an *alignment* (or a *line*) is a path Γ from a starting node $o \in V^{\mathrm{PUB}}$ to a terminus node $d \in V^{\mathrm{PUB}}$ on the public network G_{PUB}

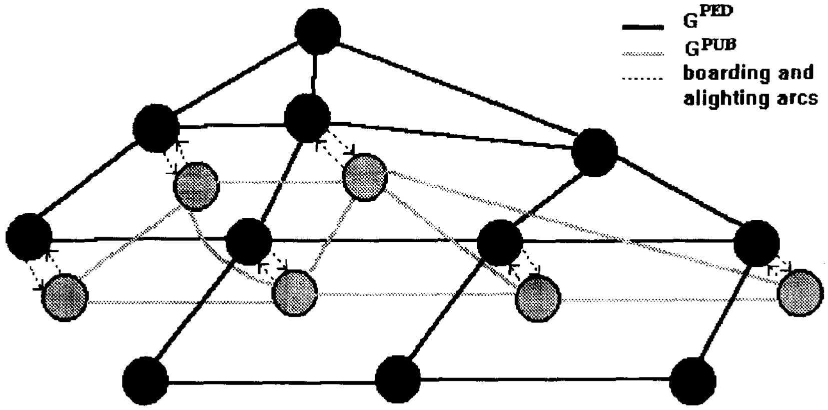

Fig. 1. An example of a bi-modal network.

(Fig. 2) and a *bi-modal path* associated to a demand d_{hk} is defined as a user route on the bi-modal network from node h to node k (Fig. 3). Starting and terminus nodes are predetermined as in most similar applications (see, e.g., Dufourd et al., 1995).

The bicriterion model we propose is based, in conclusion, on the following hypotheses:

1. mobility demand is described by an origin–destination matrix d_{hk};
2. users can use a private mode or the hybrid pedestrian–public transportation system to be designed;
3. demand d_{hk} is assigned to the transportation system corresponding to the least perceived travel cost;
4. travel costs are independent of flows;

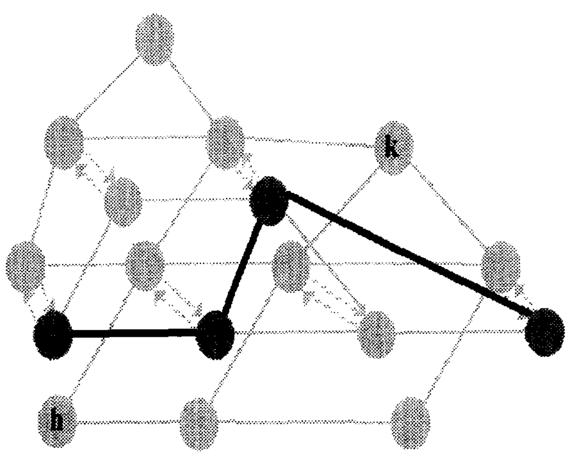

Fig. 2. An alignment.

5. mobility demand and costs are deterministic.

From these hypotheses it follows that, for each origin–destination, the users choose their own path comparing the shortest path on the private network and the shortest path on the bi-modal network to be designed. The objectives assumed are the minimization of the construction cost (z_1) and the total weighted travel cost incurred by users (z_2).

3. Further discussion of the formulation

The problem described in Section 2 provides a more realistic description of user behavior and can be modelled as a *two objective integer linear program* (see Appendix A).

In this program, z_1 and z_2 are respectively the sum of the construction costs of the public arcs belonging to the alignment and the weighted sum of demands d_{hk} multiplied by the minimum value between the private travel cost $c_{hk}{}^{PRI}$ and the minimum travel cost on the bi-modal network G^{BIM}.

In the following, some interesting relationships between the new formulation and the Median Shortest Path Problem are highlighted. In particular, it is shown that the new model is a sort of *generalization* of MSPP because, if:

- the private transportation system is not competitive;
- travel, boarding and alighting costs are equal to zero;

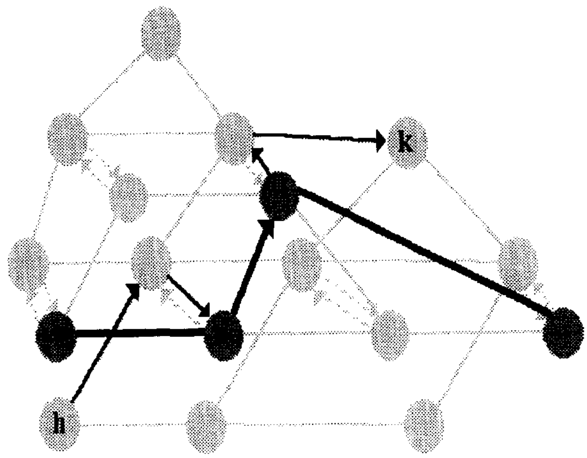

Fig. 3. A bimodal path.

- all demands are directed to the terminus node of the line path;

the efficient solutions of the two models coincide.

Proposition 1. *Let $G(V, A, c_{ij}, t_{ij}, d_i)$ be a graph for which c_{ij} and t_{ij} are respectively the arc construction cost and the travel time associated to each link $(i,j) \in A$ and d_i is the nodal demand associated to each node $i \in V$. The solutions of the new model coincide with the solutions of the Median Shortest Path Problem defined on G if the following assumptions are adopted:*

hp. 1. $V^{\mathrm{PRI}} = V^{\mathrm{PUB}} = V^{\mathrm{PED}} = V$,
hp. 2. $A^{\mathrm{PRI}} = A^{\mathrm{PUB}} = A^{\mathrm{PED}} = A$,
hp. 3. $c_{ij}^{\mathrm{C}} = c_{ij}$
hp. 4. $c_{ij}^{\mathrm{PUB}} = 0 \quad \forall (i,j) \in A^{\mathrm{PUB}}$,
hp. 5. $c_{ij}^{\mathrm{PED}} = t_{ij} \quad \forall (i,j) \in A^{\mathrm{PED}}$,
hp. 6. $c_{ij}^{\mathrm{PRI}} \geq t_{ij} \quad \forall (i,j) \in A^{\mathrm{PRI}}$,
hp. 7. $c_{ij}^{\mathrm{ALIGH}} = 0 \quad \forall (i,j) \in A^{\mathrm{ALIGH}}$,
hp. 8. $c_{ij}^{\mathrm{BOARD}} = 0 \quad \forall (i,j) \in A^{\mathrm{BOARD}}$,
hp. 9. $d_{hk} = \begin{cases} d_h & \text{if } k \equiv d, \\ 0 & \text{otherwise.} \end{cases}$

Proof. In both models the first objective is the line construction cost; furthermore, as private costs are greater than or equal to the corresponding pedestrian costs (i.e., the private transportation system is not competitive, hp. 6) and the transportation line is ideal (travel, boarding and alighting costs are equal to zero, hp. 4, 7, 8) the shortest path on the bi-modal network is always preferable to the private one ($y_{hk} = 1 \ \forall h,k \in V^{\mathrm{PED}}$). As all demands are directed to d (the terminus node of the line path, hp. 9), the travel cost of each user is the same as the accessibility to the line and consequently the second objectives of the two models coincide (Fig. 4).

4. The algorithm

To estimate the set of non-inferior solutions a method which draws its inspiration from MONET algorithm (Current et al., 1987) was developed. Referring to construction costs (z_1), the procedure identifies K shortest paths on the public network with starting node "o" and terminus node "d" using a label correcting technique (Azevedo et al., 1993). For each generated path, the corresponding bi-modal network is built by augmenting the pedestrian network with the arcs and nodes of the path and the corresponding boarding and alighting arcs. Furthermore, for each origin–destination demand d_{hk}, the comparison between the shortest path calculated on the private network and the shortest path on the bi-modal network is performed. Then, the second objective value z_2 associated with that path is calculated. Finally, the efficient solutions are selected in the set of the generated paths using the dominance relationship. A

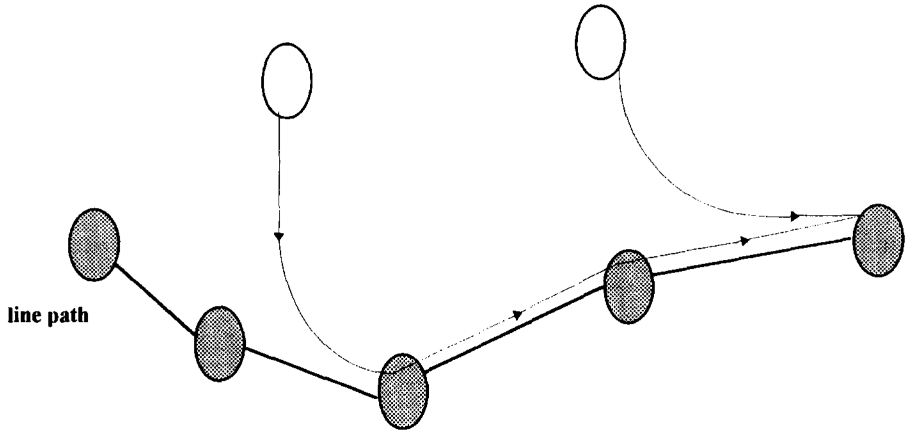

Fig. 4. Bi-modal shortest paths under assumptions hp1–hp9.

more formal description of the algorithm is provided:

STEP 1: For a given positive integer K, referring to the construction costs $c_{ij}{}^{C}$, the K shortest paths Γ_s (on G^{PUB}) from node $o \in G^{PUB}$ to node $d \in G^{PUB}$ are determined (the length of Γ_s corresponds to the value of z_1);

STEP 2: For each path Γ_s ($s = 1...K$) a bi-modal network is built in the following way:
- the arcs and nodes of Γ_s are added to the pedestrian network G^{PED};
- boarding and alighting arcs between the nodes of Γ_s and the corresponding nodes of G^{PED} are added;
- z_2 is initially set to 0 ($z_2 = 0$) and
- for each origin–destination (h,k), the weighted length of the shortest path from node h to node k on the bi-modal network is computed; if such a travel cost is less than the weighted corresponding private cost ($c_{hk}{}^{PRI}$), it is added to z_2; otherwise $c_{hk}{}^{PRI}$ is added to z_2;

STEP 3: Dominated paths Γ_s are eliminated by comparison.

Step 1 requires O(K $|V^{PUB}|$ log$|V^{PUB}|$) operations once the shortest path tree has been determined (Azevedo et al., 1993). Step 2 calculates k times the shortest paths between all origin–destination nodes and consequently requires O(K $|V^{PED}|$ ($|A^{PED}|$ + $|V^{PED}|$ log$|V^{PED}|$)) operations (Ahuja et al., 1993).

5. Computational results

The problem was solved using the sample network introduced by Current et al. (1987) formed by $n = 21$ nodes and $m = 39$ undirected arcs (Fig. 5); K was set equal to 250. The proposed algorithm was implemented in C language; computational time on an IBM 486 (33 Mhz) was equal to about 40 s.

Let G (V, A, c_{ij}, t_{ij}) be the sample network with V node set, A arc set, c_{ij} construction cost and t_{ij} travel cost; two different cases were considered. In the first (case 1) the following hypotheses are assumed:

hp. 1. $V^{PRI} = V^{PUB} = V^{PED} = V$,

hp. 2. $A^{PRI} = A^{PUB} = A^{PED} = A$,

hp. 3. $c_{ij}^{C} = c_{ij} \quad \forall (i,j) \in A^{PUB}$,

hp. 4. $c_{ij}^{PUB} = 0 \quad \forall (i,j) \in A^{PUB}$,

hp. 5. $c_{ij}^{PRI} = c_{ij}^{PED} = t_{ij} \quad \forall (i,j) \in A^{PRI}, A^{PED}$,

hp. 6. $c_{ij}^{ALIGH} = 0 \quad \forall (i,j) \in A^{ALIGH}$,

hp. 7. $c_{ij}^{BOARD} = c^{W} \quad \forall (i,j) \in A^{BOARD}$ (c^{W} is a parameter),

hp. 8. $$d_{hk} = \begin{cases} d_h & \text{if } k \equiv d \\ 0 & \text{otherwise} \end{cases} \quad \forall h,k \in V^{PED}.$$

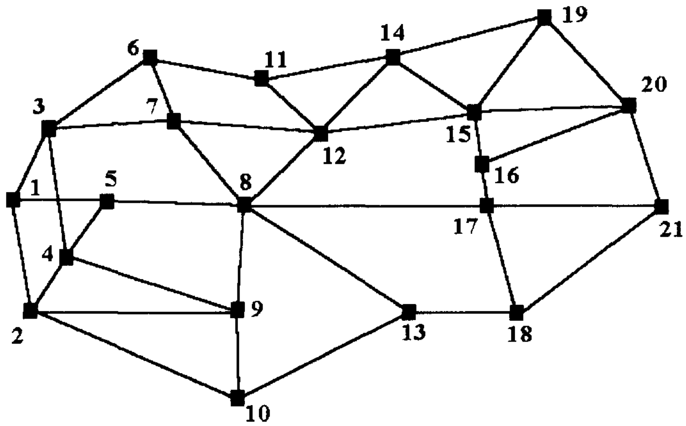

Fig. 5. Sample network.

For $c^{W}=0$ (no waiting costs), the assumptions are the same as in Proposition 1. The set of non-inferior solutions was determined by increasing c^{W} from 0 with a step equal to 5. In Fig. 6 the graphical representation in the space (z_1,z_2) of such sets is shown. In Appendix B the efficient paths together with the corresponding values of the objective functions are reported.

As mentioned, the case $c^{W}=0$ provides the same solutions of the MSPP problem (all the users take the public line to reach the destination entering at the closest node to the origin of each user trip). It can be observed that, as the waiting cost increases, the set of non-inferior solutions is characterized by a lower number of solutions with higher values of z_2. This is due to the fact that, in this situation, the number of users who choose the public line decreases. For $c^{W}=135$ (*critical waiting cost*) there is only one non inferior solution.

This case corresponds to the situation in which all the users choose the private mode and hence the total travel cost associated with each path becomes equal to

$$z_2=\sum_{h,k} d_{hk}\, c_{hk}^{\mathrm{PRI}}.$$

Consequently the presence of the public path does not affect user choice, so its construction is

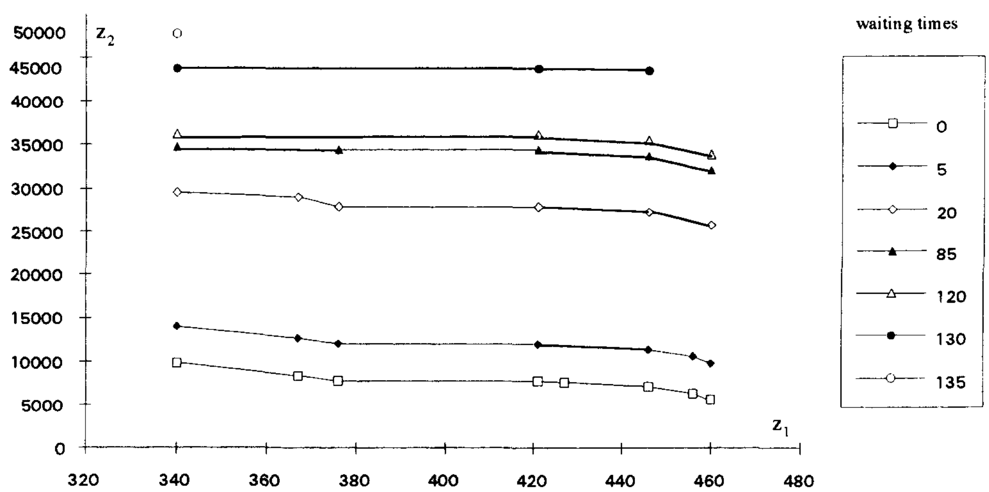

Fig. 6. The graphical representation of the non-inferior sets for different values of c^{w} (case 1).

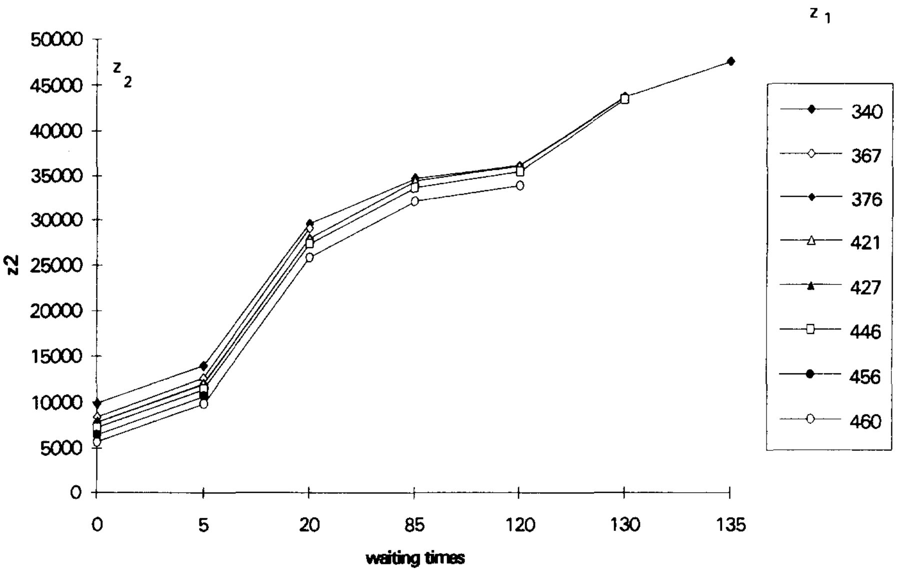

Fig. 7. Total travel times over waiting times for different values of z_1 (case 1).

unnecessary. Moreover, it can be noted that by varying the waiting cost c^{W} (for a given value of z_1), the total travel time (z_2) is characterized by considerable variations (Fig. 7).

The second case was performed using the following hypotheses:

hp. 1. $V^{\mathrm{PRI}} = V^{\mathrm{PUB}} = V^{\mathrm{PED}} = V$,
hp. 2. $A^{\mathrm{PRI}} = A^{\mathrm{PUB}} = A^{\mathrm{PED}} = A$,
hp. 3. $c_{ij}^{\mathrm{C}} = c_{ij} \quad \forall (i,j) \in A^{\mathrm{PUB}}$,
hp. 4. $c_{ij}^{\mathrm{PRI}} = t_{ij} \quad \forall (i,j) \in A^{\mathrm{PRI}}$,
hp. 5. $c_{ij}^{\mathrm{PED}} = 5\, t_{ij} \quad \forall (i,j) \in A^{\mathrm{PED}}$,
hp. 6. $c_{ij}^{\mathrm{PUB}} = 0.5\, t_{ij} \quad \forall (i,j) \in A^{\mathrm{PUB}}$,
hp. 7. $c_{ij}^{\mathrm{ALIGH}} = 0 \quad \forall (i,j) \in A^{\mathrm{ALIGH}}$,
hp. 8. $c_{ij}^{\mathrm{BOARD}} = c^{\mathrm{W}} \quad \forall (i,j) \in A^{\mathrm{BOARD}}$,
hp. 9. $d_{ij} = d_i/(n-1) \quad \forall i,j \in V^{\mathrm{PED}} (i \neq j)$.

This case describes a more realistic situation in which:
1. travel costs are considered to be travel times;
2. the pedestrian cost is supposed to be five times the private cost and 10 times the public cost.

Furthermore, the demand d_i associated with each node is assumed uniformly distributed over the destinations (hp. 9). As in the previous example, the set of non-inferior solutions was determined by varying c^{W} from 0 with a step equal to 5. The results (Figs. 8–9 and Appendix B) show characteristics similar to those underlined in the first case.

It can be noted that now the model shows less sensitivity to waiting times, which corresponds to a more realistic real-world situation. Waiting times can be used to model some combined measure of different factors (line frequency, accessibility of interchange areas, ...). Hence, the concept of waiting times can be considered, in our scheme, as a measure of the public transportation supply. It is well known that, in general, a great increase in public transportation supply is necessary to modify user modal choice. In both cases it was shown that parameters like waiting times usually play a fundamental role in the definition of efficient solutions.

6. Concluding remarks

In this paper a new bicriterion model, which evaluates in a more realistic way the attractivity of a RTL was introduced. The model considers an origin–destination demand matrix and assumes that each user chooses for his/her trip the path of least perceived cost from origin to destination by comparing the shortest path by private mode with

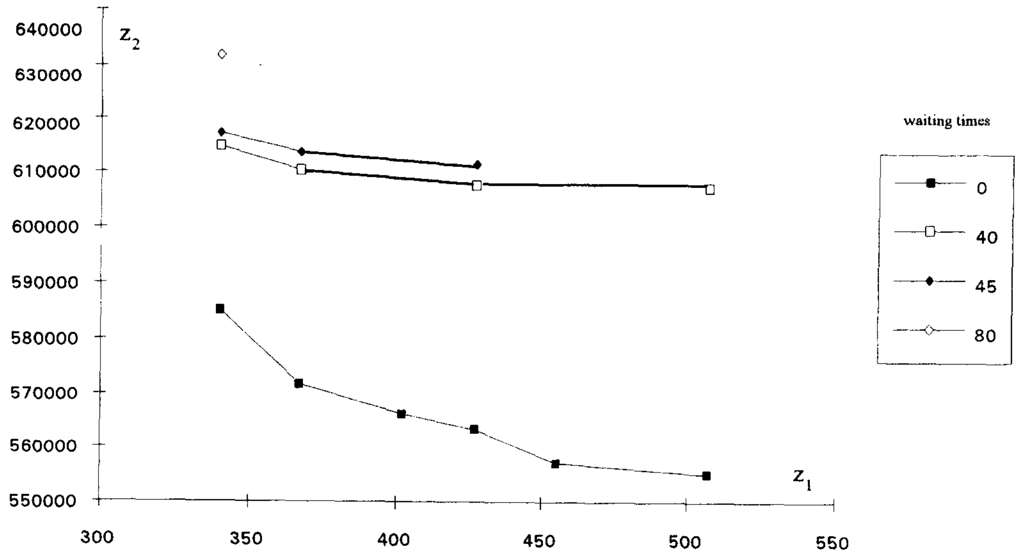

Fig. 8. The graphical representation of the non-inferior sets for different values of c^w (case 2).

the hybrid pedestrian–public shortest path which includes the use of the line to be located.

A procedure based on a k shortest path algorithm was developed to estimate the set of non-inferior solutions. It was applied to a well-known test problem and the results were discussed and compared with those obtained using a Median Shortest Path model.

The model represents a first fundamental step in order to design a decision support system able to give a more realistic evaluation in terms of costs and benefits of the various line paths.

In the present formulation, the model assumes some simplifying hypotheses which should be gradually removed. In particular, the construction cost of the stations and\or an upper bound on the

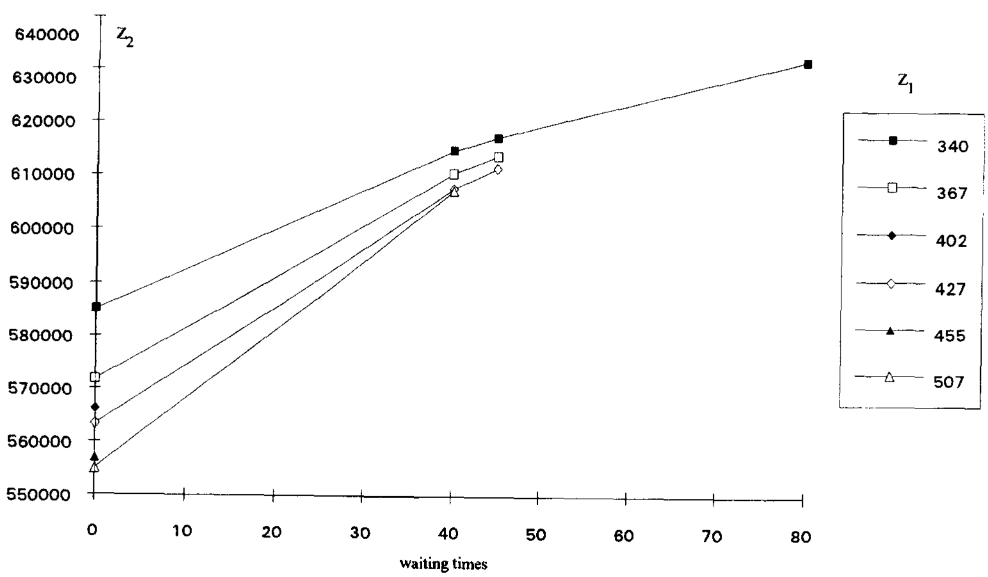

Fig. 9. Total travel times over waiting times for different values of z_1 (case 2).

number the stations (Dufourd et al., 1995) are to be considered. Furthermore, the possibility of a general multi-modal path (i.e. pedestrian–private–public–pedestrian) to model different interchange situations should be taken into account. Moreover, it could be interesting to formulate a general model within the context of different kinds of concurrent public transportation lines (for instance, bus lines) by introducing hyperpath assignment models (see, e.g., Nguyen and Pallottino, 1988; Hao Wu et al., 1994). The introduction of cost-flow functions to calculate the assignment of the flows on the private network should also be required.

Appendix A. A bicriterion integer linear model

$$Z_1 = \sum_{(i,j)\in A^{\mathrm{PUB}}} c^{\mathrm{c}}_{ij} \cdot x_{ij} \quad \min!$$

$$Z_2 = \sum_{h\in V^{\mathrm{PED}}} \sum_{k\in V^{\mathrm{PED}}} d_{hk} \left\{ c^{\mathrm{PRIV}}_{hk}(1-y_{hk}) + \sum_{(s,t)\in A^{\mathrm{BIM}}} c^{\mathrm{BIM}}_{st} u^{hk}_{st} \right\} \quad \min!$$

s.t.
$$\sum_{j\in N^{\mathrm{PUB}}_o} x_{oj} = 1, \tag{A.1}$$

$$\sum_{i\in M^{\mathrm{PUB}}_d} x_{id} = 1, \tag{A.2}$$

$$\sum_{j\in N^{\mathrm{PUB}}_k} x_{kj} - \sum_{i\in M^{\mathrm{PUB}}_k} x_{ik} = 0, \quad k \in V^{\mathrm{PUB}}(k \neq o,d), \tag{A.3}$$

$$\sum_{i\in B}\sum_{j\in B} x_{ij} \leq |B| - 1 \quad \forall B \subset V^{\mathrm{PUB}}, \tag{A.4}$$

$$\sum_{(s,t)\in A^{\mathrm{BIM}}} c^{\mathrm{BIM}}_{st} w^{hk}_{st} - c^{\mathrm{PRI}}_{hk} + M\cdot y_{hk} \geq 0, \quad h,k \in V^{\mathrm{PED}} \equiv V^{\mathrm{PRI}}, \tag{A.5}$$

$$\sum_{(s,t)\in A^{\mathrm{BIM}}} c^{\mathrm{BIM}}_{st} w^{hk}_{st} - c^{\mathrm{PRI}}_{hk} + M\cdot (1-y_{hk}) \leq 0, \quad h,k \in V^{\mathrm{PED}} \equiv V^{\mathrm{PRI}}, \tag{A.6}$$

$$\sum_{t\in Nh^{\mathrm{BIM}}} w^{hk}_{ht} = 1, \quad h,k \in V^{\mathrm{PED}}, h \neq k, \tag{A.7}$$

$$\sum_{s\in Mk^{\mathrm{BIM}}} w^{hk}_{sk} = 1, \quad h,k \in V^{\mathrm{PED}}, h \neq k, \tag{A.8}$$

$$\sum_{t\in Nr^{\mathrm{BIM}}} w^{hk}_{rt} - \sum_{s\in Mr^{\mathrm{BIM}}} w^{hk}_{sr} = 0, \quad h,k \in V^{\mathrm{PED}}, \quad r \in V^{\mathrm{BIM}}, r \neq h,k, \tag{A.9}$$

$$w^{hk}_{st} \leq x_{st} \quad (s,t) \in A^{\mathrm{PUB}}; h,k \in V^{\mathrm{PED}} \equiv V^{\mathrm{PRI}}, \tag{A.10}$$

$$u^{hk}_{st} \geq y_{hk} + w^{hk}_{st} - 1 \quad (s,t) \in A^{\mathrm{PUB}}; \quad h,k \in V^{\mathrm{PED}} \equiv V^{\mathrm{PRI}}, \tag{A.11}$$

$$x_{ij} = 0/1 \quad (i,j) \in A^{\mathrm{PUB}}, \tag{A.12}$$

$$y_{hk} = 0/1, \quad h,k \in V, \tag{A.13}$$

$$w^{hk}_{st} = 0/1 \quad (s,t) \in A^{\mathrm{BIM}}, \ h,k \in V, \tag{A.14}$$

$$u^{hk}_{st} = 0/1 \quad (s,t) \in A^{\mathrm{BIM}}, \ h,k \in V, \tag{A.15}$$

where:

- binary variable x_{ij} is equal to 1 if arc (i,j) belongs to the alignment, 0 otherwise;
- binary variable y_{hk} is equal to 1 if the demand d_{hk} uses the shortest path on the bi-modal network from h to k, 0 otherwise;
- binary variable $w^{hk_{st}}$ is equal to 1 if arc (s,t) belongs to the shortest path ($\in G_{\mathrm{BIM}}$) from node h to node k, 0 otherwise;
- binary variable $u^{hk_{st}}$ is equal to 1 if the demand d_{hk} uses the shortest path on the bi-modal network from h to k and such path contains arc (s,t), 0 otherwise;
- N_i^{PUB} is the set of nodes j such that arc $(i,j) \in A^{\mathrm{PUB}}$ exists;
- M_j^{PUB} is the set of nodes i such that arc $(i,j) \in A^{\mathrm{PUB}}$exists;
- N_i^{BIM} is the set of nodes j such that arc $(i,j) \in A^{\mathrm{BIM}}$ exists;

Table 2
Efficient paths together with the corresponding values of the objective functions

k	z_1	z_2	Path
Case 1			
Non inferior solutions for $c^W = 0$			
1	340	9879	2-4-5-8-12-14-19
5	367	8370	2-4-5-8-12-15-14-19
9	376	7813	2-4-5-8-12-11-14-19
43	421	7799	2-4-9-8-12-11-14-19
44	427	7664	2-4-5-8-12-14-15-20-19
62	446	7201	2-4-5-8-7-12-14-19
74	456	6394	2-4-5-8-12-11-14-15-19
79	460	5660	2-4-5-8-7-6-11-14-19
Non inferior solutions for $c^W = 5$			
1	340	13989	2-4-5-8-12-14-19
5	367	12636	2-4-5-8-12-15-14-19
9	376	12013	2-4-5-8-12-11-14-19
43	421	11999	2-4-9-8-12-11-14-19
62	446	11401	2-4-5-8-7-12-14-19
74	456	10660	2-4-5-8-12-11-14-15-19
79	460	9860	2-4-5-8-7-6-11-14-19
Non inferior solutions for $c^W = 20$			
1	340	29612	2-4-5-8-12-14-19
5	367	29102	2-4-5-8-12-15-14-19
9	376	27978	2-4-5-8-12-11-14-19
43	421	27964	2-4-9-8-12-11-14-19
62	446	27339	2-4-5-8-7-12-14-19
79	460	25825	2-4-5-8-7-6-11-14-19
Non inferior solutions for $c^W = 85$			
1	340	34738	2-4-5-8-12-14-19
9	376	34434	2-4-5-8-12-11-14-19
43	421	34420	2-4-9-8-12-11-14-19
62	446	33690	2-4-5-8-7-12-14-19
79	460	32141	2-4-5-8-7-6-11-14-19
Non inferior solutions for $c^W = 120$			
1	340	36188	2-4-5-8-12-14-19
43	421	36174	2-4-9-8-12-11-14-19
62	446	35490	2-4-5-8-7-12-14-19
79	460	33915	2-4-5-8-7-6-11-14-19
Non inferior solutions for $c^W = 130$			
1	340	43749	2-4-5-8-12-14-19
43	421	43735	2-4-9-8-12-11-14-19
62	446	43470	2-4-5-8-7-12-14-19
Non inferior solutions for $c^W = 135$			
1	340	47627	2-4-5-8-12-14-19
Case 2			
Non inferior solutions for $c^W = 0$			
1	340	585142	2-4-5-8-12-14-19
5	367	571769	2-4-5-8-12-15-14-19
22	402	566195	2-4-5-8-12-15-14-19
44	427	563311	2-4-5-8-12-14-15-20-19
73	455	557141	2-4-5-8-12-14-15-16-20-19
202	507	555073	2-4-3-7-6-11-14-15-16-20-19
Non inferior solutions for $c^W = 45$			
1	340	617104	2-4-5-8-12-14-19
5	367	613624	2-4-5-8-12-15-14-19
44	427	611373	2-4-5-8-12-14-15-20-19
Non inferior solutions for $c^W = 40$			
1	340	614697	2-4-5-8-12-14-19
5	367	610352	2-4-5-8-12-15-14-19
44	427	607581	2-4-5-8-12-14-15-20-19
202	507	607164	2-4-3-7-6-11-14-15-16-20-19
Non inferior solutions for $c^W = 80$			
1	340	631840	2-4-5-8-12-14-19

- M_j^{BIM} is the set of nodes i such that arc $(i,j) \in A^{\mathrm{BIM}}$ exists;
- M is a large positive number.

Objective functions z_1 and z_2 are respectively the construction cost of the path and the total user travel cost. In particular, z_2 is expressed by the weighted sum of demand d_{hk} multiplied by: the private travel cost $c_{hk}{}^{\mathrm{PRI}}$ if $y_{hk} = 0$ or the minimum travel cost on the bi-modal network G^{BIM} (i.e. $\sum_{(s,t)\in A^{\mathrm{BIM}}} c_{st}^{\mathrm{BIM}} u_{st}^{hk}$) if $y_{hk} = 1$.

Constraints (1) and (2) respectively ensure that the starting node "*o*" and the terminus node "*d*" are on the line path. Constraint set (3) ensures that, if an arc on the line path enters node k, then one will exit node k, unless k is the starting or the terminus node. Constraint set (4) imposes the absence of tours or subtours in the solution.

Constraints (5) and (6) set modal variables y_{hk} to 1 if the private travel cost $c_{hk}{}^{\mathrm{PRI}}$ is higher than the corresponding bi-modal path cost (0 otherwise).

Constraints (7)–(9) are similar to constraints (1)–(3) for the shortest bi-modal network from node h (origin) to node k (destination).

Constraints (10) impose that, for each h, $k \in V^{\mathrm{PED}}$, arc $(s,t) \in A^{\mathrm{PUB}}$ does not belong to the shortest path from h to k on the bi-modal network (i.e. $w^{hk_{st}} = 0$) if (s,t) is not included in the alignment (i.e. $x_{st} = 0$).

Constraints (11) impose that variable u_{st}^{hk} is equal to 1 if both w_{st}^{hk} and y_{hk} are equal to 1 (i.e. if users travelling from h to k choose the transit line and arc $(s,t) \in G^{\mathrm{PUB}}$ belongs to the shortest path from h to k on the bi-modal network).

Appendix B. Non inferior solutions for the two sample problems

In Table 2 the efficient paths together with the corresponding values of the objective functions are given.

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