MATH1034OL1 Pre-Calculus Mathematics Notes from Sections 4.9, 3.6, 3.4 (Wednesday)

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1 Factors of Polynomials

If a polynomial has a zero of $x = a$ whose multiplicity is b, $(x - a)^b$ is a factor.

2 Included Angles

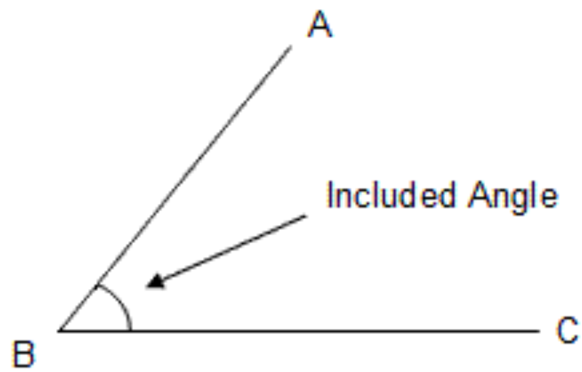

Figure 1: Credit: https://www.mathopenref.com/angleincluded.html

3 Law of Cosines

We use the law of cosines to ”solve” (find all angles and sides) of a triangle when we are given either a) three sides or b) two sides and the included angle.

Consider the triangle

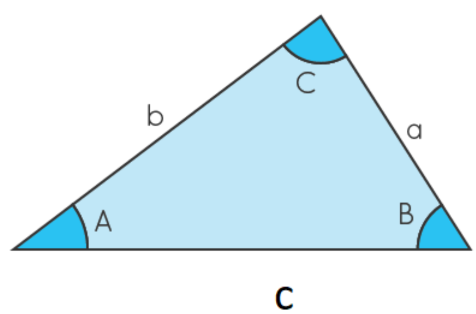

Figure 2: Credit: https://www.cuemath.com/questions/for-triangle-abc-with-sides-a-b-and-c-the-law-of-cosines-states-the-following/

with sides a, b, and c and angles A, B, and C. We define the law

$$a^2 = b^2 + c^2 - 2bc\cos A$$

which can be rewritten by swapping any two of the variables:

$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

These can also be rewritten algebraically to solve for angles A, B, or C.

4 Law of Sines

The law of sines relates the proportions of sides to the sine values of the angles opposite the sides:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

The rule is useful when we are given either a) two angles and one side, or b) two sides and a non-included angle.

5 Complex and Imaginary Numbers

Let z be the complex number $a + bi$. The conjugate of z is $\bar{z} = a - bi$. One useful property to remember is that $z\bar{z}$ is a real number.

6 Asymptotes and x-Intercepts of Rational Functions

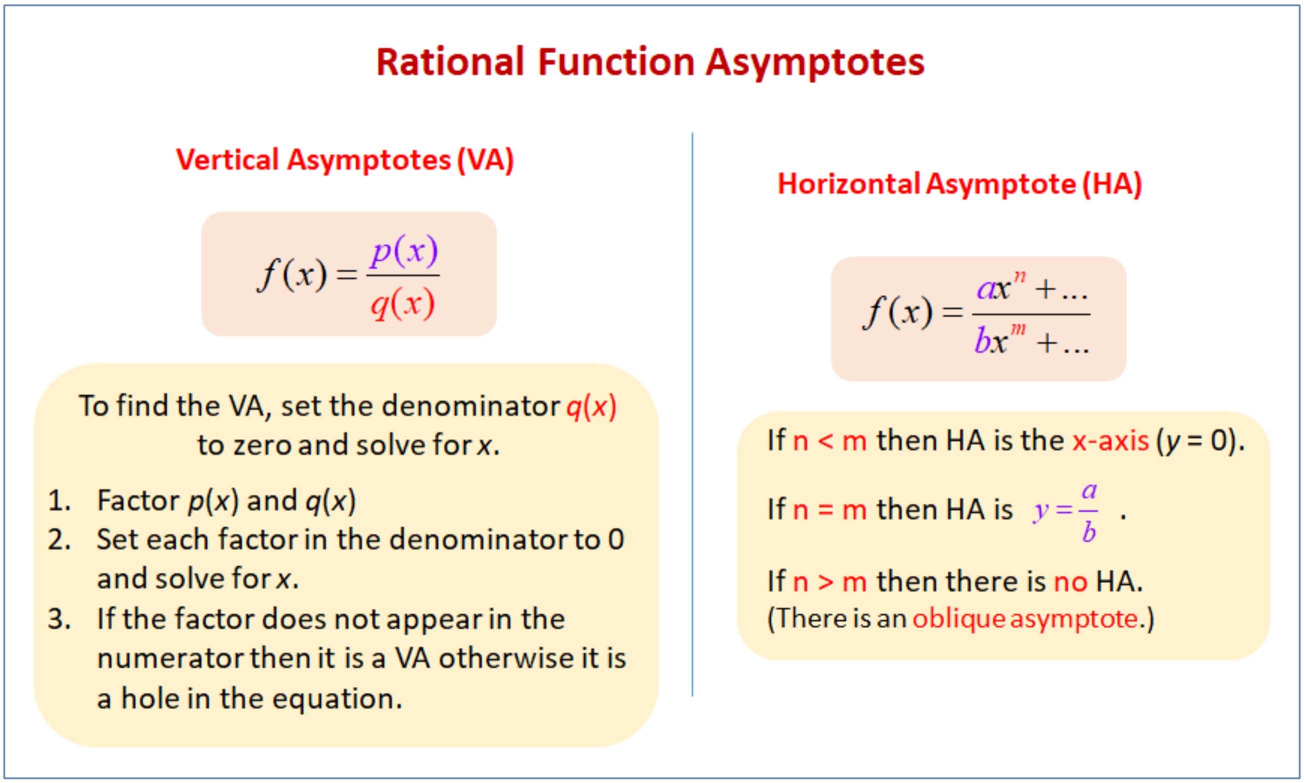

Figure 3: Credit: https://www.onlinemathlearning.com/rational-functions.html

The x-intercepts of a rational function are the values that make the numerator equal to zero.

If the degree of the top polynomial is one greater than the bottom polynomial, there exists an oblique or slant asymptote:

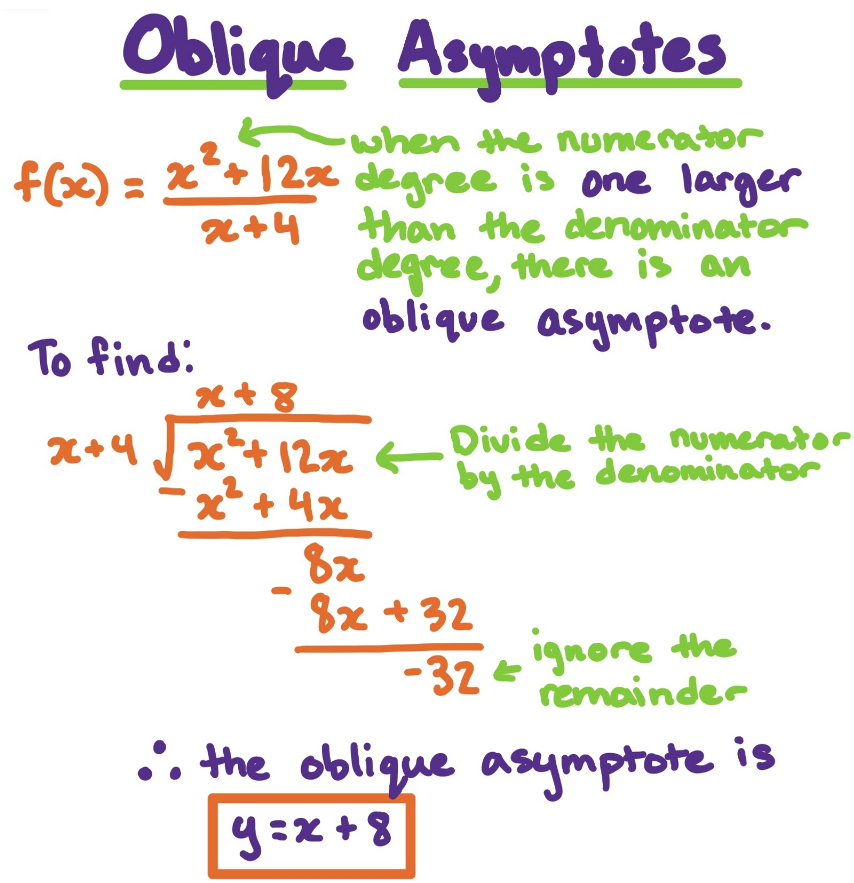

Figure 4: Credit: https://www.expii.com/t/oblique-asymptotes-of-rational-functions-5138

Some final notes about asymptotes:

Horizontal asymptotes and oblique asymptotes may be crossed by the rational function, but vertical asymptotes may not (because they're the values where the function is undefined).

To determine if some asymptote intersects a rational function, set them equal and look for a contradiction. If there isn't a contradiction, the rational function intercepts that asymptote at the value of x in that equality.