Lineare Algebra 1, Blatt 3

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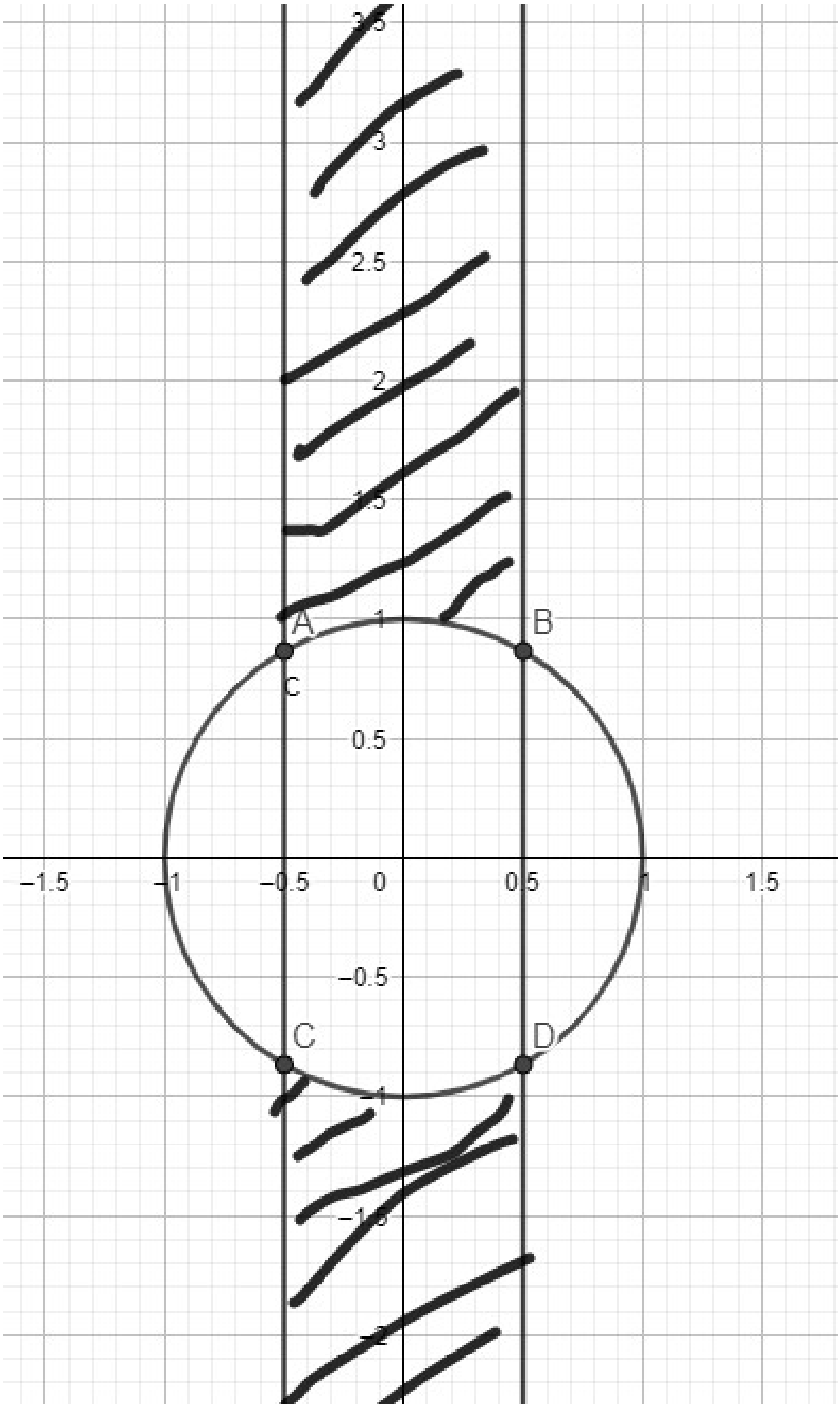
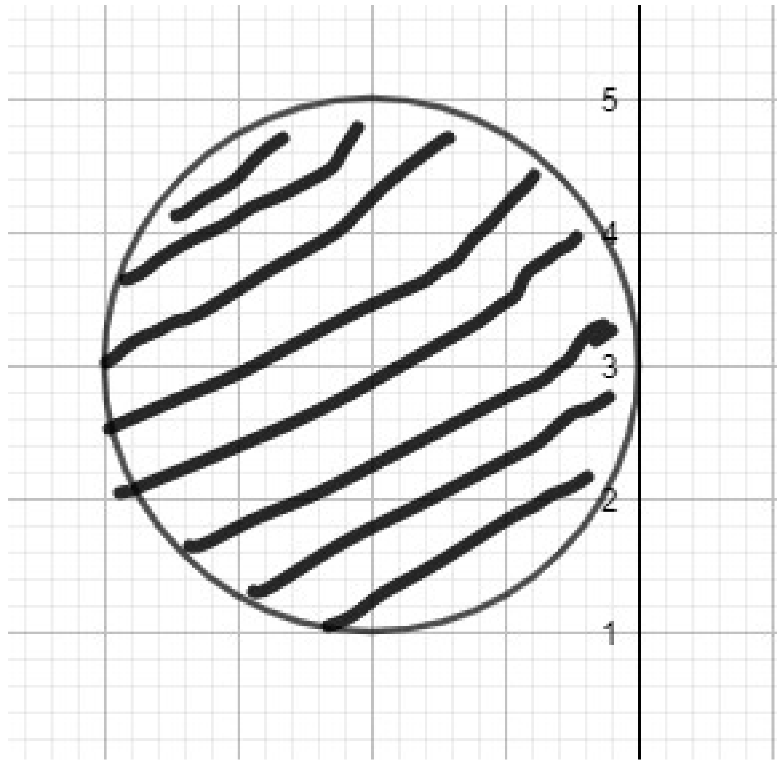
Aufgabe 12



- 1.

- 2. 

Aufgabe 13

- $(1+i)^{10} = ((1+i)^2)^5 = (1+2i-1)^5 = 2^5 i^5 = 2^5 i$
- $(1-\sqrt{3}i)^6 = (2(\frac{1}{2} - \frac{\sqrt{3}}{2}i))^6 = 2^6(\cos(\frac{5\pi}{3}) + i\sin(\frac{5\pi}{3}))^6 = 2^6(\cos(10\pi) + i\sin(10\pi)) = 2^6(\cos(0) + i\sin(0)) = 2^6(1) = 2^6$
- $(\frac{\sqrt{3}+i}{1-i})^{12} = (\frac{(\sqrt{3}+i)(1+i)}{(1-i)(1+i)})^{12} = (\frac{\sqrt{3}+i+\sqrt{3}i+i^2}{2})^{12} = (\frac{(\sqrt{3}-1)+i(1+\sqrt{3})}{2})^{12}$
  $= \frac{1}{2^{12}}((\sqrt{3}-1) + i(\sqrt{3}+1))^{12} \overset{*}{=} \frac{\sqrt{8}^{12}}{2^{12}}(\frac{\sqrt{3}+1}{\sqrt{8}} + \frac{\sqrt{3}-1}{\sqrt{8}}i)^{12}$
  $= \frac{8^6}{2^{12}}(\cos(\frac{\pi}{12}) + i\sin(\frac{\pi}{12}))^{12} = \frac{(2^3)^6)}{2^{12}}(\cos(\pi) + i\sin(\pi)) = \frac{2^{18}}{2^{12}}(i) = 2^6 i$
- $\sqrt{(\sqrt{3}+1)^2 + (\sqrt{3}-1)^2} = \sqrt{3+2\sqrt{3}+1+3-2\sqrt{3}+1} = \sqrt{8}$
- $(\frac{\sqrt{3}-i}{\sqrt{3}+i})^{2021} = (\frac{(\sqrt{3}-i)(\sqrt{3}-i)}{(\sqrt{3}+i)(\sqrt{3}-i)})^{2021} = (\frac{3-2\sqrt{3}i-1}{4})^{2021} = (\frac{1-\sqrt{3}i}{2})^{2021} = (\cos(\frac{5\pi}{3}) + i\sin(\frac{5\pi}{3}))^{2021} = \cos(\frac{10105\pi}{3}) + i\sin(\frac{10105\pi}{3}) = \cos(3368\pi + \frac{\pi}{3}) + i\sin(3368\pi + \frac{\pi}{3}) = \cos(\frac{\pi}{3}) + i\sin(\frac{\pi}{3}) = \frac{1}{2} + \frac{\sqrt{3}}{2}i$

Aufgabe 14 $z \in \mathbb{C} : z^2 = \overline{z}$

$\{z \in \mathbb{C} : |z| = 1\}$

Aufgabe 15 $K := \{a + \sqrt{3}ib | a, b \in \mathbb{Q}\} \subseteq \mathbb{C}$

Aufgabe 16 $\forall \theta \in \mathbb{R}, z \in \mathbb{C}\backslash\{0\}, n \in \mathbb{N} : z + \frac{1}{z} = 2\cos(\theta)$

z.z. $z^n + z^{-n} = 2\cos(n\theta)$

Aufgabe 17

(1)

$$\begin{pmatrix} 3-i & 4+2i & | & 2+6i \\ 4+2i & -2-3i & | & 5+4i \end{pmatrix}$$
$$\overset{I\cdot(3+i),II\cdot(4-2i)}{\rightsquigarrow} \begin{pmatrix} (3-i)(3+i) & (4+2i)(3+i) & | & (2+6i)(3+i) \\ (4+2i)(4-2i) & (-2-3i)(4-2i) & | & (5+4i)(4-2i) \end{pmatrix}$$
$$\rightsquigarrow \begin{pmatrix} 10 & 12+6i+4i-2 & | & 6+18i+2i-6 \\ 20 & -8-12i+4i-6 & | & 20+16i-10i+8 \end{pmatrix}$$
$$\overset{I\cdot(\frac{1}{10}),II\cdot(\frac{1}{2})}{\rightsquigarrow} \begin{pmatrix} 1 & 1+i & | & 2i \\ 10 & -7-4i & | & 14+3i \end{pmatrix}$$
$$\overset{II-10I}{\rightsquigarrow} \begin{pmatrix} 1 & 1+i & | & 2i \\ 0 & -17-14i & | & 14-17i \end{pmatrix}$$
$$\overset{II\cdot(-17+14i)}{\rightsquigarrow} \begin{pmatrix} 1 & 1+i & | & 2i \\ 0 & (-17-14i)(-17+14i) & | & (14-17i)(-17+14i) \end{pmatrix}$$
$$\overset{II-2I}{\rightsquigarrow} \begin{pmatrix} 1 & 1+i & | & 2i \\ 0 & 485 & | & 485i \end{pmatrix}$$
$$\overset{II-2I}{\rightsquigarrow} \begin{pmatrix} 1 & 1+i & | & 2i \\ 0 & 1 & | & i \end{pmatrix}$$
$$\overset{I-(1+i)II}{\rightsquigarrow} \begin{pmatrix} 1 & 0 & | & 1+i \\ 0 & 1 & | & i \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1+i \\ i \end{pmatrix}$$

$$\begin{pmatrix} 1 & i & -2 & | & 10 \\ 1 & -1 & 2i & | & 20 \\ i & 3i & -1-i & | & 30 \end{pmatrix}$$

$$\overset{II-I, III-i\cdot I}{\rightsquigarrow} \begin{pmatrix} 1 & i & -2 & | & 10 \\ 0 & -1-i & 2i+2 & | & 10 \\ 0 & 3i+1 & -1+i & | & 30-10i \end{pmatrix}$$

$$\overset{II\cdot(-1+i)}{\rightsquigarrow} \begin{pmatrix} 1 & i & -2 & | & 10 \\ 0 & 2 & -4 & | & -10+10i \\ 0 & 3i+1 & -1+i & | & 30-10i \end{pmatrix}$$

$$\overset{II\cdot\frac{1}{2}}{\rightsquigarrow} \begin{pmatrix} 1 & i & -2 & | & 10 \\ 0 & 1 & -2 & | & -5+5i \\ 0 & 3i+1 & -1+i & | & 30-10i \end{pmatrix}$$

$$\overset{III-(3i+1)II}{\rightsquigarrow} \begin{pmatrix} 1 & i & -2 & | & 10 \\ 0 & 1 & -2 & | & -5+5i \\ 0 & 0 & 7i+1 & | & 50 \end{pmatrix} \tag{2}$$

$$\overset{III\cdot(1-7i)}{\rightsquigarrow} \begin{pmatrix} 1 & i & -2 & | & 10 \\ 0 & 1 & -2 & | & -5+5i \\ 0 & 0 & 50 & | & 50-350i \end{pmatrix}$$

$$\overset{III\cdot\frac{1}{50}}{\rightsquigarrow} \begin{pmatrix} 1 & i & -2 & | & 10 \\ 0 & 1 & -2 & | & -5+5i \\ 0 & 0 & 1 & | & 1-7i \end{pmatrix}$$

$$\overset{I+2III, II+2III}{\rightsquigarrow} \begin{pmatrix} 1 & i & 0 & | & 12-14i \\ 0 & 1 & 0 & | & -3-9i \\ 0 & 0 & 1 & | & 1-7i \end{pmatrix}$$

$$\overset{I-i\cdot II}{\rightsquigarrow} \begin{pmatrix} 1 & 0 & 0 & | & 3-11i \\ 0 & 1 & 0 & | & -3-9i \\ 0 & 0 & 1 & | & 1-7i \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3-11i \\ -3-9i \\ 1-7i \end{pmatrix}$$