

MATH1034OL1 Pre-Calculus Mathematics Notes from Sections 5.2, 5.3 (Friday)

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1 Compound Interest and the Natural Base

For an initial amount P , interest rate r , compounding rate per time period t , and time periods elapsed t , we define the final amount A as

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

If the compounding is continuous, meaning it's calculate infinitely many times in a given compounding period, the base is $e \approx 2.718$. Let's derive e :

$$e = \lim_{x \rightarrow \infty} \left(1 + \frac{1}{n}\right)^{1 \cdot n}$$

Hence, to calculate the new amount given an initial amount P after t time periods with interest rate r , we use $A = Pe^{rt}$

For $P = 1000$, $t = 3$, and $r = 13\%$: $A = 1000e^{0.13 \cdot 3} \approx 1,476.98$

2 Logarithms

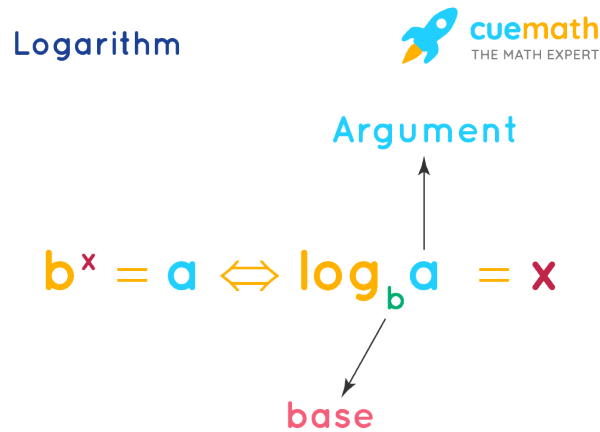


Figure 1: Credit: <https://www.cuemath.com/log-formulas/>

In plain language: the value of a logarithm is the power the base must be raised to in order to get the argument.

Logarithms are also the inverses of exponential functions. This means that for $f(x) = b^x$, $f^{-1}(x) = \log_b x$. Given this property, we know that an exponential function and its corresponding inverse logarithmic function will reflect over the line $y = x$:

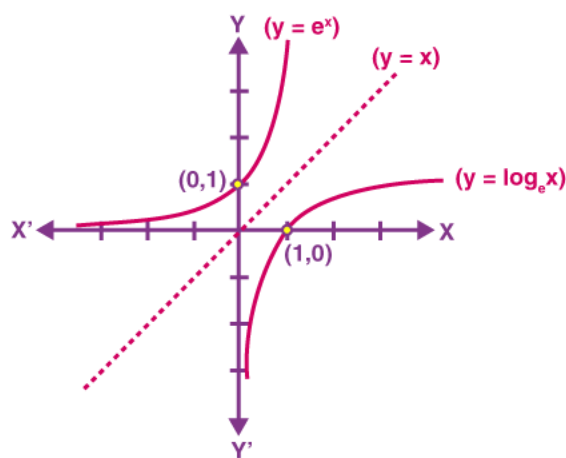


Figure 2: Credit: <https://byjus.com/maths/exponential-and-logarithmic-functions/>

Here are the rules to find the inverse of a logarithmic or exponential function:

π

To find the inverse of a logarithmic function, follow these steps:

Example: Find the inverse of $f(x) = \log_2(x+3)$.

Step 1: write as $y = \log_2(x+3)$.

Step 2: swap x and y : $x = \log_2(y+3)$.

Step 3: make y the subject by changing to an exponential form first :

$$2^x = y + 3$$

Step 4: then subtract 3 from both sides:

$$2^x - 3 = y$$

Step 5: replace y with $f^{-1}(x)$.

Step 6: write down the final answer

$$f^{-1}(x) = 2^x - 3$$

Figure 3: Credit: <https://slideplayer.com/slide/12938368/>

π

To find the inverse of an exponential function, follow these steps:

Example: Find the inverse of $f(x) = 3^{x-1} + 5$.

Step 1: write as $y = 3^{x-1} + 5$

Step 2: swap x and y : $x = 3^{y-1} + 5$

Step 3: make y the subject by subtracting 5 from both sides first :

$$x - 5 = 3^{y-1}$$

Step 4: then take log base 3 of both sides:

$$\log_3(x - 5) = \log_3 3^{y-1}$$

$$\log_3(x - 5) = y - 1$$

Step 5: add 1 to both sides

$$\log_3(x - 5) + 1 = y$$

Step 5: replace y with $f^{-1}(x)$.

Step 6: write down the final answer

$$f^{-1}(x) = \log_3(x - 5) + 1$$

Figure 4: Credit: <https://slideplayer.com/slide/12938368/>

We didn't discuss logarithmic graphs in class, but here are some important properties to understand:

$|a| > 1 \rightarrow$ vertical stretch by a factor of a

$0 < |a| < 1 \rightarrow$ vertical compression by a factor of a

$a < 0 \rightarrow$ reflection over the x-axis

$h \rightarrow$ horizontal translation

$k \rightarrow$ vertical translation

$$y = a \cdot \log_b(x - h) + k$$

Transformations
of
Logarithmic
Functions

| Domain | Range | Vertical Asymptote |
|-----------------------------------|---------------------|--------------------|
| $[h, \infty)$ or $x \geq h$ | $(-\infty, \infty)$ | $x = h$ |

Figure 5: Credit: <https://goodsifyet.shop/product-details/7113278.html>

There exists an asymptote at $x = h$ because logarithmic functions may not have an argument less than 0. Similarly, they also may not have a negative base.

There are some rules for logarithms, too:

Rule of Logarithms



| Rule Name | Property |
|--------------------------------|---|
| Log of 1 | $\log_b 1 = 0$ |
| Log of the same number as base | $\log_b b = 1$ |
| Product Rule | $\log_b(mn) = \log_b m + \log_b n$ |
| Quotient Rule | $\log_b\left(\frac{m}{n}\right) = \log_b m - \log_b n$ |
| Power Rule | $\log_b m^n = n \log_b m$ |
| Change of Base Rule | $\log_a b = \frac{\log_c b}{\log_c a}$ (OR) $\log_a b \cdot \log_c a = \log_c b$ |
| Equality Rule | $\log_b a = \log_b c \Rightarrow a = c$ |
| Number Raised to Log | $b^{\log_b x} = x$ |
| Other Rules | $\log_b a^m = \frac{m}{n} \log_b a$ $-\log_b a = \log_b \frac{1}{a}$ (OR) $= \log_{\frac{1}{b}} a$ |

Figure 6: Credit: <https://www.cuemath.com/algebra/log-rules/>

3 Fraction Exponents

$$x^{\frac{a}{b}} = \sqrt[b]{x^a} = (\sqrt[b]{x})^a$$