Institution	Number of Cases (approximate)
Duke University	680
University of California San Francisco	600
University of Missouri Columbia	400
University of California San Diego	350
Heidelberg University Hospital	300
University of Michigan	100
Indiana University	70
Total	**2200**

Table 1: Institutional contributions to the BraTS 2024 dataset.

1 Diffusion Processes

1.1 Forward Diffusion Process

The forward diffusion process gradually adds Gaussian noise to the data over T timesteps, defined as a Markov chain:

$$q(\mathbf{x}_t \mid \mathbf{x}_{t-1}) = \mathcal{N}\left(\mathbf{x}_t; \sqrt{\alpha_t}\mathbf{x}_{t-1}, (1-\alpha_t)\mathbf{I}\right) \tag{1}$$

where:

[leftmargin=*]$\mathbf{x}_0$ is the original data (e.g., an MRI image). $\mathbf{x}_t$ is the data at timestep t. α_t is a predefined noise schedule parameter for timestep t. $\mathbf{I}$ is the identity matrix.

The cumulative forward process can be expressed in closed form as:

$$q(\mathbf{x}_t \mid \mathbf{x}_0) = \mathcal{N}\left(\mathbf{x}_t; \sqrt{\bar{\alpha}_t}\mathbf{x}_0, (1-\bar{\alpha}_t)\mathbf{I}\right) \tag{2}$$

where $\bar{\alpha}_t = \prod_{s=1}^{t} \alpha_s$.

1.2 Reverse Diffusion Process

The reverse diffusion process aims to remove the added noise, recovering the original data from $\mathbf{x}_T$ to $\mathbf{x}_0$:

$$p_\theta(\mathbf{x}_{t-1} \mid \mathbf{x}_t) = \mathcal{N}\left(\mathbf{x}_{t-1}; \mu_\theta(\mathbf{x}_t, t), \Sigma_\theta(\mathbf{x}_t, t)\right) \tag{3}$$

where:

[leftmargin=*]$\mu_\theta(\mathbf{x}_t, t)$ is the mean predicted by the neural network at timestep t. $\Sigma_\theta(\mathbf{x}_t, t)$ is the covariance matrix predicted by the neural network at timestep t.

During the reverse process, noise is explicitly subtracted over T timesteps as follows:

$$\mathbf{x}_{t-1} = \mu_\theta(\mathbf{x}_t, t) + \Sigma_\theta(\mathbf{x}_t, t) \cdot \epsilon, \quad \epsilon \sim \mathcal{N}(0, \mathbf{I}) \tag{4}$$

This iterative denoising continues from $t = T$ down to $t = 1$, ultimately reconstructing the clean data $\mathbf{x}_0$.

1.3 Noise Schedule and Subtraction

The noise schedule $\{\alpha_t\}$ controls the amount of noise added at each timestep in the forward process and consequently the noise subtracted in the reverse process. A common choice is a linear or cosine schedule that defines α_t decreasing over time.

$$\alpha_t = 1 - \beta_t \tag{5}$$

where β_t is a small positive constant representing the noise variance at timestep t.

Example Illustration

Given: Flow Rate (FR) = $1.00 \times 10^3\, ft^3/min$
$Time = 1\, day = 24\, hours = 1440\, minutes$

Find:

- Mass of water in kilograms (kg)
- Weight of water in pounds (lb)

Calculations:

1. **Total Volume in ft^3:**

$$TotalVolume = FlowRate \times Time = 1.00 \times 10^3\, ft^3/min \times 1440\, min = 1.44 \times 10^6\, ft^3$$

2. **Convert ft^3 to Liters (L):**

$$1\, ft^3 = 28.3168\, L$$

$$TotalVolume(L) = 1.44 \times 10^6\, ft^3 \times 28.3168\, L/ft^3 \approx 4.08 \times 10^7\, L$$

3. **Calculate Mass in Kilograms (kg):**

$$Mass(kg) = Volume(L) \times DensityofWater = 4.08 \times 10^7\, L \times 1\, kg/L = 4.08 \times 10^7\, kg$$

4. **Convert Mass to Weight in Pounds (lb):**

$$1\,kg = 2.20462\,lb$$

$$Weight(lb) = 4.08 \times 10^7\,kg \times 2.20462\,lb/kg \approx 9.01 \times 10^7\,lb$$

Answers: Mass of water = 4.08 $\times 10^7\,kg$
$Weight of water = 9.01 \times 10^7\,lb$

1. **Total Volume in ft³:**

$$TotalVolume = FlowRate \times Time = 1.00 \times 10^3\,ft^3/min \times 1440\,min = 1.44 \times 10^6\,ft^3$$

2. **Convert ft³ to Liters (L):**

$$1\,ft^3 = 28.3168\,L$$

$$TotalVolume(L) = 1.44 \times 10^6\,ft^3 \times 28.3168\,L/ft^3 \approx 4.08 \times 10^7\,L$$

3. **Calculate Mass in Kilograms (kg):**

$$Mass(kg) = Volume(L) \times DensityofWater = 4.08 \times 10^7\,L \times 1\,kg/L = 4.08 \times 10^7\,kg$$

4. **Convert Mass to Weight in Pounds (lb):**

$$1\,kg = 2.20462\,lb$$

$$Weight(lb) = 4.08 \times 10^7\,kg \times 2.20462\,lb/kg \approx 9.01 \times 10^7\,lb$$

Answers: Mass of water = 4.08 $\times 10^7\,kg$
$Weight of water = 9.01 \times 10^7\,lb$