18, November 2021

Schohn L. Shannon, Ph.D.

Swanson School of Engineering

106 Benedum Hall

University of Pittsburgh, Pittsburgh, PA 15261

Dear Dr. Shannon:

Team B has completed developing a closed-loop model with control, based upon set point changes on the control variable for the small liquid level tank. The enclosed progress report summarizes the work done thus far and future experimental plans. The attached table shows the distribution of responsibilities for each team member.

Sincerely,

Logan Dailey

Jake Goodwin

Bret Leydig

Eli Wissenbach

Kimberly Xue

RESPONSBILTIES	NAMES
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Small Tank Liquid-Level

Progress Report
18 November 2021
ChE 0501 Team B

Logan Dailey
Jake Goodwin
Bret Leydig
Eli Wissenbach
Kimberly Xue

Contents

Nomenclature

Variable	Definition	Unit
h	Height of the fluid	cm
q_{in}	Inlet flow rate to the tank	LPM
q_2	Outlet flow rate from SV2	LPM
q_3	Outlet flow rate from SV3	LPM
C_{v2}	Valve constant from SV2	$\frac{mL}{min * cm^x}$
C_{v3}	Valve constant from SV3	$\frac{mL}{min * cm^x}$
K_c	Controller gain	$\frac{mA}{cm}$
τ_I	Integral time	s
t	Time	s
θ	Time delay	s
A	Cross sectional area	cm^2
x	Exponential term	No unit
τ_{rt}	Rise time	s
τ_{st}	Settling time	s
$\%_{os}$	Percent overshoot	%

I. Introduction

In the chemical industry, it is crucial to control and model certain aspects of a system to maintain process safety and to meet design specifications. This is done by defining the different process variables and determining how changes in their values affect the operation and outcome of this system. The goal of this experiment is to expand upon experimentation from previous reports in order to develop a feedback control model to account for any deviation in the process control variables. Open loop simulation results from previous reports were analyzed to create an optimized control system. The revised simulation results were compared to experimental results obtained from the lab to validate the model. Once the system is considered from an open-loop perspective, a closed-loop model can be developed to determine a feedback control system. The labeled small-tank liquid level system can be seen in Figure A-1.1.

The purpose of this system is to measure liquid height which is impacted by the volumetric flow rate into the tank, and the volumetric flow rate of liquid out of the tank. For this experiment, the control variable in the tank height (h, cm), the manipulated variable is the flow into the tank (F_{in}, LPM), while the disturbance variable considered is the flow out of the tank via a solenoid valve. (F_3, LPM). To date, disturbance variables have not been considered, as the flow out of the tank was held constant.

For the initial part of this experiment, the liquid height was controlled using proportional control, as well as proportion integral (PI) control. Experimental results for the closed-loop system were recorded and used to determine the effects of set point changes in flow rate at different controller gain (K_c) and integral time (τ_I) settings. Once the results were obtained and analyzed for time delay (Θ), percent overshoot ($\%_{os}$), rise time (τ_{rt}), settling time (τ_{st}), and offset error (Δe_{ss}), a feedback control simulation model was determined using MATLAB and SIMULINK. The model simulation was then compared to experimental results for the same varying controller parameters. Modeling will be discussed in detail in the following section.

II. Modeling

To determine the closed loop model results a previous report that experimented with open loop model was referenced. The open loop model experiment determined the valve constant parameters that were used in this report. The model and values used for this experiment were based on the report from Team Q [2]. Their Simulink model was a close fit to their experimental results except for session five, as shown in Figure A-2.19, where only the linear model was a good fit to the data. For other sessions, the model produced a close fit of the data with variable offset. Due to the accuracy of the simulation model, those parameters were used.

There were also three other reports that were analyzed: Team A, Team I, and Team Y. Team A's models followed the trend of water height with the tank rising or falling with the set point changes, solved for the two valve constants and the power relationship, and followed the assumption that Bernoulli's relationship didn't apply. However, the s-function and the transfer function did not line up with the experimental data, so the report was not used [3].

Team I also solved for two valve constants and the power relationship, however they assumed Bernoulli's relationship could be applied. The models followed the same trend of height increasing and decreasing with step changes. Both the transfer function models from sessions 3 to 5 and the s-function models deviated from the experimental data. Thus, the report was not used [1].

Team Y found two valve constants, two power relationships, and applied Bernoulli's relationship [5]. The model was a close fit for experimental Session 1 results but deviated from other result step changes. The team also did not construct a transfer function; therefore, the report was not used [5].

Team Q's selected parameters were the valve constants, (C_v and C_{v2}), and power relationship (x). The initial ODE Team Q used defined a general mass balance for the system assuming constant liquid density (ρ) and cross-sectional area (A_c):

$$(1) \qquad \rho A_c \frac{dh}{dt} = \rho(F_{in} - F_{out})$$

The flow rate in (F_{in}) and the height (h) were each measured by the system. The flow rate out was calculated by Bernoulli's relation given the assumption of constant density, flow, and pressure. Equation 1 was simplified and solved for $\frac{dh}{dt}$.

$$(2) \qquad \frac{dh}{dt} = \frac{F_{in}}{A_c} - C_v\frac{\sqrt{h}}{A_c} - C_{v2}\frac{\sqrt{h}}{A_c}$$

Equation 2 was used at steady state to solve for the valve constants, C_v and C_{v2}, which refer to solenoid valves 3 and 2 respectively. Once the valve constants were determined, a new fit for $\frac{dh}{dt}$ was determined by solving for the power relationship as seen in Equation 3.

$$(3) \qquad \frac{dh}{dt} = \frac{F_{in}}{A_c} - C_v\frac{h^x}{A_c} - C_{v2}\frac{h^x}{A_c}$$

The final parameterized values from the report were C_{v2} =157.5 ml/min* $cm^{0.293}$
C_{v3} = 930.6 ml/min*$cm^{0.293}$ and X = 0.293. The parameters offered the best fit of their results.

The ODE and parameter values from Team Q were used in a s-function that was implemented with Simulink to model the closed-loop system. In the nonlinear Simulink diagram, the servo valve current (Amp) was added to the PID output to adjust units. Saturation blocks with the flow and current limits of the system were implemented after they were determined from A-2.19. The s-function output was plotted against the set point change and measured height.

III. Experimental Results to Date

All of Day 1 and most of Day 2 data represents proportional control only. The results are represented in Figures A-2.1, A-2.3, A-2.5, and A-2.7 and Tables A-3.1 and A-3.2. When proportional control is implemented, there should not be oscillatory behavior which is confirmed by the results and represented in the figures. A lack of overshoot and settling time is also shown in the proportional control figures. This is due to the lack of oscillatory behavior in proportional only control. Finally, an inverse trend of proportional control and rise time is seen in Tables A-3.1 and A-3.2. As the value of controller gain, K_c (mA/cm) increased from 1 mA/cm to 2 mA/cm the rise time decreased as the controller is more aggressive which equated to a faster response. The opposite is seen when the controller gain was set to 0.5 mA/cm. The rise time increased due to the gain being less aggressive.

Some results of Day 2 and all of Day 3 had PI control implemented. These are represented in Figures A-2.9, A-2.11, A-2.13, A-2.15, and A-2.17 and Tables A-3.2 and A-3.3. PI control introduces oscillatory behavior as integral control (τ_I, s) is included. This created settling time (τ_{st}, s) and overshoot ($\%_{os}$) for the respective step changes. As integral control increased the amount of oscillatory behavior also increased. The settling time which was calculated by using a range of 2%, increased as integral control time increased. This can be seen in Tables A-3.2 and A-3.3 as time between a step-change and settling time increases with more aggressive integral control values. This is due to the more aggressive oscillatory behavior with higher integral control values. Percent overshoot values also increased when higher integral control time was used. This is because of the increased oscillatory behavior and is shown in Tables A-3.2 and A-3.3. Finally, when compared to proportional only control the values of offset error are closer to zero in proportional integral control. This is due to the addition of integral control that continually sums the controller error and acts based on the summed error.

It is important to note that the data for the step-change in the tank height from 6.99 cm to 9 cm on Day 3, shown in Figure A-2.15 and Table A-3.3, displays odd behavior as a large drop in tank height occurs that is not oscillatory. This is believed to have occurred because of a system error and not because of the controller conditions. This conclusion is drawn because the drop in tank height is not of oscillatory behavior and returns to steady state with no additional oscillatory behavior or overshoot.

IV. Model Validation

An s-function and simulation flow was developed to model the experimental results. Simulation graphs are show in appendix A-2 alongside the experimental results. When comparing the figures for each respective step change, the results for the non-linear model does follow the general trend of the experimental results. The simulation increased or decreased with the changes in set point similar to the experimental results. There is significant offset between the experimental data and the Simulink model for each noted step change. The simulation model highly depends on the valve parameters that were take from Team Q’s report. This may be a potential reason why the simulation data did not match this reports experimental data. Changes in the valve parameters may imapact and optimize the Simulink models to fit the experimental results better. While a transfer function was developed, to date it has not been implemented into the Simulink workspace. Therefore, its model does not appear in the graphs in Appendix A-2.

Session 1 and the majority of Session 2 used a proportional only controller. In session 1, the trend of the Simulink model follows the experimental data gathered from that day. However, all of the proportional control models experienced some form of offset between experimental and model results. The amount of offset changed with the increase or decrease of K_c, but the Simulink model was never able to fit the experimental data from the day. As for Day 2, the same issue with session 1 was present, where there was a noticeable offset between the Simulink model and the experimental data gathered from that day. The offset increases with a decrease in K_c.

Day 3, as well as the last step changes from session 2, used a PI controller. Much like the proportional only controller, the Simulink model follows the general trend of increasing and decreasing with changes in set point. The model continously displays significant offset between experimental and simulation results. Although it is not as egregious as the offset for the proportional controller. In Figure A-2.10, the simulation model fit the experimental results the best when compared to the other step changes. In Figures A-2.12 and A-2.18, the Simulink model does not fit the experimental results very closely when the step-change was intially made, but as time went on, both results ended with very similar steady state values with miuscule offset. However, this cannot be said for Figures A-2.14. In Figure A-2.14, the offset between the Simulink model and the experimental results is far greater than the other graphs from previous step-changes. This offset seemed to occur with an increase in K_c the intergal time was also changed between Day 2 to Day 3 from 100 seconds to 75 seconds. As for Figure A-2.15 and A-2.16, the experimental data seemed to experience extreme oscillation, as shown by the sudden dip. The controller gain was 1 mA/cm and the intergral time was set to 125 seconds. A determination of the sudden dip has not been concluded but may be due to some sort of system error. These results will not be considered for validation. Due to this error more results were not used with those specific controller parameters.

Based on the figures in Appendix A-2, the models are not very accurate to the experimental results throughout each day. However, the models using a PI controller seem to fit the experimental data better than the proportional only controller. Optimization of the model will occur in the future. Also, In the future more simulations will be run to determine validation for the implementation of PID control.

V. Recommandations

Considering this report is not finalized, some material needs to be fixed. Overall, the entire model needs tuning and some errors fixed to make it optimized. Specifically, the valve constant parameters that were used and referenced from Team Q's report do not create a perfect model fit when the simulations were run. As shown in each simulation figures, it is evident the model results do not accurately represent the measured results from the system. Manipulation of these and running more simulations proved that the model is sensitive to the valve constant parameters. Changing these parameters could improve the model simulations. One possible solution is to look more closely at Team Q's report and to determine if there are better valve parameters that fit the model simulations represented in this report.

In the future, more data will need to be collected to run model simulations for PI, and PID controls. To date only some data was collected for PID control. One thing not included in this report is the transfer function model. This will be addressed in upcoming reports. Furthermore, future work will entail the implementation of disturbance variables, the collection of those results, and model simulations for those results. Another item that will be explored is to incorporate controller tuning using Simulink to obtain the best values to use for each control system. To this date, manual tuning of the controller has been implemented.

VI. References

[1] A. Laird, S. Oldenburg, A. Griffiths, L. Rattay, and K. Ruslavage, “Team I Small Tank Liquid Level Experiment ,” Oct. 2021.

[2] B. Bailey, K. Carlson, J. Coffelt, B. Pilsbury, and J. Young, “Team Q Small Liquid Level Tank,” Oct. 2021.

[3] E. Akabua, T. Brighton, E. Ganter, and N. Keck, “Team A Small Liquid Level Tank Final Report,” *CHE 501:Team A Small Liquid Level Tank Final Report*, Oct. 2021.

[4] “Systems Engineering I: Dynamics and Modeling LIQUID LEVEL TANK,” May 2008.

[5] W. Chen, M. Eppley, M. Miller, A. Schmidt, and K. Wintruba, “Team Y Small Tank Liquid Level: Open-Loop Model,” Oct. 2021.

Appendices

A-1. Experiment Schematic

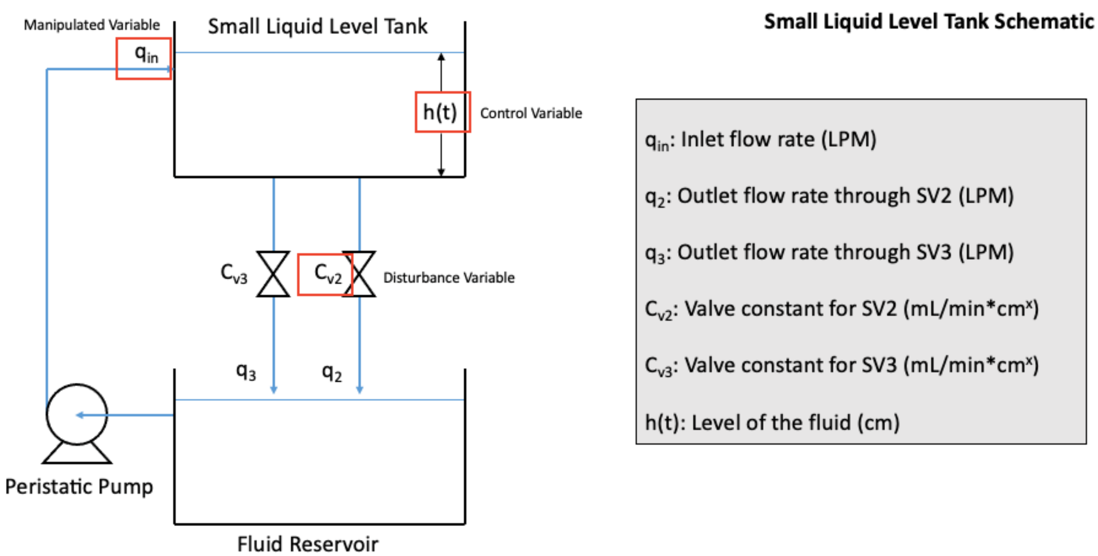


Figure A-1.1 Shows the schematic of the small liquid level tank. The different components and instrumentation of the system are depicted as well as the control, manipulated, and disturbance variables. The different variables as well are their meanings and units are also displayed

A-2. Figures

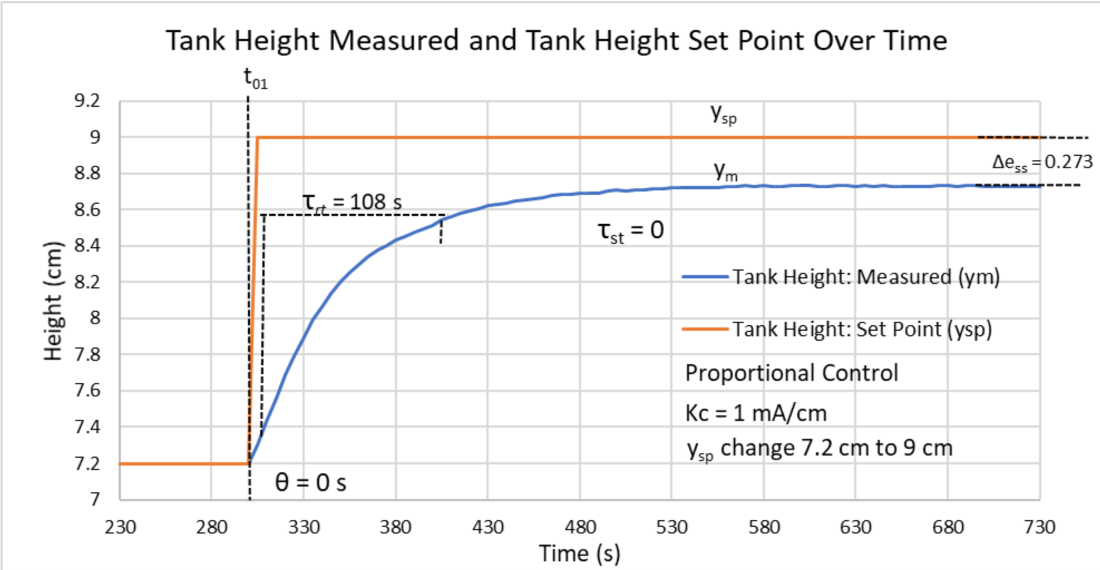


Figure A-2.1 Illustrates measured tank height and the tank height set point over time. The results displayed are from experimental Day 1. This figure shows a step-change in the tank height from 7.2 cm to 9 cm. The controller gain (Kc, mA/cm) was held constant at 1 mA/cm. t_{01} (s) notes the initial time when the step change was initiated. All other parameters such as time delay (ϴ, s), percent overshoot ($\%_{os}$), rise time (τ_{rt}, s), settling time (τ_{st}, s, 2%), and the offset error (Δe_{ss}) are displayed on the graph with their respective values.

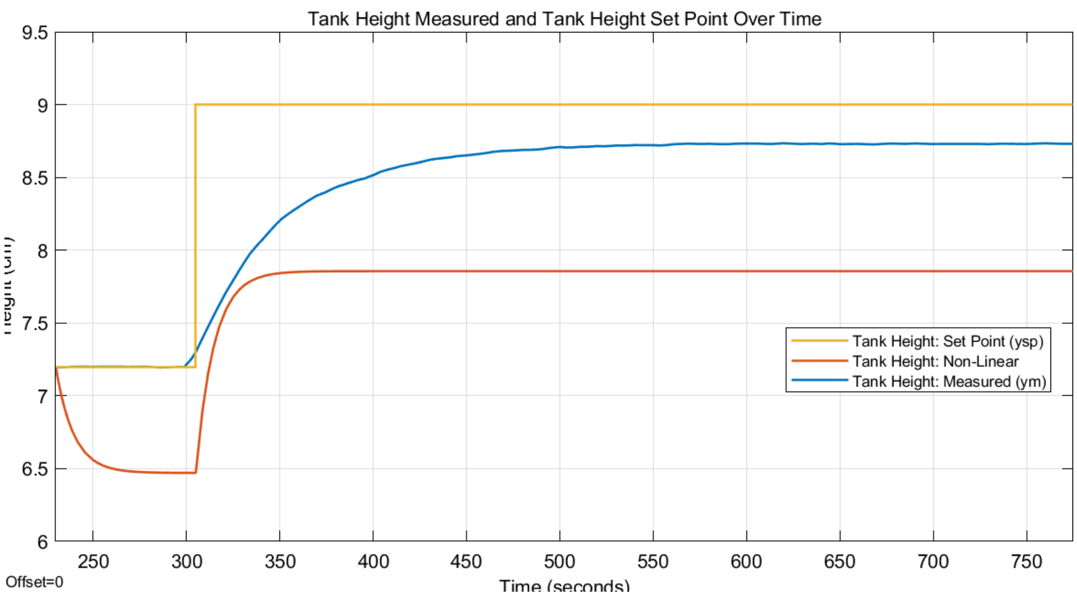


Figure A-2.2 Illustrates measured tank height and the tank height set point over time as well as results from the Simulink model. The results displayed are from experimental Day 1. The tank height set point is noted by the yellow line, the experimental results are noted by the blue line and the Simulink results are noted by the red line. This figure shows a step-change in the tank height from 7.2 cm to 9 cm. The controller gain (Kc, mA/cm) was held constant at 1 mA/cm.

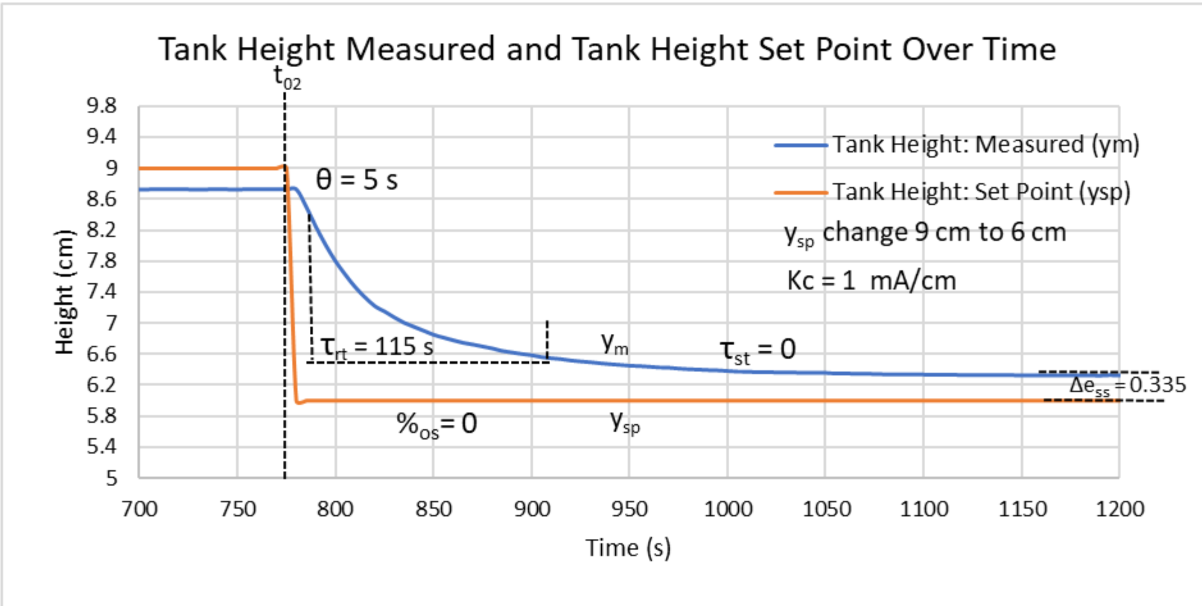


Figure A-2.3 Illustrates measured tank height and the tank height set point over time as well as results from the Simulink model. The results displayed are from experimental Day 1. The tank height set point is noted by the yellow line, the experimental results are noted by the blue line and the Simulink results are noted by the red line. This figure shows a step-change in the tank height from 9 cm to 6 cm. The controller gain (Kc, mA/cm) was held constant at 1 mA/cm.

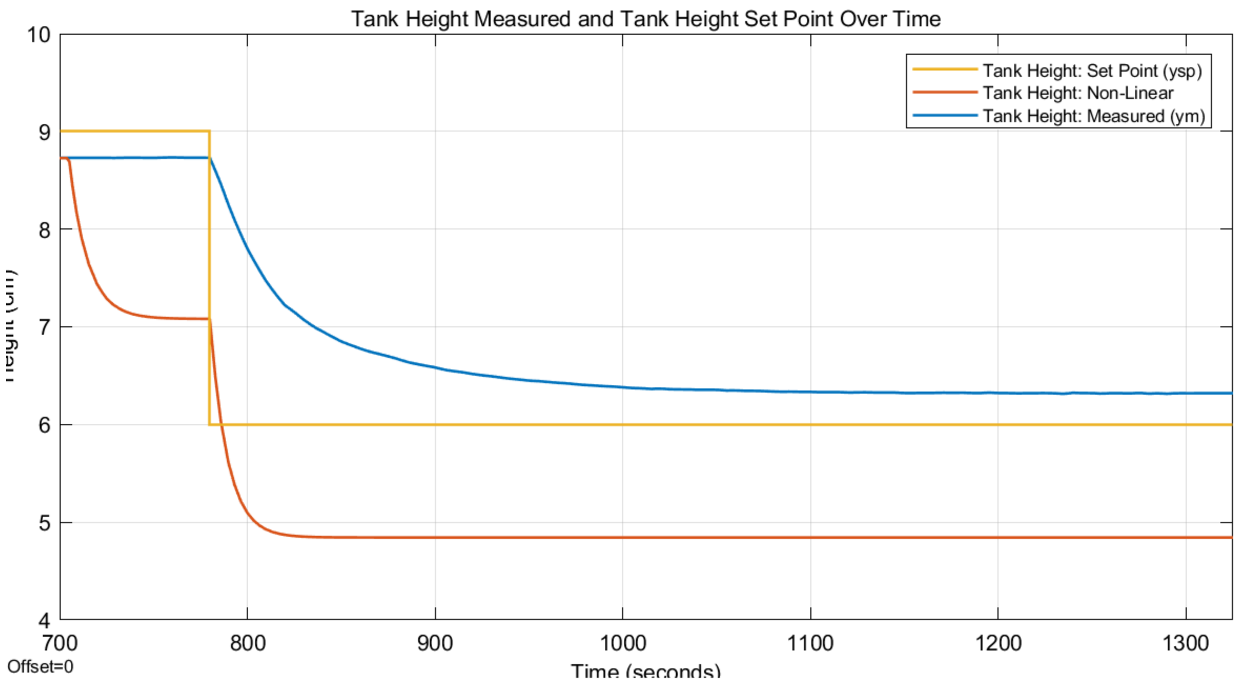


Figure A-2.4 Illustrates measured tank height and the tank height set point over time as well as results from the Simulink model. The results displayed are from experimental Day 1. The tank height set point is noted by the yellow line, the experimental results are noted by the blue line and the Simulink results are noted by the red line. This figure shows a step-change in the tank height from 9 cm to 6 cm. The controller gain (Kc, mA/cm) was held constant at 1 mA/cm.

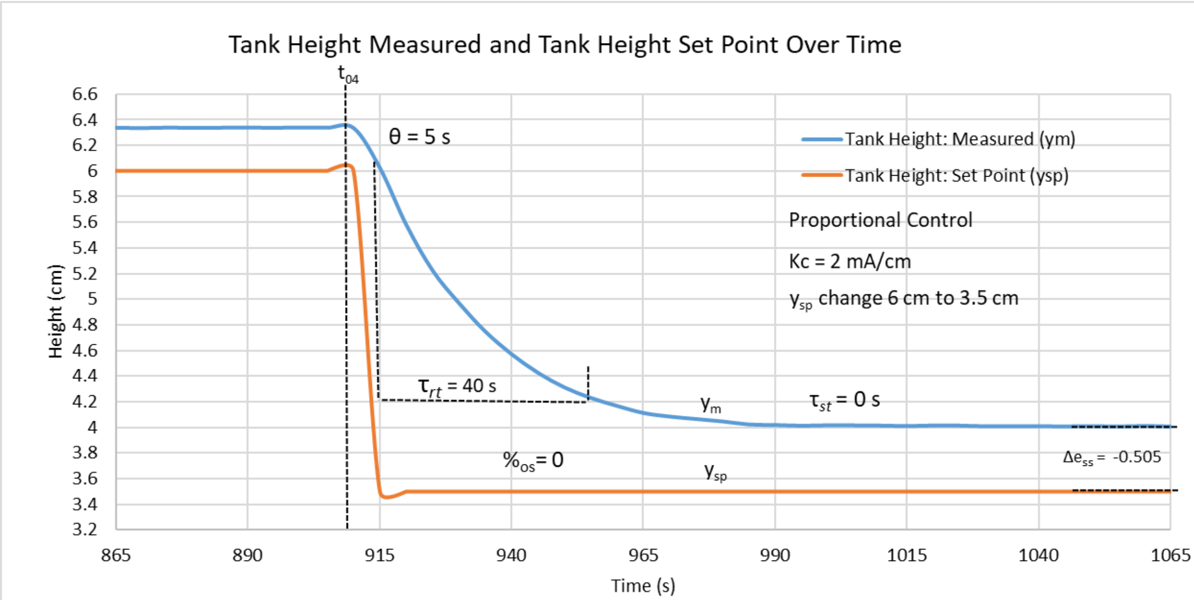


Figure A-2.5 Illustrates measured tank height and the tank height set point over time. The results displayed are from experimental Day 2. This figure shows a step-change in the tank height from 6 cm to 3.5 cm. The controller gain (Kc, mA/cm) was held constant at 2 mA/cm. t_{04} (s) notes the initial time when the step change was initiated. All other parameters such as time delay (Θ, s), percent overshoot ($\%_{os}$), rise time (τ_{rt}, s), settling time (τ_{st}, s, 2%), and the offset error (Δe_{ss}) are displayed on the graph with their respective values.

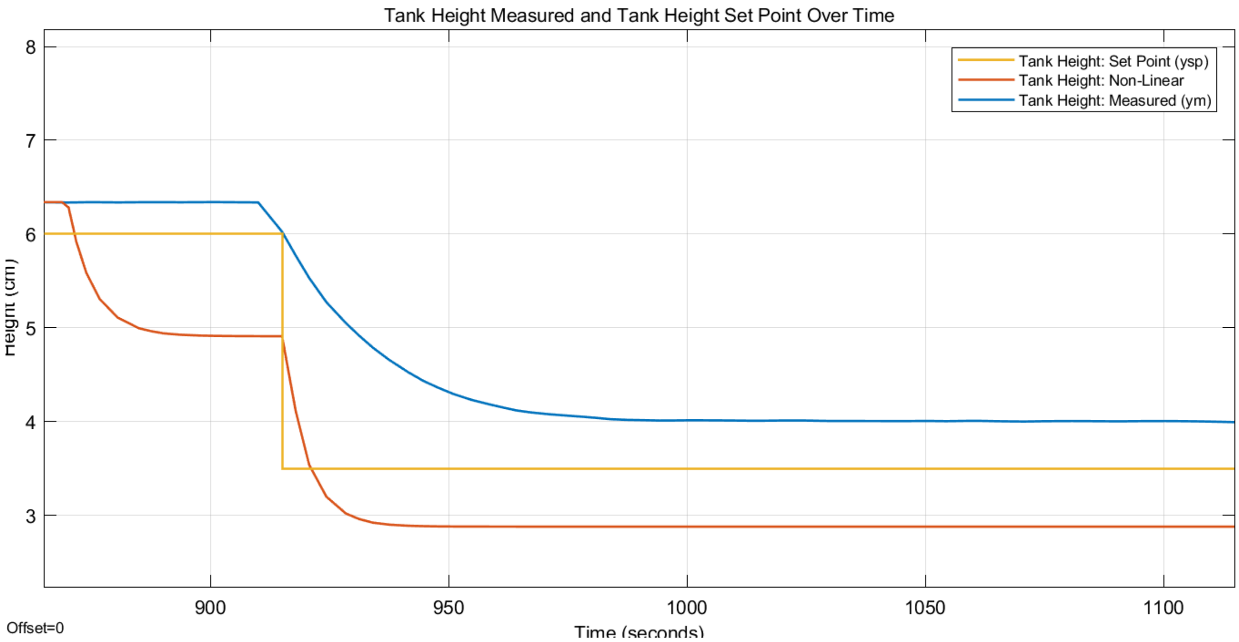


Figure A-2.6 Illustrates measured tank height and the tank height set point over time as well as results from the Simulink model. The results displayed are from experimental Day 2. The tank height set point is noted by the yellow line, the experimental results are noted by the blue line and the Simulink results are noted by the red line. This figure shows a step-change in the tank height from 6 cm to 3.5 cm. The controller gain (Kc, mA/cm) was held constant at 2 mA/cm.

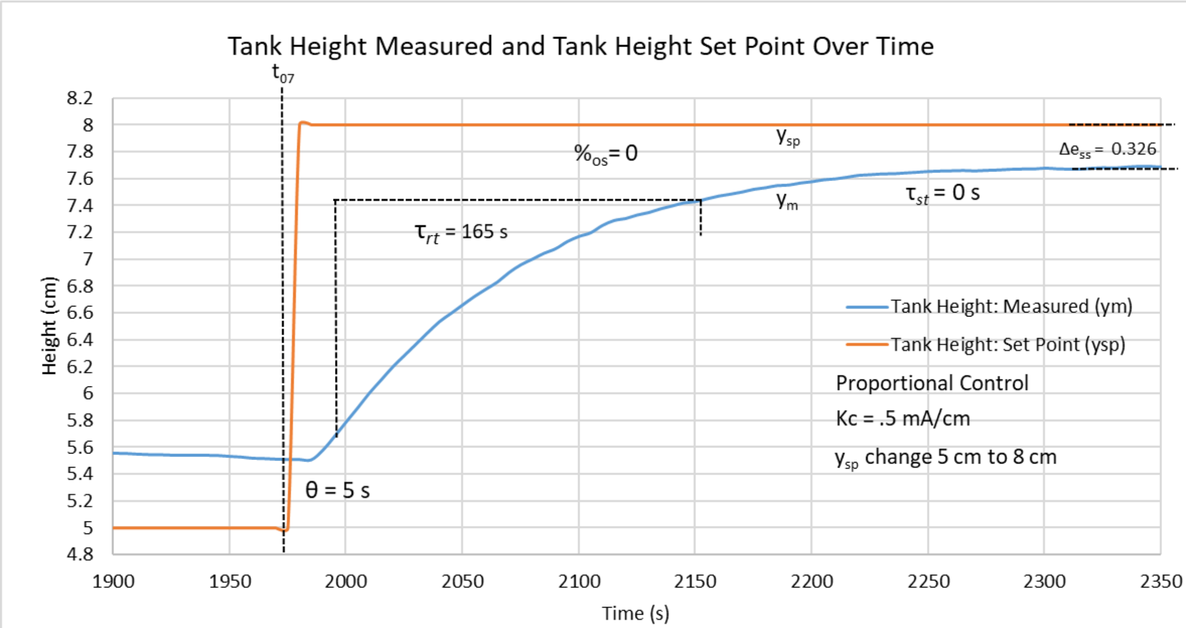


Figure A-2.7 Illustrates measured tank height and the tank height set point over time. The results displayed are from experimental Day 2. This figure shows a step-change in the tank height from 5 cm to 8 cm. The controller gain (Kc, mA/cm) was held constant at 0.5 mA/cm. t_{07} (s) notes the initial time when the step change was initiated. All other parameters such as time delay (ϴ, s), percent overshoot ($\%_{os}$), rise time (τ_{rt}, s), settling time (τ_{st}, s, 2%), and the offset error (Δe_{ss}) are displayed on the graph with their respective values.

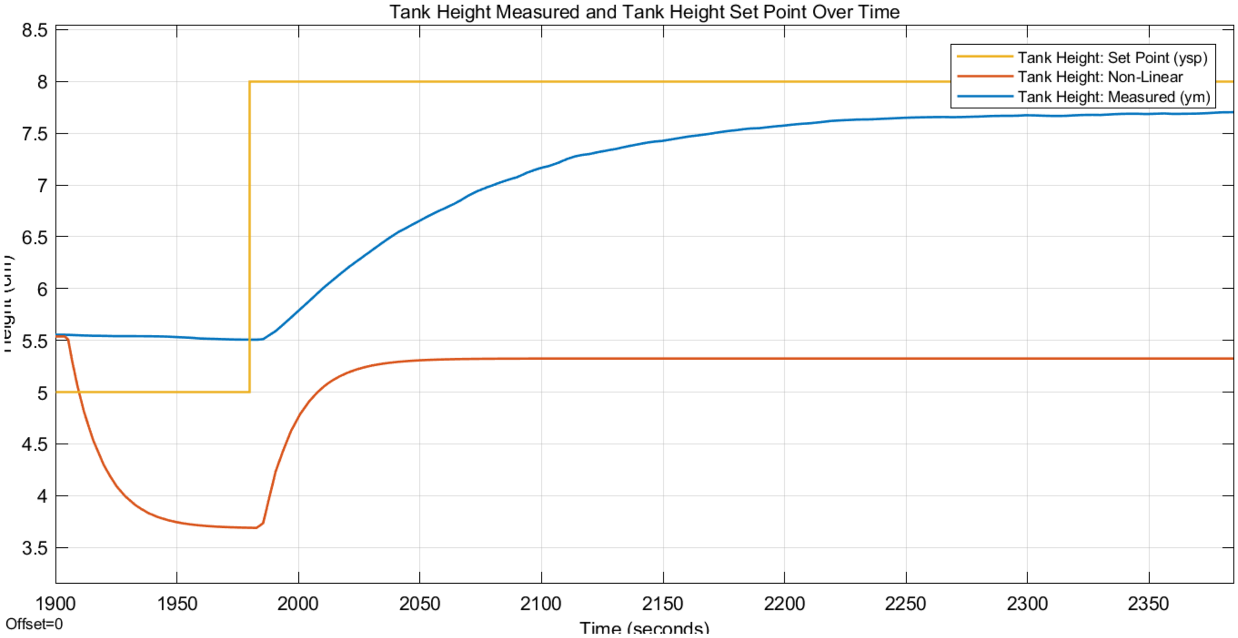


Figure A-2.8 Illustrates measured tank height and the tank height set point over time as well as results from the Simulink model. The results displayed are from experimental Day 2. The tank height set point is noted by the yellow line, the experimental results are noted by the blue line and the Simulink results are noted by the red line. This figure shows a step-change in the tank height from 5 cm to 8 cm. The controller gain (Kc, mA/cm) was held constant at .5 mA/cm.

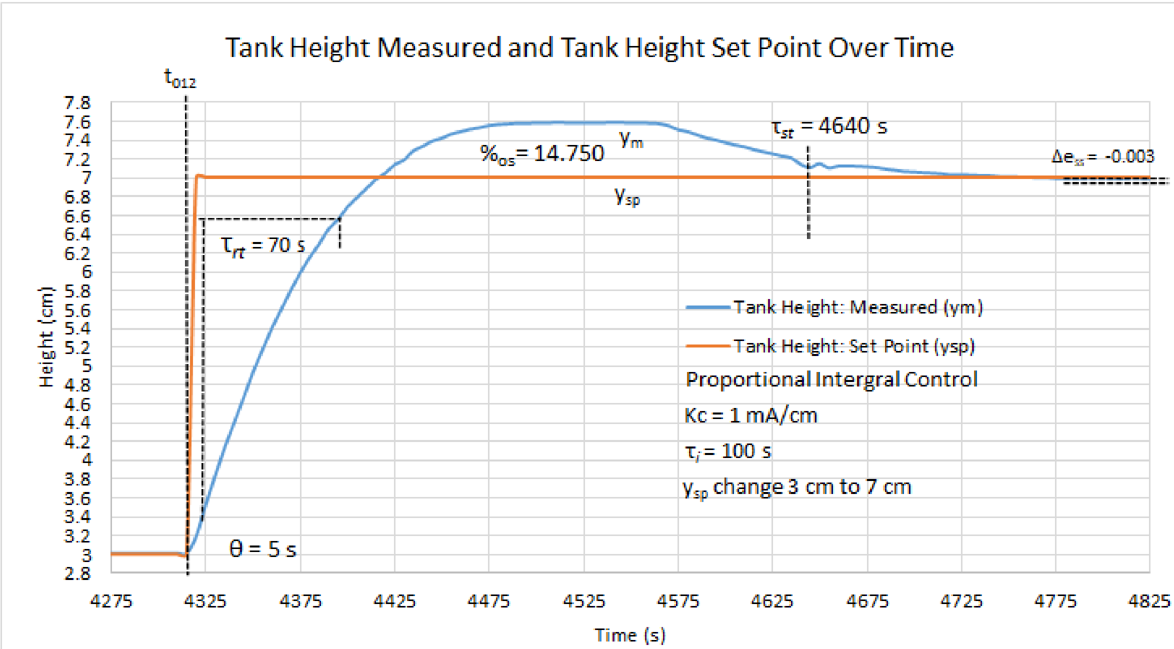


Figure A-2.9 Illustrates measured tank height and the tank height set point over time. The results displayed are from experimental Day 2. This figure shows a step-change in the tank height from 3 cm to 7 cm. The controller gain (Kc, mA/cm) was held constant at 1 mA/cm. Integral time (τ_i,s) was held constant at 100 seconds. t_{012} (s) notes the initial time when the step change was initiated. All other parameters such as time delay (Θ, s), percent overshoot ($\%_{os}$), rise time (τ_{rt}, s), settling time (τ_{st}, s, 2%), and the offset error (Δe_{ss}) are displayed on the graph with their respective values.

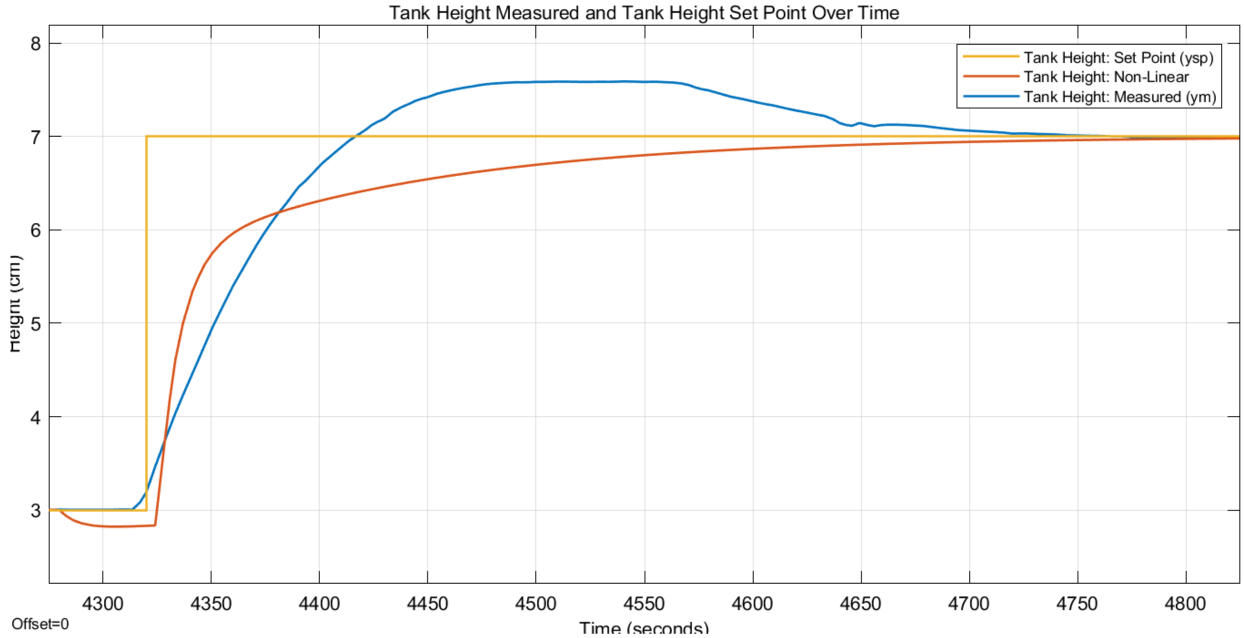


Figure A-2.10 Illustrates measured tank height and the tank height set point over time as well as results from the Simulink model. The results displayed are from experimental Day 2. The tank height set point is noted by the yellow line, the experimental results are noted by the blue line and the Simulink results are noted by the red line. This figure shows a step-change in the tank height from 3 cm to 7 cm. The controller gain (Kc, mA/cm) was held constant at 1 mA/cm. Integral time (τ_i,s) was held constant at 100 seconds

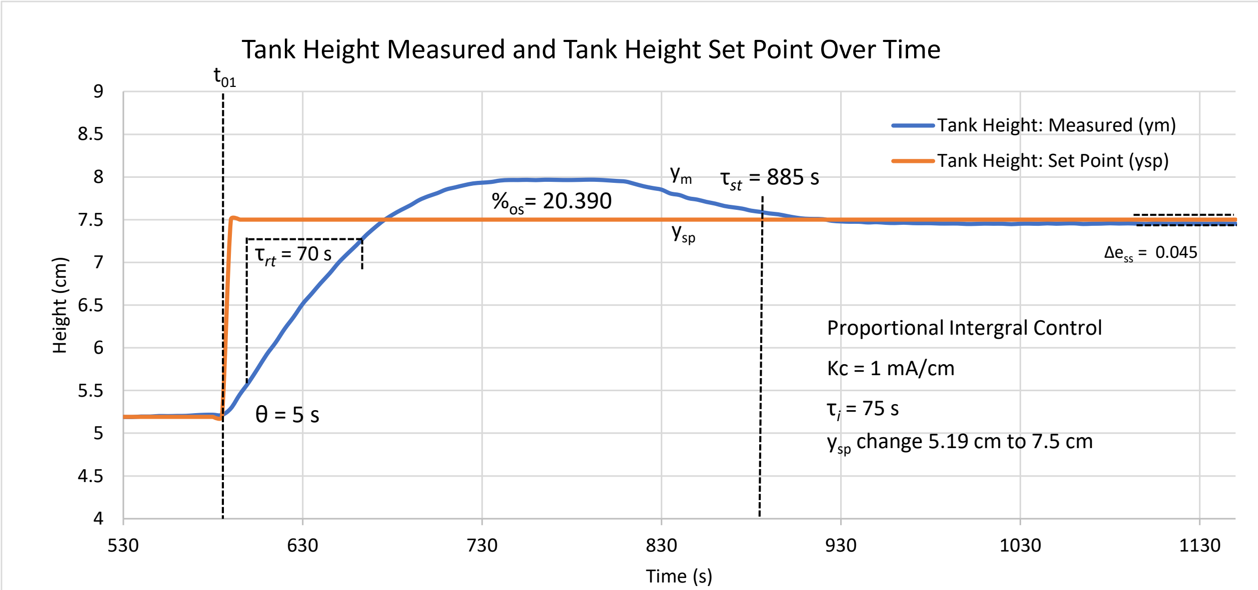


Figure A-2.11 Illustrates measured tank height and the tank height set point over time. The results displayed are from experimental Day 3. This figure shows a step-change in the tank height from 5.19 cm to 7.5 cm. The controller gain (Kc, mA/cm) was held constant at 1 mA/cm. Integral time (τ_i,s) was held constant at 75 seconds. t_{01} (s) notes the initial time when the step change was initiated. All other parameters such as time delay (Θ, s), percent overshoot ($\%_{os}$), rise time (τ_{rt}, s), settling time (τ_{st}, s, 2%), and the offset error (Δe_{ss}) are displayed on the graph with their respective values.

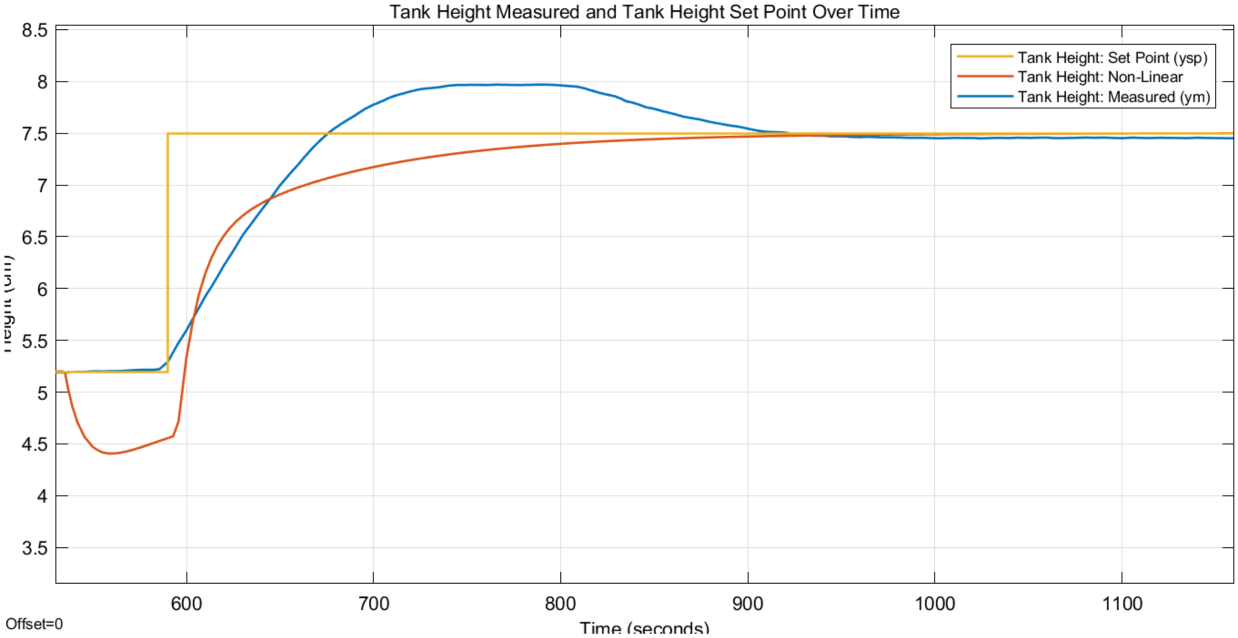


Figure A-2.12 Illustrates measured tank height and the tank height set point over time as well as results from the Simulink model. The results displayed are from experimental Day 3. The tank height set point is noted by the yellow line, the experimental results are noted by the blue line and the Simulink results are noted by the red line. This figure shows a step-change in the tank height from 5.19 cm to 7.5 cm. The controller gain (Kc, mA/cm) was held constant at 1 mA/cm. Integral time (τ_i,s) was held constant at 75 seconds.

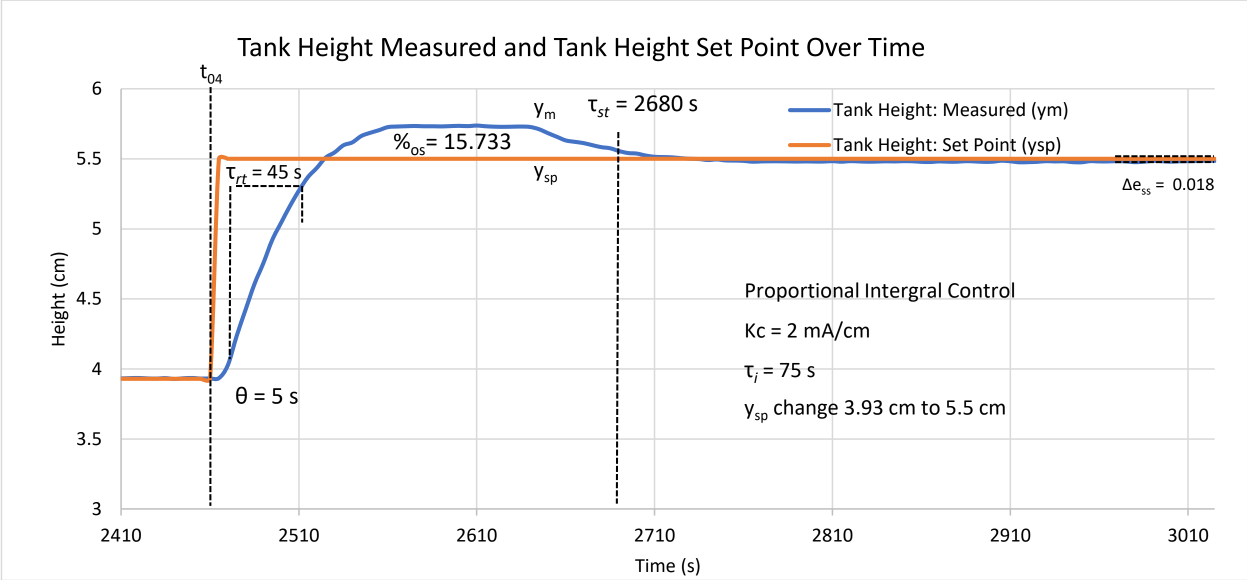


Figure A-2.13 Illustrates measured tank height and the tank height set point over time. The results displayed are from experimental Day 3. This figure shows a step-change in the tank height from 3.93 cm to 5.5 cm. The controller gain (Kc, mA/cm) was held constant at 2 mA/cm. Integral time (τ_i,s) was held constant at 75 seconds. t_{04} (s) notes the initial time when the step change was initiated. All other parameters such as time delay (Θ, s), percent overshoot ($\%_{os}$), rise time (τ_{rt}, s), settling time (τ_{st}, s, 2%), and the offset error (Δe_{ss}) are displayed on the graph with their respective values.

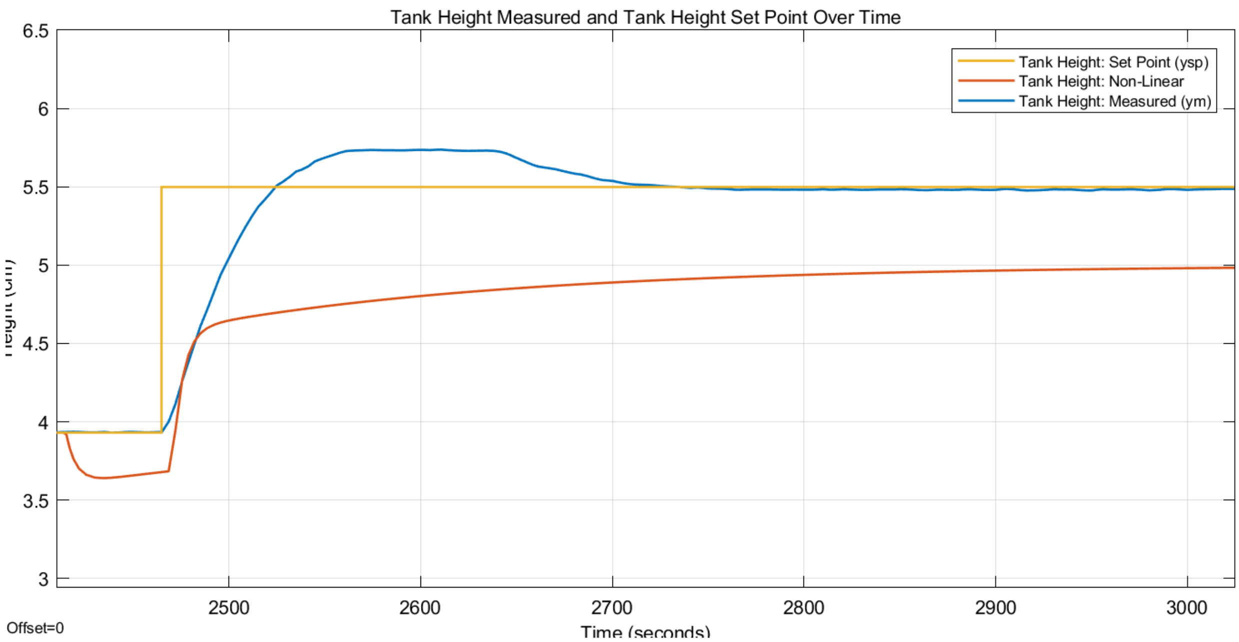


Figure A-2.14 Illustrates measured tank height and the tank height set point over time as well as results from the Simulink model. The results displayed are from experimental Day 3. The tank height set point is noted by the yellow line, the experimental results are noted by the blue line and the Simulink results are noted by the red line. This figure shows a step-change in the tank height from 3.93 cm to 5.5 cm. The controller gain (Kc, mA/cm) was held constant at 2 mA/cm. Integral time (τ_i,s) was held constant at 75 seconds.

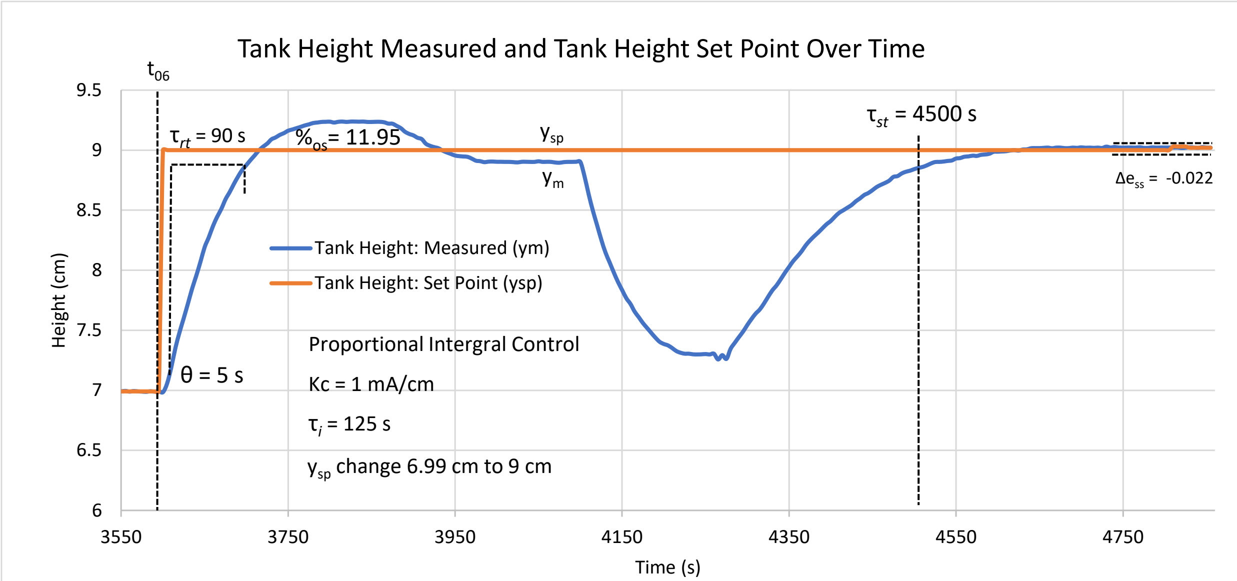


Figure A-2.15 Illustrates measured tank height and the tank height set point over time. The results displayed are from experimental Day 3. This figure shows a step-change in the tank height from 6.99 cm to 9 cm. The controller gain (Kc, mA/cm) was held constant at 1 mA/cm. Integral time (τ_i,s) was held constant at 125 seconds. t_{04} (s) notes the initial time when the step change was initiated. All other parameters such as time delay (Θ, s), percent overshoot ($\%_{os}$), rise time (τ_{rt}, s), settling time (τ_{st}, s, 2%), and the offset error (Δe_{ss}) are displayed on the graph with their respective values.

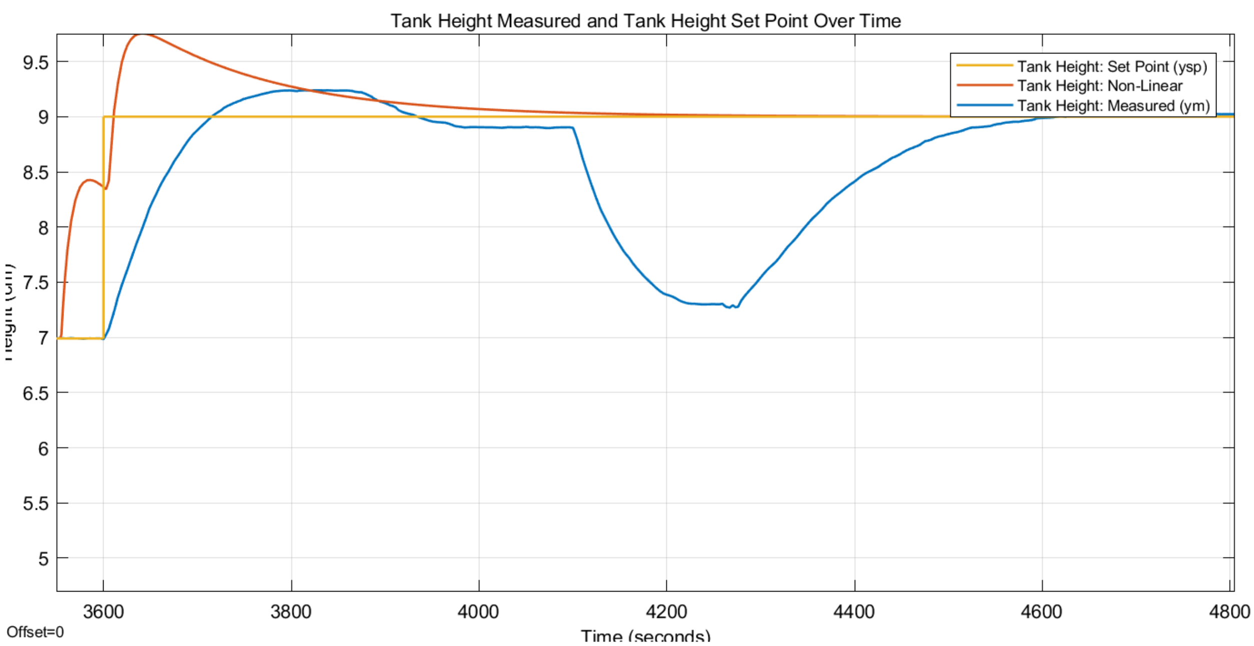


Figure A-2.16 Illustrates measured tank height and the tank height set point over time as well as results from the Simulink model. The results displayed are from experimental Day 3. The tank height set point is noted by the yellow line, the experimental results are noted by the blue line and the Simulink results are noted by the red line. This figure shows a step-change in the tank height from 6.99 cm to 9 cm. The controller gain (Kc, mA/cm) was held constant at 1 mA/cm. Integral time (τ_i,s) was held constant at 125 seconds.

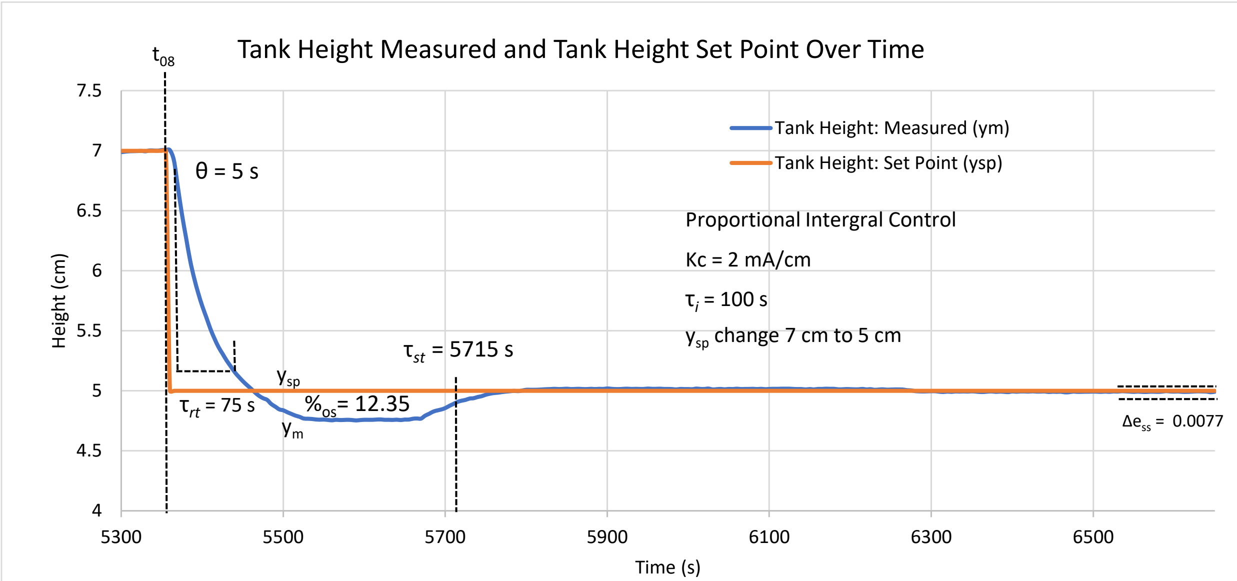


Figure A-2.17 Illustrates measured tank height and the tank height set point over time. The results displayed are from experimental Day 3. This figure shows a step-change in the tank height from 7 cm to 5 cm. The controller gain (Kc, mA/cm) was held constant at 2 mA/cm. Integral time (τ_i,s) was held constant at 100 seconds. t_{04} (s) notes the initial time when the step change was initiated. All other parameters such as time delay (Θ, s), percent overshoot ($\%_{os}$), rise time (τ_{rt}, s), settling time (τ_{st}, s, 2%), and the offset error (Δe_{ss}) are displayed on the graph with their respective values.

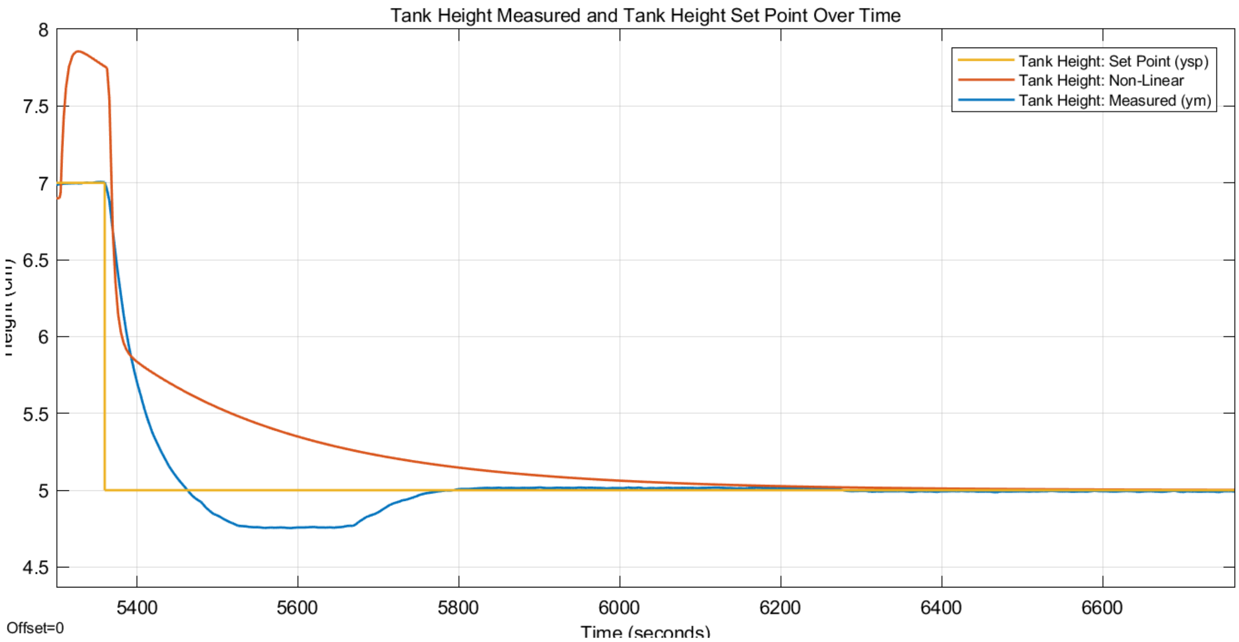


Figure A-2.18 Illustrates measured tank height and the tank height set point over time as well as results from the Simulink model. The results displayed are from experimental Day 3. The tank height set point is noted by the yellow line, the experimental results are noted by the blue line and the Simulink results are noted by the red line. This figure shows a step-change in the tank height from 7 cm to 5 cm. The controller gain (Kc, mA/cm) was held constant at 2 mA/cm. Integral time (τ_i,s) was held constant at 100 seconds.

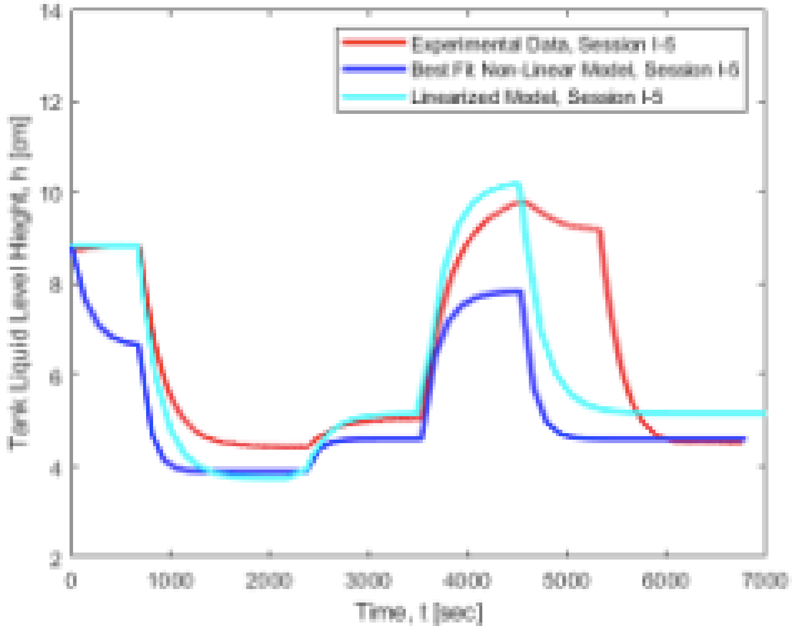


Figure A-2.19 The model from Team Q's fifth session. This figure displays the data that was used to calculate the valve parameters.

A-3. Tables

Table A-3.1 Displays calculated results from experimental day one. The table shows the full results for all day one step changes as notated by t_{0n}. Figures A-2.1 and A-2.3 correspond and reflect the values in this table.

Notation	Time (s)	Set Point (cm)	Kc (mA/cm)	θ (s)	ym_{ss} (cm)	Δe_{ss}	τ_{rt} (s)	%os	τ_{st} (s, 2%)
t_{01}	305	9	1	5	8.727	0.273	108	0	0
t_{02}	780	6	1	5	6.335	0.335	115	0	0
t_{03}	1320	3.5	1	5	4.397	0.897	80	0	0
t_{04}	1765	6	1	5	6.278	0.278	80	0	0
t_{05}	2525	9	1	5	8.752	0.248	105	0	0
t_{06}	3175	11	1	5	10.452	0.548	100	0	0

Table A-3.2 Displays calculated results from experimental day two. The table shows the full results for all day three step changes as notated by t_{0n}. Figures A-2.5, A-2.7, and A-2.9 correspond and reflect the values in this table. Note for experimental day two step changes 10 through 13 had proportional integral control implemented.

Notation	Time (s)	Set Point (cm)	Kc (mA/cm)	τ_i (s)	θ (s)	ym_{ss} (cm)	Δe_{ss}	τ_{rt} (s)	%os	τ_{st} (s, 2%)
t_{01}	145	5	2	-	5	5.389	-0.389	45	0	0
t_{02}	380	7.5	2	-	5	7.711	-0.211	60	0	0
t_{03}	665	6	2	-	5	6.340	-0.340	35	0	0
t_{04}	915	3.5	2	-	5	4.005	-0.505	40	0	0
t_{05}	1120	7	2	-	5	7.265	-0.265	60	0	0
t_{06}	1690	5	0.5	-	5	5.537	-0.537	115	0	0
t_{07}	1980	8	0.5	-	5	7.674	0.326	165	0	0
t_{08}	2390	9.5	0.5	-	5	8.842	0.658	185	0	0
t_{09}	2860	7	0.5	-	5	7.070	-0.069	140	0	0
t_{010}	3390	5	1	100	5	4.990	0.010	50	10.050	3630
t_{011}	3785	3	1	100	5	3.002	-0.002	45	12.150	4065
t_{012}	4320	7	1	100	5	7.003	-0.003	70	14.750	4640
t_{013}	4905	10	1	100	5	9.971	0.029	70	12.933	5250

Table A-3.3 Displays calculated results from experimental day three. The table shows the full results for all day three step changes as notated by t_{0n}. Figures A-2.11, A-2.13, A-2.15, and A-2.17 correspond and reflect the values in this table. Note for experimental day two step changes 10 through 13 had proportional integral control implemented.

Notation	Time (s)	Set Point (cm)	Kc (mA/cm)	τ_i (s)	θ (s)	ym_{ss} (cm)	Δe_{ss}	τ_{rt} (s)	%os	τ_{st} (s) 2%
t_{01}	590	7.5	1	75	5	7.455	0.045	70	20.390	885
t_{02}	1155	6	1	75	5	6.040	-0.040	70	17.066	1475
t_{03}	1730	4	1	75	5	4.049	-0.049	65	10.1	2460
t_{04}	2465	5.5	2	75	5	5.482	0.018	45	15.733	2680
t_{05}	3030	7	2	75	5	6.990	0.010	40	17.066	3210
t_{06}	3600	9	1	125	5	9.022	-0.022	90	11.95	4500
t_{07}	4860	7	2	100	5	6.994	0.006	60	16.2	5130
t_{08}	5360	5	2	100	5	4.992	0.008	75	12.35	5715

A-4. Code

```
function [sys,x0,str,ts] = S_function1(t,x,u,flag)

%SFUNTMPL General M-file S-function template
%   With M-file S-functions, you can define you own ordinary differential
%   equations (ODEs), discrete system equations, and/or just about
%   any type of algorithm to be used within a Simulink block diagram.

%   Copyright (c) 1990-97 by The MathWorks, Inc.
%   $Revision: 1.9 $
%
switch flag

  %%%%%%%%%%%%%%%%%%
  % Initialization %
  %%%%%%%%%%%%%%%%%%
  case 0
    [sys,x0,str,ts]=mdlInitializeSizes;

  %%%%%%%%%%%%%%%
  % Derivatives %
  %%%%%%%%%%%%%%%
  case 1
    sys=mdlDerivatives(t,x,u);

  %%%%%%%%%%
  % Update %
  %%%%%%%%%%
  case 2
    sys=mdlUpdate(t,x,u);

  %%%%%%%%%%%
  % Outputs %
  %%%%%%%%%%%
  case 3
    sys=mdlOutputs(t,x,u);

  %%%%%%%%%%%%%%%%%%%%%%%
  % GetTimeOfNextVarHit %
  %%%%%%%%%%%%%%%%%%%%%%%
  case 4
    sys=mdlGetTimeOfNextVarHit(t,x,u);

  %%%%%%%%%%%%%
  % Terminate %
  %%%%%%%%%%%%%
  case 9
    sys=mdlTerminate(t,x,u);

  %%%%%%%%%%%%%%%%%%%%
  % Unexpected flags %
  %%%%%%%%%%%%%%%%%%%%
  otherwise
```

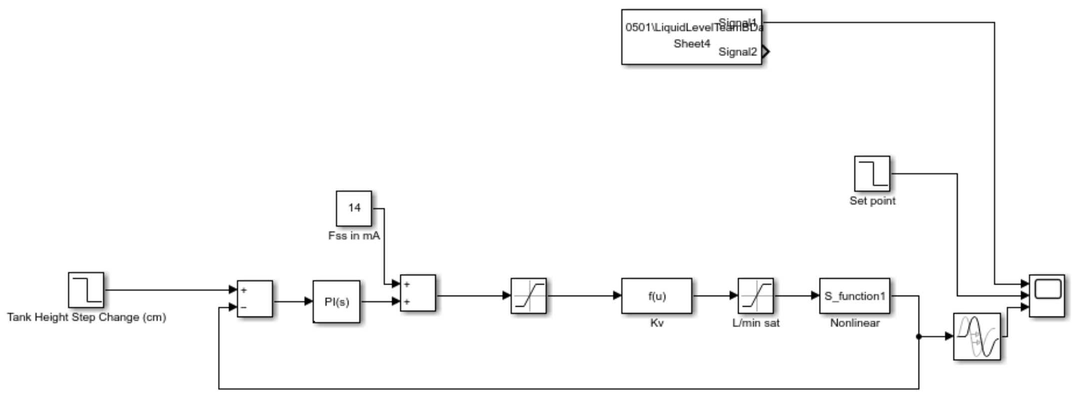
```
    error(['Unhandled flag = ',num2str(flag)]);

end

%
%=============================================================================
=
% mdlInitializeSizes
% Return the sizes, initial conditions, and sample times for the S-function.
%=============================================================================
=
%
function [sys,x0,str,ts]=mdlInitializeSizes

sizes = simsizes;

% ChE 0500
sizes.NumContStates  = 1; % number of ordinary differential
                          % equations in the model
sizes.NumDiscStates  = 0; % number of discrete difference equations
                          % in the model [typically 0 for ChE 0500/0501]
sizes.NumOutputs     = 1; % number of outputs FROM THE S-FUNCTION.
                          % this is equal to the number of
                          % measured variables plus any other
                          % outputs you want to yield.
sizes.NumInputs      = 1; % number of inputs TO THE S-FUNCTION
                          % this is equal to the number of
                          % manipulated variables plus the number
                          % of disturbance variables, plus the
                          % number of (constant) parameter values
                          % you are passing to the model
sizes.DirFeedthrough = 0; % value 0, if u(i) does NOT appear in the
                          % output block (flag = 3); value 1 if
                          % u(i) (for any i) IS used in the output block
sizes.NumSampleTimes = 1; % number of sample times [typically 1 for 500/501]

sys = simsizes(sizes);

%
% ChE 0500:  initial values for the ordinary differential equations
%
x0  = [6.9];

%
% str is always an empty matrix
%
str = [];

%
% initialize the array of sample times
%
ts  = [0 0];

% end mdlInitializeSizes
```

```
%
%==============================================================================
=
% mdlDerivatives
% Return the derivatives for the continuous states.
%==============================================================================
=
%
function sys=mdlDerivatives(t,x,u)

% input the nonlinear model, where w is the state and v is the input
  Cv2= .1575; %L/(min*cm^.293)
  Cv3= .9306; %L/(min*cm^.293)
  X= 0.293;
  Ac= 199.6; %cm^2
  dx(1) = (1/Ac*(u(1)-Cv2*x(1).^X-Cv3*x(1).^X))*60;

  sys=[dx];

% end mdlDerivatives

%
%==============================================================================
=
% mdlUpdate
% Handle discrete state updates, sample time hits, and major time step
% requirements.
%==============================================================================
=
%
function sys=mdlUpdate(t,x,u)

sys = [];

% end mdlUpdate

%
%==============================================================================
=
% mdlOutputs
% Return the block outputs.
%==============================================================================
=
%
function sys=mdlOutputs(t,x,u)

% ChE 0500:  place outputs here - x means all states (ODEs)
sys = [x];

% end mdlOutputs

%
%==============================================================================
=
% mdlGetTimeOfNextVarHit
```

```
% Return the time of the next hit for this block.  Note that the result is
% absolute time.  Note that this function is only used when you specify a
% variable discrete-time sample time [-2 0] in the sample time array in
% mdlInitializeSizes.
%============================================================================
=
%
function sys=mdlGetTimeOfNextVarHit(t,x,u)

sampleTime = 1;    %  Example, set the next hit to be one second later.
sys = t + sampleTime;

% end mdlGetTimeOfNextVarHit

%
%============================================================================
=
% mdlTerminate
% Perform any end of simulation tasks.
%============================================================================
=
%
function sys=mdlTerminate(t,x,u)

sys = [];

% end mdlTerminate




]
```

A-5. Simulation Flow Diagram

Figure A-5.1 Displays the simulation flow diagram used to obtain the model results that are represented in A-2. The simulation diagram was constructed using Simulink.



A-6. Experimental Data

Additional Experimental data can be provided upon request.

Run 1											
Time (s)	Servo Valve (Amps)	Height (cm)	Flow Rate (L/min)	Proportional	Integral	Derivative	Set-point	Kc	Tau I	Tau D	
0	0.011	7.167	1.619	0	0	0	5	0	1	0	5
5	0.011	7.167	1.619	0	0	0	5	0	1	0	5
10	0.011	7.173	1.619	0	0	0	5	0	1	0	5
15	0.011	7.172	1.619	0	0	0	5	0	1	0	5
20	0.011	7.176	1.619	0	0	0	5	0	1	0	5
25	0.011	7.171	1.619	0	0	0	5	0	1	0	5
30	0.011	7.175	1.619	0	0	0	5	0	1	0	5
35	0.011	7.178	1.619	0	0	0	5	0	1	0	5
40	0.011	7.18	1.619	0	0	0	5	0	1	0	5
45	0.011	7.182	1.619	0	0	0	5	0	1	0	5
50	0.011	7.179	1.619	0	0	0	5	0	1	0	5
55	0.011	7.179	1.619	0	0	0	5	0	1	0	5
60	0.011	7.182	1.619	0	0	0	5	0	1	0	5
65	0.011	7.178	1.619	0	0	0	5	0	1	0	5
70	0.011	7.178	1.619	0	0	0	5	0	1	0	5
75	0.011	7.18	1.619	0	0	0	5	0	1	0	5
80	0.011	7.179	1.619	0	0	0	5	0	1	0	5
85	0.011	7.18	1.619	0	0	0	5	0	1	0	5
90	0.011	7.181	1.619	0	0	0	7.2	0	1	0	7.2
95	0.011	7.18	1.619	0	0	0	7.2	0	1	0	7.2
100	0.011	7.182	1.619	0	0	0	7.2	0	1	0	7.2
105	0.011	7.179	1.619	0	0	0	7.2	0	1	0	7.2
110	0.011	7.175	1.627	1	0	0	7.2	1	1	0	7.2
115	0.011	7.18	1.626	1	0	0	7.2	1	1	0	7.2
120	0.011	7.178	1.626	1	0	0	7.2	1	1	0	7.2
125	0.011	7.176	1.627	1	0	0	7.2	1	1	0	7.2
130	0.011	7.177	1.627	1	0	0	7.2	1	1	0	7.2
135	0.011	7.173	1.628	1	0	0	7.2	1	1	0	7.2
140	0.011	7.176	1.627	1	0	0	7.2	1	1	0	7.2
145	0.011	7.171	1.629	1	0	0	7.2	1	1	0	7.2
150	0.011	7.184	1.624	1	0	0	7.2	1	1	0	7.2
155	0.011	7.195	1.621	1	0	0	7.2	1	1	0	7.2
160	0.011	7.199	1.619	1	0	0	7.2	1	1	0	7.2
165	0.011	7.2	1.619	1	0	0	7.2	1	1	0	7.2
170	0.011	7.202	1.618	1	0	0	7.2	1	1	0	7.2
175	0.01	7.2	1.619	1	0	0	7.2	1	1	0	7.2
180	0.01	7.202	1.618	1	0	0	7.2	1	1	0	7.2
185	0.011	7.201	1.619	1	0	0	7.2	1	1	0	7.2
190	0.01	7.197	1.62	1	0	0	7.2	1	1	0	7.2
195	0.01	7.201	1.619	1	0	0	7.2	1	1	0	7.2
200	0.01	7.202	1.618	1	0	0	7.2	1	1	0	7.2
205	0.01	7.2	1.619	1	0	0	7.2	1	1	0	7.2
210	0.01	7.2	1.619	1	0	0	7.2	1	1	0	7.2
215	0.011	7.202	1.618	1	0	0	7.2	1	1	0	7.2
220	0.011	7.195	1.621	1	0	0	7.2	1	1	0	7.2
225	0.011	7.197	1.62	1	0	0	7.2	1	1	0	7.2
230	0.011	7.195	1.621	1	0	0	7.2	1	1	0	7.2
235	0.011	7.196	1.62	1	0	0	7.2	1	1	0	7.2
240	0.01	7.2	1.619	1	0	0	7.2	1	1	0	7.2
245	0.011	7.2	1.619	1	0	0	7.2	1	1	0	7.2
250	0.011	7.199	1.619	1	0	0	7.2	1	1	0	7.2
255	0.011	7.2	1.619	1	0	0	7.2	1	1	0	7.2
260	0.011	7.2	1.619	1	0	0	7.2	1	1	0	7.2
265	0.011	7.2	1.619	1	0	0	7.2	1	1	0	7.2
270	0.011	7.198	1.62	1	0	0	7.2	1	1	0	7.2
275	0.01	7.2	1.619	1	0	0	7.2	1	1	0	7.2
280	0.011	7.199	1.619	1	0	0	7.2	1	1	0	7.2
285	0.011	7.195	1.621	1	0	0	7.2	1	1	0	7.2
290	0.011	7.196	1.62	1	0	0	7.2	1	1	0	7.2
295	0.011	7.198	1.62	1	0	0	7.2	1	1	0	7.2
300	0.011	7.198	1.62	1	0	0	7.2	1	1	0	7.2
305	0.012	7.301	2.184	1	0	0	9	1	1	0	9
310	0.012	7.433	2.14	1	0	0	9	1	1	0	9
315	0.012	7.561	2.097	1	0	0	9	1	1	0	9
320	0.012	7.686	2.056	1	0	0	9	1	1	0	9
325	0.012	7.792	2.02	1	0	0	9	1	1	0	9
330	0.012	7.898	1.985	1	0	0	9	1	1	0	9
335	0.012	7.995	1.953	1	0	0	9	1	1	0	9
340	0.011	8.062	1.931	1	0	0	9	1	1	0	9
345	0.011	8.136	1.906	1	0	0	9	1	1	0	9
350	0.011	8.205	1.883	1	0	0	9	1	1	0	9
355	0.011	8.254	1.867	1	0	0	9	1	1	0	9
360	0.011	8.296	1.853	1	0	0	9	1	1	0	9
365	0.011	8.338	1.839	1	0	0	9	1	1	0	9
370	0.011	8.376	1.826	1	0	0	9	1	1	0	9
375	0.011	8.4	1.818	1	0	0	9	1	1	0	9
380	0.011	8.433	1.807	1	0	0	9	1	1	0	9
385	0.011	8.453	1.801	1	0	0	9	1	1	0	9
390	0.011	8.476	1.793	1	0	0	9	1	1	0	9
395	0.011	8.491	1.788	1	0	0	9	1	1	0	9
400	0.011	8.515	1.78	1	0	0	9	1	1	0	9
405	0.011	8.545	1.77	1	0	0	9	1	1	0	9
410	0.011	8.559	1.766	1	0	0	9	1	1	0	9
415	0.011	8.578	1.759	1	0	0	9	1	1	0	9
420	0.011	8.59	1.755	1	0	0	9	1	1	0	9
425	0.011	8.603	1.751	1	0	0	9	1	1	0	9
430	0.011	8.62	1.745	1	0	0	9	1	1	0	9
435	0.011	8.63	1.742	1	0	0	9	1	1	0	9
440	0.011	8.635	1.74	1	0	0	9	1	1	0	9