1. (a) Let $\langle \cdot, \cdot \rangle$ be an inner product defined on a vector space V. Prove the Cauchy-Schwarz inequality

$$|\langle u, v\rangle|^2 \leq \langle u, u\rangle\langle v, v\rangle \quad \forall u, v \in V. \tag{1}$$

Hint: Consider the quadratic polynomial

$$p(t) = \langle u + tv, u + tv\rangle, \quad t \in \mathbb{R} \text{ (or } \mathbb{C}). \tag{2}$$

Can this polynomial take negative values? Use your answer to conclude what should be the sign of the discriminant if you set $p = 0$.

Solution: TODO

(b) Let B be a real symmetric positive definite $n \times n$ matrix. Use the Cauchy-Schwarz inequality to prove that

$$(g^\top Bg)(g^\top B^{-1}g) \geq (g^\top g)^2 \quad \forall g \in \mathbb{R}^n. \tag{3}$$

Solution: TODO