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Abstract

This paper brings into focus the topological and geographical evaluation of metro networks through the definition of a methodological approach based on a set of indicators, a lot of which are defined in the sector literature. Once the methodology was stated, the results of an application on the metro networks of 13 big metropolitan areas were illustrated. Statistical comparative analyses are proposed to classify networks.

Keywords: metro transit network evaluation, graph indicators, geographical indicators

1. Introduction

The assessment of transit network performances involves different levels of investigation. The first level consists on the topological evaluation of a network through quantities giving indications about the degree of connection existing between the nodes; the next level concerns the geographic evaluation through quantities giving indications about the level of territorial cover of the service; the third level aims at an evaluation of the technological performances in terms of commercial speeds, frequencies and transportation capacity. These indicators, mostly taken from the sector literature (Kansky, 1963; Ore, 1963; Vaughan, 1990; Torrieri, 1990; Vuchic, 1991; Vuchic and Musso, 1991), are useful to evaluate different project alternatives, to verify the capability to serve the territory and to make comparative analysis of systems while working in different urban and metropolitan contexts.

2. Metro network analysis. Methodological approach

The approach consists on a gradually more detailed examination of public network elements including, at first, a graph level and, later, a geographical level evaluation. In the graph level evaluation the topological aspects regarding lines (number of nodes, links etc.) and network (number of loops, lines, etc.) are investigated. In the geographical level, more comprehensive information allow computing quantities such as the length of links, and therefore the average interspaces between stops, the territory involved or served by the network. The logic process leading to the computation and use of indicators is shown in figure 1.

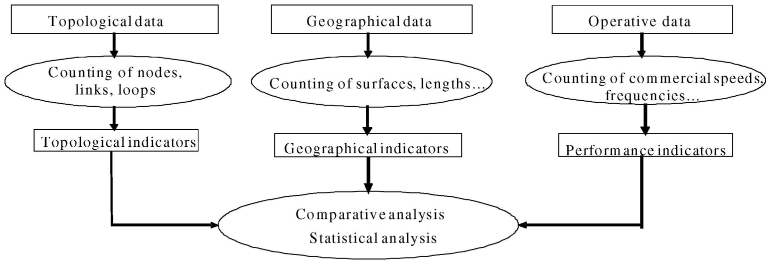


Figure 1. Methodological approach.

The analysis aims at pointing out reference meaningful parameters for the metro network comparative analysis, rather than methodologies of comparative analysis. For the possible approach of a comparative evaluation, a wide bibliography concerning Multicriteria analysis methods is available (Chankong and Haimest, 1983; Hwang and Yoon, 1981; Saaty, 2001; Vallée and Zielniewicz, 1994, Voogd, 1983).

2.1. Graph level indicators

This phase refers to three elements:

- node, point representing the spatial positions where there is the access to the metro;
- line, sequence of links and nodes representing the routes;
- network, the set of nodes, links and lines.

The following indicators are some of those existing in literature and will be used to describe the topological characteristics of the network elements.
A *i* node can be characterized by:

- δ_i = local degree, number of concurrent links on the generic node (Kansky, 1963);
- p_i = node weight, it is equal to 1 when local degree is 1, or it is a multiple of local degree when this is greater than 1 (Kansky indicates a factor 2);
- p_{ri} = relative node weight, it is the weight of the *i* node referred to the average weight of all the nodes of a line (p^L_{ri}) or of a network (p^R_{ri}):

$$p^L_{ri} = \frac{p_i}{\left(1/N^L \cdot \sum_{i=1}^{N^L} p_i\right)} \qquad p^R_{ri} = \frac{p_i}{\left(1/N \cdot \sum_{i=1}^{N} p_i\right)}$$

where N^L indicates the total number of the line nodes and *N* the total number of the network nodes;

- n_{Di} = number of destinations directly connected by *i* node.

A line *j* can be characterized by:

- p_j = line weight, amount of the weights of the nodes of line j:

  $$p_j = \sum_k p_{kj}$$

  where p_{kj} is the generic k node weight of line j;
- p_{rj} = line relative weight, it is the weight of the line j referred to the average weight of the network lines:

  $$p_{rj} = \frac{p_j}{\left(1/NL \cdot \sum_{j=1}^{NL} p_j\right)}$$

where NL indicates the total number of the network lines.

A network can be characterized by:

- N = number of nodes; generally it does not correspond to the number of network stations (two or three different line stations, placed at the same point, are represented by a unique network node);
- A = number of links (a link covered by several lines is counted only once);
- NL = number of lines;
- P = network weight, it is the amount of the weights of the network nodes

  $$P = \sum_{i=1}^{N} p_i$$

- $\bar{p}$ = average weight of the network nodes, it is equal to the ratio between the network weight and the total number of the network nodes

  $$\bar{p} = \frac{P}{N}$$

- N_D = average number of destinations connected in a direct way by the network nodes

  $$N_D = \frac{1}{N} \cdot \sum_{i=1}^{N} n_{Di}$$

- τ = connection indicator, it is the ratio between the number of network links and the highest number of links in a planar graph with the same number N of nodes (Kansky, 1963):

  $$\tau = \frac{A}{(3 \cdot (N - 2))}$$

- M = number of network loops, it is calculated through the following formula (Ore; 1963):

  $$M = A - N + 1$$

- α_M = availability of loops, it is the ratio between the number of network loops and the highest number of loops in a planar graph with the same number N of nodes (Kansky, 1963)

  $$\alpha_M = \frac{M}{(2N - 5)}$$

Table 1. Test network indicators.

		Node						Line			
Indicators	Symbol	1	2	3	4	5	6	A	B	C	Network
Local degree	δ_i	2	3	2	2	3	2	–	–	–	–
Node's weight	p_i	4	6	4	4	6	4	–	–	–	–
Node's relative weight (referred to network)	p_{ri}	0,86	1,29	0,86	0,86	1,29	0,86	–	–	–	–
Number of directly connetted destinations	N_{Di}	5	4	3	5	4	3	–	–	–	–
Line's weight	p_j	–	–	–	–	–	–	18	16	18	–
Line's relative weight	p_{rj}	–	–	–	–	–	–	1,04	0,92	1,04	–
Number of nodes	N	–	–	–	–	–	–	–	–	–	6
Number of links	A	–	–	–	–	–	–	–	–	–	7
Number of lines	NL	–	–	–	–	–	–	–	–	–	3
Network's weight	P	–	–	–	–	–	–	–	–	–	28
Nodes' average weight	$\bar{p}$	–	–	–	–	–	–	–	–	–	4,667
Average number of directly connected destinations	N_D	–	–	–	–	–	–	–	–	–	4
Connectivity	τ	–	–	–	–	–	–	–	–	–	0,583
Complexity	β	–	–	–	–	–	–	–	–	–	1,167
Number of loops	M	–	–	–	–	–	–	–	–	–	2
Availability of loops	α_M	–	–	–	–	–	–	–	–	–	0,286

- β = complexity indicator, it is the ratio between the number of links and the number of nodes (Kansky, 1963)

$$\beta = A/N$$

Table 1 shows the main topological indicators calculated on the test network of figure 2 where there are 3 lines, 6 nodes and 7 links.

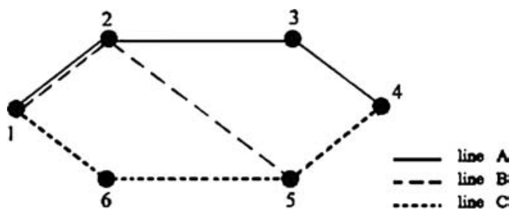


Figure 2. Test networks.

2.2. *Geographical level indicators*

Since there are better information, knowing for example the length of links, the real spreading of the lines on territory, the territory surface involved by system, it is possible to define other indicators. The following indicators are some of the most important ones existing in literature:

- L = network length, it is the amount of the lengths l_j of NL network lines

$$L = \sum_{j=1}^{NL} l_j$$

- LN = net network length, it is the network length minus the link lengths counted several times

$$LN = L - \sum_{m} (k-1) \cdot l_m^k$$

where:
k is the multiplicity of link m, that is the number of lines passing through it;
l_m^k is the length of link m with multiplicity k
- ϑ_l = line overlapping degree; $\theta_l = 1 - LN/L$;
- $\bar{L}$ = average line length, it is the ratio between the network length and the number of lines

$$\bar{L} = L/NL$$

- d_j = average interspaces between the line stops

$$d_j = \frac{l_j}{A_j}$$

where l_j is the line length and A_j is the number of links of line j.
- $\bar{L}_A$ = average link length (or average distance between two network stops), it is the ratio between the net network length and the number of network links

$$\bar{L}_A = LN/A$$

- d_i = average distance of connection referred to node i, is the average distance between node i and the nodes linked with it

$$d_i = \left(\sum_{k=1}^{N_i} d_{ik} \right) \Bigg/ N_i$$

where N_i is the number of nodes linked with the node i and d_{ik} the distance between node i and the generic node k linked with it;
- D = network diameter; it is the length of the shortest route connecting the farthest nodes of the network, as the crow flies;
- Π = network extension, it is the ratio between the network net length and the network diameter

$$\Pi = LN/D$$

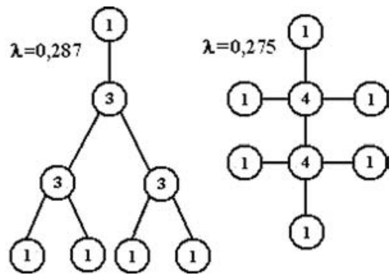


Figure 3. Graphs with different specific weights (www.hofstra.edu).

- λ = network specific weight, it is the ratio between the weight of the graph nodes and the network net length

$$\lambda = P/LN$$

A greater value of λ indicates a higher facility of movement within the network. The two graphs of figure 3, where the local degree is shown for each node, have the same length ($LN = 80$ km) and the same number of nodes ($N = 8$), but two different values of λ indicator; on the left network the movements are "easier".

- S = surface served by the network, it is equal to the territory extension where network is attractive;
- S_u = reference territory surface. The served surface S does not often correspond to the administrative borders and metro networks extend outside of them; in these cases the reference territory surface is larger than town extension. In par. 3 a specific practice is proposed to compute S_u.
- ρ_u = network density, it is the ratio between the network net length and surface S_u
- $\rho_u = LN/S_u$ (if LN is referred to served surface: $\rho = LN/S$)
- ρ_{Nu} = access density, it is the ratio between the number N of nodes and surface Su
- $\rho_{Nu} = N/S_u$ (if N is referred to served surface: $\rho_N = N/S$)
- A_S = network covering degree (or spatial accessibility), it is the ratio between the served surface and the reference territory surface

$$A_S = S/S_u$$

2.3. Proposed indicators

Two approaches exist in order to calculate the influence of a transit network on the territory: the first approach hypothesizes that a line j serves a corridor of territory, the second one hypothesizes that a stop i serves a circular surface with range of influence R_i. In the first approach the served surface will be the amount of corridors, in the second approach it will be the amount of the circular surfaces. For metro networks the second approach is preferred since interspace between stops is high and a method for computing the range of influence of the generic stop i is proposed.

2.3.1. Node's range of influence. Since it is reasonable in a network to hypothesize that nodes are not equally attractive, it will be proposed to differentiate the attractiveness of a node *i*, therefore the width of the range of influence R_i, according to three parameters:

- geographic position of node *i* (in order to establish the node position, the territory is divided into three areas through three concentric circles, with increasing radii from the town centre to suburbs, named respectively "centre", "first corona" and "second corona");
- relative node weight;
- ratio between the number of destinations directly connected by node *i* and the average number of direct connections on the network (n_{Di}/N_D). The greater is this ratio, the greater is the attractiveness of a node.

The proposed formula to compute the range of influence R_i is:

$$R_i = R_b \cdot a_1 \cdot (a_2 \cdot n_{Di}/N_D + a_3 \cdot p_{ri}) \quad [m]$$

where:

R_b, standard range, indicates the largest distance accepted on average by a walker to access to a generic metro stop ($R_b = 500$ m);

a_1 depends on the node position and it is 0.5 if the node is in "centre", 1 if it is in the "first corona", 1.5 if it is in the "second corona";

a_2 and a_3 are two coefficients used to weight the contributions of n_{Di}/N_D and p_{ri}; it has been chosen $a_2 = 0,65$ e $a_3 = 0,35$.

So, the range of influence of a metro station extends according to its geographic position (in the suburbs longer distances of access are accepted) and the importance of the station itself (this is measured by the potentiality of links and by the fraction of direct connections).

2.3.2. Network's covering. Circular surfaces defined by the range of influence can be placed one upon another. Therefore it is necessary to distinguish a theoretical served surface *ST*, that is the amount of the circular surfaces, and a real served surface *S* (net covering) which is obtained by eliminating the surfaces counted several times from *ST*:

$$S = ST - [(S_1 \cap S_2) \cup (S_2 \cap S_3) \cup \ldots] = \sum_{i=1}^{N} \pi . R_i^2 - [(S_1 \cap S_2) \cup (S_2 \cap S_3) \cup \ldots]$$

where S_1, S_2,... are the surfaces served by nodes 1, 2, ... (figure 4)

It is possible to define an indicator named "net covering level" that is the ratio between real and theoretical served surfaces:

$$\vartheta = S/ST$$

This indicator measures the level of agreement of the covering supplied by the network; the greater is ϑ indicator, the smaller is the overlapping.

2.4. Performance indicators

This paper will examine two performance indicators:

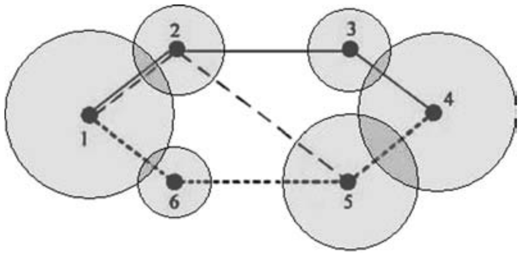


Figure 4. Example of served surfaces with overlapping.

- commercial speed;
- frequencies.

The commercial speed of a line is the ratio between the total length of line L and the amount of the average travel time t_k of the k generic sections making the line:

$$v_c = \frac{L}{\left(\sum_k t_k\right)}$$

where in t_k the times of braking, acceleration and wait on starting or final stop k are calculated.

Frequency indicates the number of transits going on a direction. This greatness can be associated to a line or to a stop; if only a line transits from a stop, line frequency will coincide with stop frequency, if more lines transit from it, stop frequency is the amount of line frequencies.

Among the studied networks there are lines with two or more terminals (respectively single branch and multi-branch lines). In the second case the line has one or more points of bifurcation where it divides into two or more branches. A line with x branches will be transformed into x lines (figure 5) and its commercial speed will be computed as if there were as many lines as branches.

On the websites of transit companies there are information systems indicating the average time of waiting t_a or the temporal space h between two transits going on a direction. Such information have been used to compute the frequencies Q of the lines.

Knowing h or t_a the frequency is computed through one of these two formulas:

$$Q = 1/h; \qquad Q = 1/2t_a$$

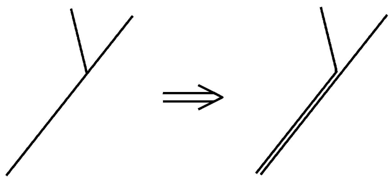


Figure 5. Example of line with two branches.

Like commercial speeds, when a line divides into more branches, it is necessary to compute as many frequency values as branches. The last chapter will deal with node frequency; if only one line transits from a stop, the line frequency coincides with stop frequency, if more lines transit from it, stop frequency is the amount of the line frequencies.

3. Analysis of indicators

Since, in most cases, metro networks extend outside urban boundaries, surfaces S_u include corridors two-kilometres wide between extra-urban sections of lines. In order to compute the radii of the city areas, city centre was placed, when possible, on the geographic barycentre of surfaces S_u using a function of the software "Autocad" (in many cases the barycentre coincides with the old city centre; when surfaces S_u had a very uneven shape, the city centre was placed on the old city centre). Fixing this point as the centre, the radii of 3 circles ("centre", "first" and "second corona") have been computed in the following way:

- the radius R_0 of "centre" has been chosen so that it includes 2/3 of urbanized surface S_u;
- the radius R_2 of "second corona" includes the most external point of surface S_u;
- the radius R_1 of "first corona" has been obtained through the expression $R_1 = 3/7 \cdot (R_0 + R_2)$

Network schemes in metro stations are often simplified. Since it does not exist any correspondence between scheme and real dimensions a scale representation has been used to show metro networks with the reference surface S_u, the barycentre position and the surface served by the network. This procedure has been adopted to compute the indicators described in the previous chapter for the networks of 12 European cities (Barcelona, Berlin, Brussels, Budapest, Lion, London, Madrid, Milan, Munich, Paris, Rome and Stockholm) and an American one (New York) (Table 2).

For example, in figure 6 the graphic representation of the territorial covering of Paris metro network is shown.

3.1. Statistical analysis

Sometimes information given by indicators on the characteristics offered by the networks are contrasting. For example, a high range of influence is, on the one hand, a positive factor since it indicates a greater level of territorial covering, on the other hand, it indicates a greater difficulty level for users who will have to walk, on average, a longer distance to reach a station.

Different indicators supply information of the same kind; that is why a set of data statistical analyses has been elaborated in order to identify possible correlations. Figure 7 shows graphically the correlations discovered between some of them: connectivity and complexity indicators resulted strongly correlated in a linear way with a high approximation ($R^2 \approx 1$); a strong correlation resulted also between the indicators of node average weight and loop availability (the representative points can be approximated by a linear curve with a very low error).

Table 2. Computed indicators.

Networks Indicators		Cities													Statistical Parameters	
Denomination	Symbol	Barcelona	Berlin	Bruxelles	Budapest	Lyon	London	Madrid	Milan	Munich	New York	Paris	Rome	Stockholm	Average	St. Dev.
Urbanized surface (km^2)	S_u	119,1	337,2	174,0	518,0	388,8	673,0	381,3	364,7	259,7	323,7	185,5	249,0	226,0	323,07	150,49
Inhabitants (millions)	Pop	3,0	3,5	1,0	2,0	1,3	8,0	4,5	3,0	1,3	8,5	9,0	4,0	1,7	3,91	2,84
Number of nodes	N	92	169	49	40	37	264	188	87	86	421	294	48	99	144,15	118,33
Number of links	A	102	182	49	39	37	368	220	90	88	485	390	47	100	169,00	151,86
Number of lines	NL	5	9	3	3	4	12	13	4	9	28	15	2	3	8,46	7,31
Number of loops	M	11	12	1	0	1	30	35	4	3	35	52	0	1	14,23	17,59
Connectivity	τ	0,387	0,363	0,348	0,342	0,352	0,468	0,394	0,353	0,349	0,386	0,445	0,341	0,344	0,37	0,04
Complexity	β	1,109	1,077	1,000	0,975	1,000	1,394	1,170	1,034	1,023	1,152	1,327	0,979	1,01	1,10	0,13
Availability of loops	α_{M}	0,061	0,036	0,011	0,000	0,014	0,057	0,094	0,024	0,018	0,042	0,089	0,000	0,005	0,03	0,03
Node average weight	$\bar{p}$	4,43	4,21	3,90	3,75	3,84	4,39	4,73	4,01	3,97	4,37	4,47	3,83	4,05	4,15	0,30
Network length (km)	L	74,50	153,00	40,10	33,00	30,00	482,50	302,00	101,61	96,97	618,78	201,80	34,00	105,28	174,89	185,86
Network net length (km)	LN	74,00	146,00	34,10	33,00	30,00	405,00	301,20	101,61	79,84	368,00	196,80	34,00	104,35	146,76	131,52
Diameter (km)	D	18,94	36,15	16,90	18,00	36,15	74,70	35,58	45,80	21,90	36,15	22,47	16,70	27,82	31,33	16,10
Network extension	Π	3,91	4,04	2,02	1,83	0,83	5,42	8,47	2,22	3,65	10,18	8,76	2,04	3,75	4,39	2,98
Network specific weight	λ	5,51	4,87	5,60	4,55	4,73	2,86	2,95	3,43	4,27	4,99	6,67	5,41	3,84	4,59	1,11
Network density (km/km^2)	ρ_u	0,62	0,43	0,20	0,06	0,08	0,60	0,79	0,28	0,31	1,14	1,06	0,14	0,46	0,47	0,35
	ρ	1,65	1,99	1,11	2,10	1,92	2,73	2,04	1,49	1,60	3,62	2,24	1,24	1,65	1,95	0,66
Access density (nodes/km^2)	ρ_{Nu}	0,77	0,50	0,28	0,08	0,10	0,39	0,49	0,24	0,33	1,30	1,58	0,19	0,44	0,52	0,46
	ρ_N	2,05	2,31	1,59	2,55	2,36	1,78	1,27	1,27	1,73	4,14	3,35	1,75	1,56	2,13	0,83
Link average length	D	725	802	696	846	811	1101	1369	1129	907	759	505	723	1044	878,21	228,61
Average range of influence (km)	R	442	373	465	338	349	405	502	492	427	323	350	478	455	415,22	62,78
Theoretical served surface (km^2)	ST	65,50	91,31	39,20	17,75	16,17	168,00	170,79	77,03	56,22	171,46	140,00	40,02	75,66	86,95	57,36
Served surface (km^2)	S	44,84	73,20	30,76	15,70	15,65	148,20	147,95	68,30	49,82	101,66	87,80	27,42	63,40	67,28	44,50
Spatial accessibility (%)	A_S	37,7	21,7	17,7	3,0	4,0	22,0	38,8	18,7	19,2	31,4	47,3	11,0	28,1	23,12	13,30
Net covering level (%)	θ	68,5	80,2	78,5	88,5	96,8	88,2	86,6	88,7	88,6	59,3	62,7	68,5	83,8	78,93	12,29

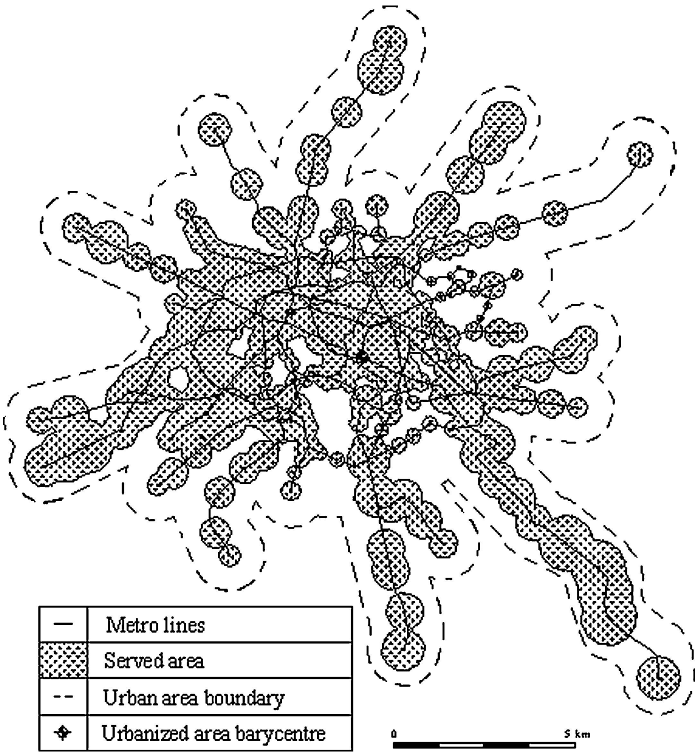


Figure 6. Territorial covering of Paris metro network.

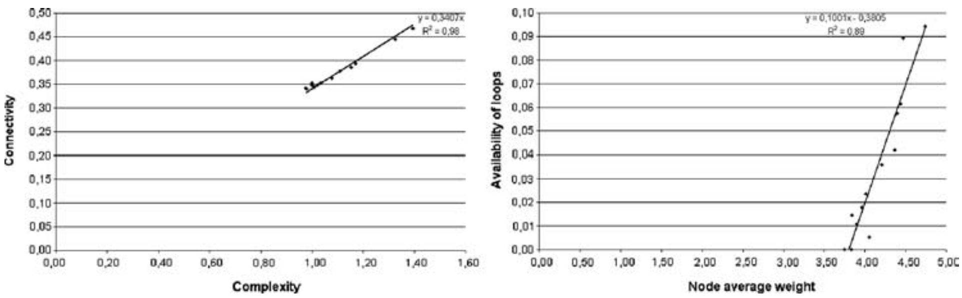
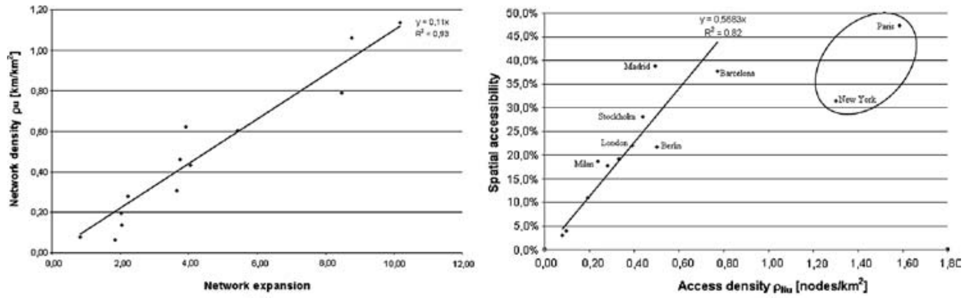


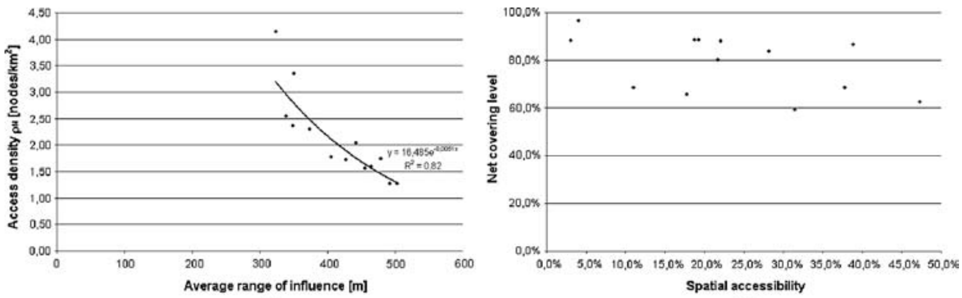
Figure 7. Correlations among topological indicators.

Other correlations resulted between network extension and network density ρ_u (figure 8(A)), and between access density and spatial accessibility (figure 8(B)). The first correlation is very strong, since the tendency curve is characterized by a R^2 value very close to one; representing graphically the data of access density and spatial accessibility it results that, except for the networks of New York and Paris, that have very high access density



(a) (b)

Figure 8. (A) Network extension/network density. (B) Access density/spatial accessibility.



(a) (b)

Figure 9. (A) Node average range of influence/access density. (B) Spatial accessibility/net covering degree.

values, the representative points are approximated by a straight line with a sufficiently acceptable error ($R^2 \approx 0.82$). The most distant points from the tendency curve concern those networks extending with a lot of corridors, or with very long corridors, towards outside the city (Milan, Madrid e Stockholm) and those rather developed networks, like Barcelona and Berlin, that do not extend uniformly in every direction within the urbanized surface.

The representative points of node average range of influence and access density ρ_N (computed referring to the served surface) are arranged according to a negative exponential function (figure 9(A)). Finally, as it is shown in figure 9(B), if spatial accessibility increases a high overlapping of the surfaces served by nodes, and therefore a reduction of net covering degree, is more likely to occur.

3.2. *Comparative analysis*

Among all the indicators of Table 2 the ones better describing the network characteristics of topology, width and density and territorial covering have been chosen, whereas the redundant ones were eliminated.

The indicators chosen to describe topological characteristics are:

- connectivity;
- node average weight;
- availability of loops.

The indicators chosen to describe width and density characteristics are:

- extension;
- network specific weight;
- network density ρ_u;
- access density ρ_N.

The indicators chosen to describe territorial covering characteristics are:

- node average range of influence;
- spatial accessibility;
- net covering level.

3.2.1. Topology indicators. The correlation between connectivity and complexity (figure 7) led to consider sufficiently indicative just one of them, that is connectivity, and to consider information coming from the other as redundant.

In spite of the existing correlation between them, both node average weight (figure 10(A)) and loop availability (figure 10(B)) have been considered significant since, even if they indicate the user level of options for their movement within the network, they depend on different factors. While loop availability depends on the number of nodes and loops, the node average weight is strongly influenced by the number of nodes existing in the lines (when in the lines there are a lot of stops, the node average weight is higher). The figures show the different information coming from the two indicators, especially when the networks are different for the presence of very long lines with many stops. For example, Stockholm network, which has 33 nodes per line on average and where there is only a loop, has a small loop availability but a higher node average weight than Milan network (21,75 nodes per line on average and 4 loops).

3.2.2. Width and density indicators. Net length is significant if referred to other indicators such as network diameter, network weight or the surface of the metropolitan area.

If net length is referred to network diameter, then network extension can be obtained (figure 11(A)), the ratio between network weight and net length is the network specific weight (figure 11(B)).

The networks which have a few lines extending outside urban boundary for many km, like Milan network, where the green line serves densely-populated suburbs, show low extension indicator. A similar thing happens for the London network: though it has the highest net length of all networks, the extension indicators is damaged by a diameter of about 75 km.

A similar argument is valid for the network specific weight; the presence of lines going from centre to suburbs, then with few intersections with other lines, reduces the network total weight, so causing a greater difficulty of movement. The network weight increases when lines are concentrated in a more compact surface.

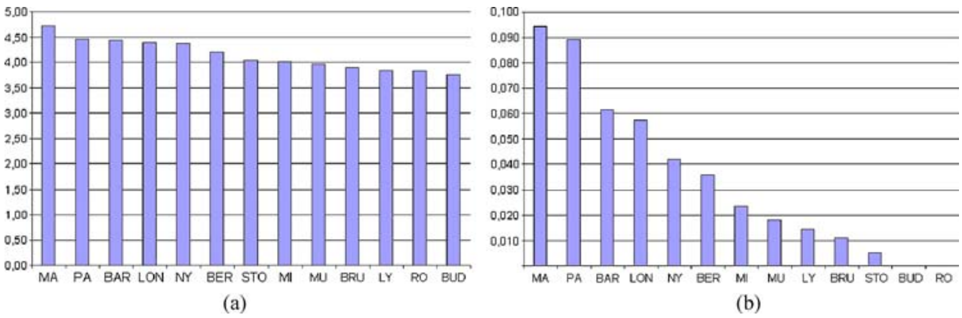


Figure 10. (A) Node average weight. (B) Availability of loops.

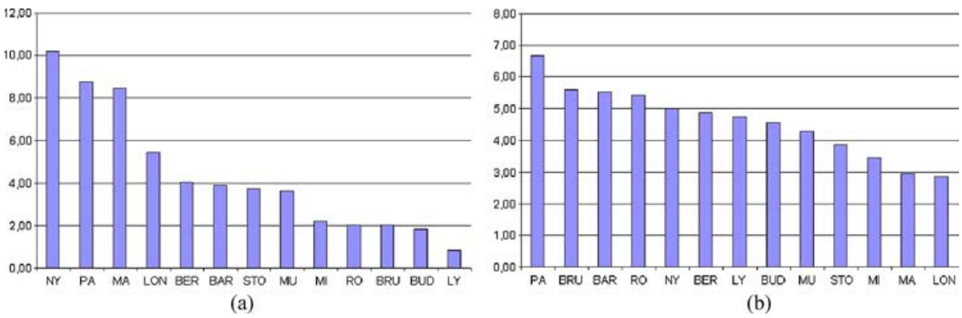


Figure 11. (A) Network extension. (B) Network specific weight.

The ratio between net length and urbanized surface indicates the network density ρ_u referred to the whole territory, while the ratio between net length and served surface indicates network density ρ referred to a portion of density. The first indicator is considered more significant since it measures density in absolute terms.

An argument analogous with the one referring to net length is valid also for the number of nodes that is significant if referred to urbanized surface (ρ_{Nu}) or to served surface (ρ_N). ρ_{Nu} is an "absolute" indicator since it is referred to the whole territory, unlike ρ_N that is a "relative" indicator. The variation in the network order, taking into account ρ_{Nu} rather than ρ_u, is due to the average interspace between stops. The networks of Madrid, London and Milan, which have the longest link average length, lose some position in favour of those networks having a shorter link average length.

3.2.3. Territorial covering indicators. Among the territorial covering indicators, spatial accessibility (figure 12(A)) and net covering degree (figure 12(B)) are considered more significant.

If territorial covering increases there is a greater probability of an overlapping of served surfaces; this trend is more marked when link average length decreases. Examining figure 12(A) and (B), it can be noticed that the networks of Paris, Barcelona and New York, which

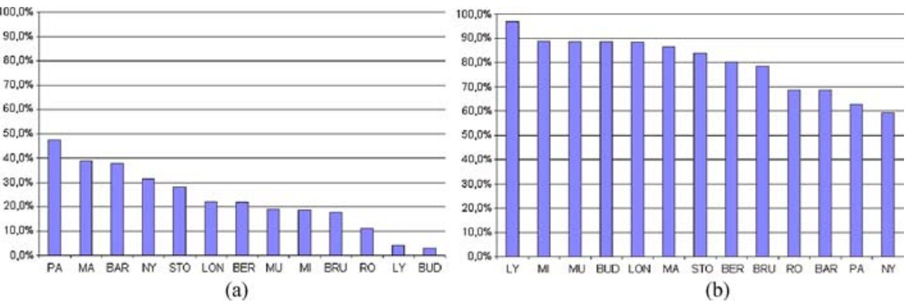


Figure 12. (A) Spatial accessibility. (B) Net covering level.

have relatively small values of link average length, have also a higher overlapping of served surfaces; Madrid and Stockholm, which have a greater link average length, preserve a very high net covering degree.

The last territorial covering indicator that has been considered is the average range of influence. If this indicator increases, the user difficulty of access to the network also increases.

3.3. Multicriteria analysis

Metro networks have been examined by means of the ten indicators presented at the beginning of par.3.2 with a multicriteria approach based on the weighed average method. Table 3 shows the valuation matrix, and Table 4 the standardized one, which has values included within the interval (0,1). The standardization operation depends on the kind of indicator; if the indicator *i* has to be maximized, then it is necessary to compute the standardized value of network *j* ($\bar{x}_{ij}$) as the ratio between its value x_{ij} and the highest value it has in all networks

$$\bar{x}_{ij} = x_{ij} \Big/ \left(\max_j(x_{ij})\right)$$

while if the indicator *i* has to be minimized the standardized value is

$$\bar{x}_{ij} = 1 - x_{ij} \Big/ \left(\max_j(x_{ij})\right)$$

Except for the average range of influence, which improves if it decreases (a higher range of influence corresponds in fact to a greater difficulty for the user in the phase of access to the network), all the other indicators improve if they increase.

After the standardization operation, a weight β_i has been assigned to the generic indicator *i*, so obtaining network utility through the expression $X_j = \sum_i (x_{ij} \cdot \beta_i)$

Assigning the identical weight to all indicators, the classification obtained is shown in figure 13(A), which put in evidence that the best structural characteristics are supplied by the networks of Paris, New York and Madrid.

Finally, it has been chosen to weight indicators in a differentiated way (figure 13(B)):

- the topological indicators of node average weight and loop availability, that are somehow correlated, have been assigned weight 0,5;

Table 3. Evaluation matrix.

Indicators		Cities												
Denomination	Symbol	Barcelona	Berlin	Bruxelles	Budapest	Lyon	London	Madrid	Milan	Munich	New York	Paris	Rome	Stockholm
Connectivity	τ	0,378	0,363	0,348	0,342	0,352	0,468	0,394	0,353	0,349	0,386	0,445	0,341	0,344
Availability of loops	α_M	0,061	0,036	0,011	0,000	0,014	0,057	0,094	0,024	0,018	0,042	0,089	0,000	0,005
Node average weight	$\bar{p}$	4,43	4,21	3,90	3,75	3,84	4,39	4,73	4,01	3,97	4,37	4,47	3,83	4,05
Network extension	Π	3,91	4,04	2,02	1,83	0,83	5,42	8,47	2,22	3,65	10,18	8,76	2,04	3,75
Network specific weight	λ	5,51	4,87	5,60	4,55	4,73	2,86	2,95	3,43	4,27	4,99	6,67	5,41	3,84
Network density (km/km^2)	ρ_u	0,62	0,43	0,20	0,06	0,08	0,60	0,79	0,28	0,31	1,14	1,06	0,14	0,46
Access density (nodes/km^2)	ρ_{Nu}	0,77	0,50	0,28	0,08	0,10	0,39	0,49	0,24	0,33	1,30	1,58	0,19	0,44
Average range of influence (m)	R	442	373	465	339	349	405	502	492	427	323	350	478	455
Spatial accessibilità	A_S	0,38	0,22	0,18	0,03	0,04	0,22	0,39	0,19	0,19	0,31	0,47	0,11	0,28
Net covering level	θ	0,69	0,80	0,79	0,89	0,97	0,88	0,87	0,89	0,89	0,59	0,63	0,69	0,84

Table 4. Standardized evaluation matrix.

Indicators		Cities												
Denomination	Symbol	Barcelona	Berlin	Bruxelles	Budapest	Lyon	London	Madrid	Milan	Munich	New York	Paris	Rome	Stockholm
Connectivity	τ	0,807	0,776	0,742	0,731	0,753	1,000	0,842	0,754	0,746	0,824	0,951	0,727	0,734
Availability of loops	α_M	0,651	0,382	0,114	0,000	0,154	0,608	1,000	0,251	0,190	0,443	0,945	0,000	0,055
Node average weight	$\bar{p}$	0,937	0,889	0,823	0,792	0,811	0,927	1,000	0,847	0,838	0,922	0,943	0,810	0,856
Network extension	Π	0,384	0,397	0,198	0,180	0,082	0,533	0,832	0,218	0,358	1,000	0,860	0,200	0,368
Network specific weight	λ	0,826	0,730	0,840	0,681	0,709	0,429	0,443	0,515	0,640	0,749	1,000	0,811	0,576
Network density (km/km^2)	ρ_u	0,547	0,381	0,172	0,056	0,068	0,529	0,695	0,245	0,270	1,000	0,933	0,120	0,406
Access density (nodes/km^2)	ρ_{Nu}	0,488	0,316	0,178	0,049	0,060	0,248	0,311	0,151	0,209	0,821	1,000	0,122	0,276
Average range of influence (m)	R	0,121	0,257	0,075	0,327	0,306	0,194	0,000	0,021	0,151	0,357	0,303	0,047	0,094
Spatial accessibilità	A_S	0,797	0,459	0,374	0,063	0,085	0,465	0,820	0,395	0,406	0,664	1,000	0,233	0,594
Net covering level	θ	0,708	0,829	0,811	0,914	1,000	0,911	0,895	0,916	0,915	0,613	0,648	0,708	0,866

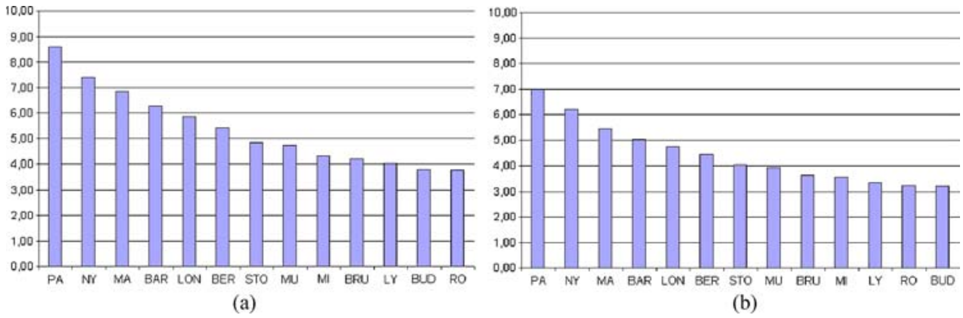


Figure 13. (A) Identical weights. (B) Differentiated weights.

- since spatial accessibility increases if the average range of influence increases, both have been assigned weight 0,5;
- all the other indicators have been assigned weight 1.

The classification does not change except for the recovery of a position for Rome and Bruxelles networks.

4. Conclusions and future developments of research

A first attempt at determining some correlations between structural characteristics and network performances, based on graph and geographic level indicators, has led to interesting results, therefore it could be extended to the other performance indicators. The future research will be addressed on operational performances and on their relationships with graph indicators. As example of the field of interest, Table 5 shows the average values of commercial speed (v_c) and of frequencies (data are referred to 7.30 a.m.). The average commercial speed is the average of the commercial speeds of the lines, the average frequency on the nodes is the average value of the frequencies of the network nodes.

The highest value of average commercial speed is in Milan network where, however, the railway link, which has a commercial speed of 51.2 km/h, has been also taken into account; if this value is not considered, then the average commercial speed falls down to 41.1 km/h. Among the largest networks, the one of Madrid has a good value of average commercial speed (35.7 km/h). The lowest commercial speeds have been computed on the networks of Paris, where there is a short average link length, and New York, where there is a high line overlapping degree. Just to get out of this problem, on this network an “Express” service exists that, every hour, has a lower number of stops than “local” service. Figure 14(A) shows a positive proportionality ratio between average commercial speed and link average length, figure 14(B) shows a negative tendency of average commercial speed when the line overlapping degree is significant.

Table 5. Average values of commercial speed and nodes' frequencies.

Performance's Indicators	Cities												
	Barcelona	Berlin	Bruxelles	Budapest	Lyon	London	Madrid	Milan	Munich	New York	Paris	Rome	Stockholm
Average comm.. speed (km/h)	27,8	30,7	28,7	31,8	28,5	34,4	35,7	42,8	34,1	21,8	25,2	28,4	35,5
Average frequency	16,9	14,2	14,4	11,1	14,1	8,6	19,2	14,5	11,4	15,2	24,7	18,1	15,9

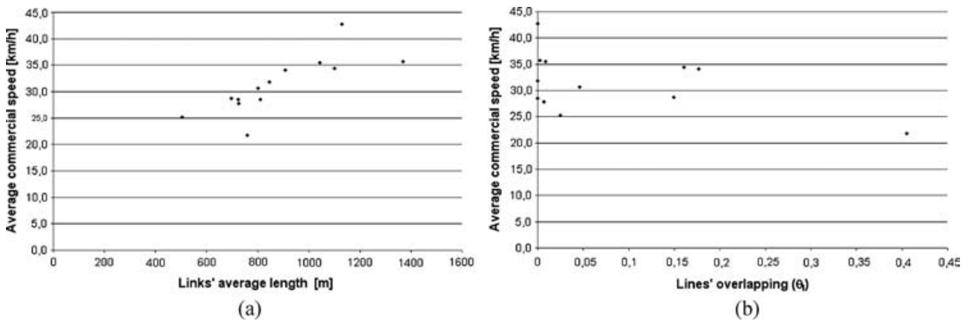


Figure 14. (A) Average v_c/links' average length. (B) Average v_c/lines' overlapping degree

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