Vollstaendige Induktion Ueben

a)

Zu beweisen:

$$\forall n \in \mathbb{N} \sum_{i=0}^{n}(i) = \frac{n^2+n-2}{2}$$

IS:

ISV:

$$\sum_{i=0}^{n}(i) = \frac{n^2+n-2}{2}$$

ZZ:

$$\forall n \in \mathbb{N} \quad \textbf{ISV} \implies \sum_{i=0}^{n+1}(i) = \frac{(n+1)^2+(n+1)-2}{2}$$

ISF:

$$\begin{aligned}
&\sum_{i=0}^{n+1}(i) \\
&= \sum_{i=0}^{n}(i) + (n+1) \\
&\overset{ISV}{=} \frac{n^2+n-2}{2} + (n+1) \\
&= \frac{n^2+n-2+2n+2}{2} \\
&= \frac{n^2+2n+1+n-1}{2} \\
&= \frac{(n+1)^2+(n+1)-2}{2}
\end{aligned}$$

IA:

ZZ:

$$\sum_{i=0}^{1}(i) = \frac{1^2+1-2}{2}$$

$$\sum_{i=0}^{1}(i) = 1 \neq 0 = \frac{1^2+1-2}{2}$$

b)

Zu Beweisen:

$$\forall n \in \mathbb{N} \sum_{k=1}^{2n}((-1)^k \cdot k) = n$$

IS:

ISV:

$$\sum_{k=1}^{2n}((-1)^k \cdot k) = n$$

ZZ:

$$\forall n \in \mathbb{N} \quad \text{ISV} \implies \sum_{k=1}^{2n+2}((-1)^k \cdot k) = n+1$$

ISF:

$$\begin{aligned}
&\sum_{k=1}^{2n+2}((-1)^k \cdot k) \\
=&\sum_{k=1}^{2n}((-1)^k \cdot k) + ((-1)^{2n+1} \cdot (2n+1)) + ((-1)^{2n+2} \cdot (2n+2)) \\
\overset{\text{ISV}}{=}& n - 2n - 1 + 2n + 2 \\
=& n+1
\end{aligned}$$

IA:

ZZ:

$$\sum_{k=1}^{2}((-1)^k \cdot k) = 1$$

$$\sum_{k=1}^{2}((-1)^k \cdot k) = -1 + 2 = 1$$

c)

Zu Beweisen:

$$\forall n \in \mathbb{N} \sum_{k=0}^{n}(x^k) = \frac{x^{n+1} - 1}{x - 1}$$

IS:

ISV:

$$\sum_{k=0}^{n}(x^k) = \frac{x^{n+1} - 1}{x - 1}$$

ZZ:

$$\forall n \in \mathbb{N} \quad \text{ISV} \implies \sum_{k=0}^{n+1}(x^k) = \frac{x^{(n+1)+1} - 1}{x - 1}$$

ISF:

$$
\begin{aligned}
&\sum_{k=0}^{n+1}(x^k) \\
&= \sum_{k=0}^{n}(x^k) + x^{n+1} \\
&\overset{\text{ISV}}{=} \frac{x^{n+1}-1}{x-1} + x^{n+1} \\
&= \frac{x^{n+1}-1}{x-1} + \frac{(x-1)\cdot x^{n+1}}{x-1} \\
&= \frac{x^{n+1}-1}{x-1} + \frac{x\cdot x^{n+1}-x^{n+1}}{x-1} \\
&= \frac{x^{n+1}-1}{x-1} + \frac{x^{(n+1)+1}-x^{n+1}}{x-1} \\
&= \frac{x^{(n+1)+1}-1}{x-1}
\end{aligned}
$$

IA:

ZZ:

$$\sum_{k=0}^{0}(x^k) = \frac{x^{0+1}-1}{x-1}$$

$$\sum_{k=0}^{0}(x^k) = x^0 = 1 = \frac{x-1}{x-1} = \frac{x^{0+1}-1}{x-1}$$

□

AB: Ein letztes AB zur vollstaendigen Induktion

a)

$$a^2 + b^2 \neq a^2 + 2ab + b^2 = (a+b)^n$$

b)

$$$$