# Summary of "Universal Statistical Simulator"
## Project 1 - Quantum Walks and Monte Carlo

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## 1 Introduction

Randomness describes processes where outcomes occur unpredictable, often with equal or well-defined probabilities. To study randomness, we examine the distribution of outcomes over many trials. The Galton Board (GB) is a physical device consisting of balls, pegs and bins (Fig 1). At the top, balls are released and fall through a grid of pegs arranged in a triangular pattern. The first row has a single peg, and each subsequent row increases by one peg. As each ball hits a peg, it randomly bounces left or right with equal probability. After passing through all rows, the balls fall into bins at the bottom. With a large number of balls and trials, the distribution of balls across the bins follows a binomial distribution, which closely approximates a normal distribution.

This classical method around randomness and probability carries over to quantum systems in many ways. When measuring multiple non-entangled qubits in the computational basis, their outcomes also follow a binomial distribution. This parallel opens doors to quantum analogs of the GB, where interference and superposition alter the output distributions.

The purpose of this paper is to analyze the quantum GB (QGB), and explore further into the types of QGBs implemented.

## 2 Quantum Galton Board

When approaching a QGB, the goal is to model the action of each individual peg. With each peg having a probability of 50% going left or right, one peg will use three working qubits ($q_1$, $q_2$, $q_3$) and one control qubit ($q_0$), all initialized to $|0\rangle$ (Fig 2). The resulting measurements indicate the probability that one ball goes into a specific bin.

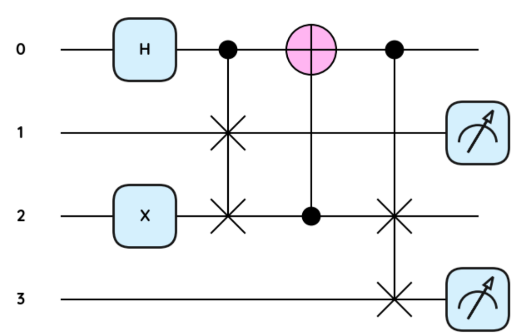


Figure 1: Circuit of One Peg (n=2)

For an n-layer QGB, it is split into two sections. First, a Hadamard is applied on $q_0$, and a Pauli-X Gate is applied on $q_{(n)}$. CSWAP & CNOT gates are then applied, starting with two pairs of CSWAP & CNOT gates, and then a certain number of CSWAP & CNOT gates in a loop until it reaches the $n$-th wire. This repeats all the way until the $n$-th Hadamard.

After applying the $n$-th Hadamard, CSWAP and CNOT gates are applied repeatedly, starting from CSWAP$(q_0, [q_1, q_2])$ and CNOT$(q_2, q_0)$, all the way until the final SWAP gate at CSWAP$(q_0, [q_{2n-1}, q_{2n}])$.

## 3 Biased QGB and Universal Simulation

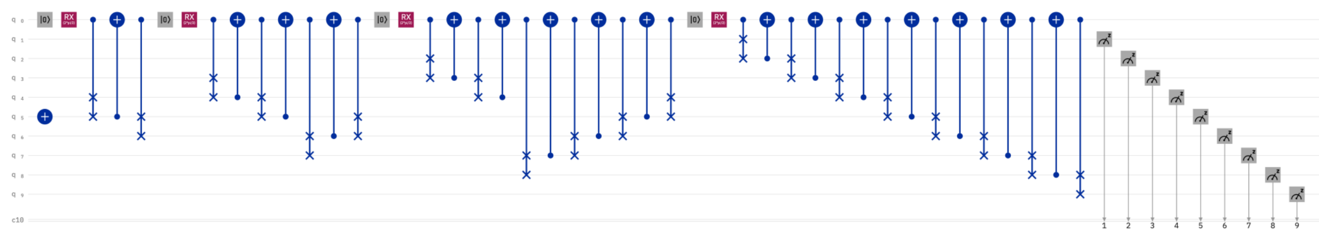

Figure 2: Circuit of a Biased QGB at n=5

To simulate biased distributions, such as exponential distributions, the Hadamard gate is replaced with a rotation gate $R_x(\theta)$ on the control qubit. This biases the "coin flip" at each peg, skewing the output. The angle $\theta$ controls the bias, for example setting $\theta = \frac{2\pi}{3}$ creates a 75% chance of going right and 25% of going left. This approach generalizes the QGB into a universal statistical simulator able to produce distributions beyond Gaussian.

## 4 Fine-grained QGB

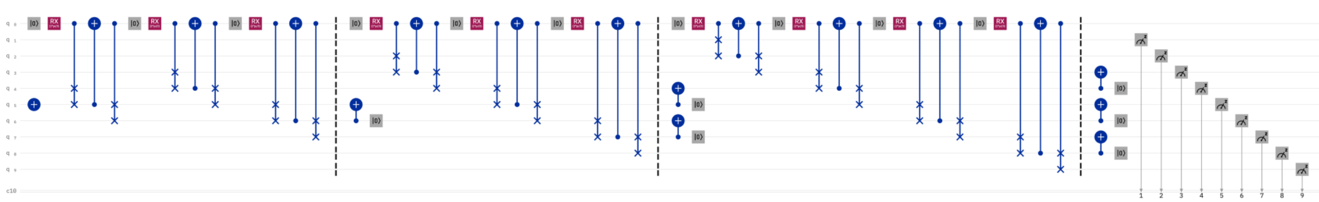

Figure 3: Circuit of a Fine-Grained QGB at n=5

To enable fine-grained control over the output distribution, the circuit is extended so that each individual peg has its own rotation gate $R_x(\theta)$ with a unique angle. This allows every peg to have a different left-right bias, rather than applying the same bias across a full row. Additional CNOT and RESET operations are used to preserve proper circuit behavior between pegs. This fine-grained approach allows for highly customized trajectory control, further generalizing the QGB into a universal statistical simulator capable of producing complex, non-uniform distributions.

For example, setting $\theta_1 = \frac{\pi}{3}$, $\theta_2 = \frac{2\pi}{3}$, and $\theta_3 = \frac{\pi}{4}$ across three pegs (n=3) introduces different probabilities at each step, shaping the overall distribution in a highly controlled way. This fine-grained approach allows for customized trajectory control, further generalizing the QGB into a universal statistical simulator capable of producing complex, non-uniform distributions.

## 5 Quantum Walks and Optimization

The authors also relate the QGB to quantum walks. A quantum walk involves a coin qubit (Hadamard gate) and conditional movement across positions. Unlike classical walks, quantum walks exhibit interference and can be implemented efficiently on quantum hardware.

To optimize QGB circuits for real hardware, the authors explore reducing circuit depth and gate count. Their version achieves quadratic resource scaling: for $n$ levels, the QGB requires at most $2n^2 + 5n + 2$ gates and $2n$ qubits (including ancillae). Despite the need for mid-circuit resets and ancilla qubits, the reduced depth improves noise resilience on current NISQ devices.

## 6 Conclusion

This paper introduces a compact, intuitive quantum implementation of the Galton board and extends it to simulate arbitrary statistical distributions. By modeling each peg as a quantum module with superposition and controlled operations, the QGB can replicate classical behavior and go beyond it. The result is a general purpose statistical simulator which uses quantum principles such as interference and entanglement to compute distributions.