Datenstrukturen und Algorithmen
Heimübung 5

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Aufgabe 1

Algorithm 1 Quicksort($A[1...n], l, r$) $O(f(n))$

if p<r **then** ▷ $O(1)$
    $q \leftarrow$ Partition(A, p, r) ▷ $O(n)$
    Quicksort(A,p,q-1) ▷ $O(f(n \cdot (1-\alpha)))$
    Quicksort(A,q+1,r) ▷ $O(f(n \cdot \alpha))$

$$f(n) = n + f(n \cdot \alpha) + f(n \cdot (1-\alpha))$$

Da die Rekursionstiefe für $\alpha > \frac{1}{2}$ nicht Fest ist können wir keine exakte Geschlossene Formel angeben. Aber eine Analyse im O-Kalkühl ist möglich:

$$\begin{aligned}
O(f(n)) &= O(n + f(n \cdot (1-\alpha)) + f(n \cdot \alpha)) \\
&= O(n + 2 \cdot f(n \cdot \alpha)) \quad \text{da} n \cdot (1-\alpha) \leq n \cdot \alpha \\
&= O(n + 2 \cdot n \cdot \alpha + \cdots + 2^{k-1} \cdot n \cdot \alpha^{k-1} + 2^k \cdot f(n \cdot \alpha^k)) \\
&\text{für } k = \log_{\frac{1}{\alpha}}(n) \\
&= O(n + 2 \cdot n \cdot \alpha + \cdots + 2^{k-1} \cdot n \cdot \alpha^{k-1}) \\
&= O(n) + O(2 \cdot n \cdot alpha) + \cdots + O(2^{k-1} \cdot n \cdot alpha^{k-1}) \\
&= O(n) + \underbrace{O(n) + \cdots + O(n)}_{\text{k-mal}} \\
&= O(k \cdot n) \\
&= O(n \cdot \log_{\frac{1}{\alpha}}(n))
\end{aligned}$$

Aufgabe 2

Nr.2)

$A = [6, 0, 2, 0, 1, 3, 4, 6, 1, 3, 2]$ (bei $k=6$)

a)

$C = [0, 0, 0, 0, 0, 0, 0]$

nach Zeile 1-2

$$C = \begin{array}{c} \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 2 & 2 & 2 & 2 & 1 & 0 & 2 \end{bmatrix} \end{array}$$

nach Zeile 3-4

$$C = [2 \ 4 \ 6 \ 8 \ 9 \ 9 \ 11]$$

Nach Zeile 6-7

b)
$$A = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \end{matrix} \\ [6, 0, 2, 0, 1, 3, 4, 6, 1, 3, 2] \end{array} \quad C = [2 \ 4 \ 6 \ 8 \ 9 \ 9 \ 11]$$

$$B = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \end{matrix} \\ [\qquad\qquad\qquad\qquad\qquad\qquad] \end{array}$$

$$C' = [2 \ 4 \ 6 \ 8 \ 9 \ 9 \ 11]$$

$$A = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \end{matrix} \\ [6, 0, 2, 0, 1, 3, 4, 6, 1, 3, \boxed{2}] \end{array} \quad C = [2 \ 4 \ 6 \ 8 \ 9 \ 9 \ 11]$$

$$B = \begin{array}{c} \begin{matrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \end{matrix} \\ [\begin{matrix} \ & \ & \ & \ & \ & 2 & \ & \ & \ & \ & \ \end{matrix}] \end{array}$$

$$C' = \begin{array}{c} \begin{matrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 \end{matrix} \\ \begin{bmatrix} 2 & 4 & 5 & 8 & 9 & 9 & 11 \end{bmatrix} \end{array}$$

1 2 3 4 5 6 7 8 9 10 11

$$\begin{array}{lccccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\
A= & [6, & 0, & 2, & 0, & 1, & 3, & 4, & 6, & 1, & \boxed{3}, & 2]
\end{array} \quad C = [2\ 4\ 6\ 8\ 9\ 9\ 11]$$

$$\begin{array}{lccccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\
B= & [& & & & & 2 & & 3 & & &]
\end{array}$$

$$\begin{array}{lccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
C'= & [2 & 4 & 5 & 7 & 9 & 9 & 11]
\end{array}$$

$$\begin{array}{lccccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\
A= & [6, & 0, & 2, & 0, & 1, & 3, & 4, & 6, & \boxed{1}, & 3, & 2]
\end{array} \quad C = [2\ 4\ 6\ 8\ 9\ 9\ 11]$$

$$\begin{array}{lccccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\
B= & [& & & 1 & & 2 & & 3 & & &]
\end{array}$$

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 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
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 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\
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\end{array} \quad C = [2\ 4\ 6\ 8\ 9\ 9\ 11]$$

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$$\begin{array}{lccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
C'= & [2 & 3 & 5 & 7 & 9 & 9 & 10]
\end{array}$$

$$\begin{array}{lccccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\
A= & [6, & 0, & 2, & 0, & 1, & 3, & \boxed{4}, & 6, & 1, & 3, & 2]
\end{array} \quad C = [2\ 4\ 6\ 8\ 9\ 9\ 11]$$

$$\begin{array}{lccccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\
B= & [& & & 1 & & 2 & & 3 & 4 & & 6]
\end{array}$$

$$\begin{array}{lccccccc}
 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\
C'= & [2 & 3 & 5 & 7 & 8 & 9 & 10]
\end{array}$$

$$\begin{array}{lccccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\
A= & [6, & 0, & 2, & 0, & 1, & \boxed{3}, & 4, & 6, & 1, & 3, & 2]
\end{array} \quad C = [2\ 4\ 6\ 8\ 9\ 9\ 11]$$

$$\begin{array}{lccccccccccc}
 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\
B= & [& & & 1 & & 2 & 3 & 3 & 4 & & 6]
\end{array}$$

$$\begin{array}{ccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6
\end{array}$$

$$\begin{array}{cccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ B = [& & & & 1 & & 2 & 3 & 3 & 4 & & 6\,] \end{array}$$

$$\begin{array}{cccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ C' = [& 2 & 3 & 5 & 6 & 8 & 9 & 10\,] \end{array}$$

$$\begin{array}{cccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ A = [& 6, & 0, & 2, & 0, & \boxed{1}, & 3, & 4, & 6, & 1, & 3, & 2\,] \end{array} \quad C = [2\ 4\ 6\ 8\ 9\ 9\ 11]$$

$$\begin{array}{cccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ B = [& & & 1 & 1 & & 2 & 3 & 3 & 4 & & 6\,] \end{array}$$

$$\begin{array}{cccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ C' = [& 2 & 2 & 5 & 6 & 8 & 9 & 10\,] \end{array}$$

$$\begin{array}{cccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ A = [& 6, & 0, & 2, & \boxed{0}, & 1, & 3, & 4, & 6, & 1, & 3, & 2\,] \end{array} \quad C = [2\ 4\ 6\ 8\ 9\ 9\ 11]$$

$$\begin{array}{cccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ B = [& & 0 & 1 & 1 & & 2 & 3 & 3 & 4 & & 6\,] \end{array}$$

$$\begin{array}{cccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ C' = [& 1 & 2 & 5 & 6 & 8 & 9 & 10\,] \end{array}$$

$$\begin{array}{cccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ A = [& 6, & 0, & \boxed{2}, & 0, & 1, & 3, & 4, & 6, & 1, & 3, & 2\,] \end{array} \quad C = [2\ 4\ 6\ 8\ 9\ 9\ 11]$$

$$\begin{array}{cccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ B = [& & 0 & 1 & 1 & 2 & 2 & 3 & 3 & 4 & & 6\,] \end{array}$$

$$\begin{array}{cccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ C' = [& 1 & 2 & 4 & 6 & 8 & 9 & 10\,] \end{array}$$

$$\begin{array}{cccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ A = [& 6, & \boxed{0}, & 2, & 0, & 1, & 3, & 4, & 6, & 1, & 3, & 2\,] \end{array} \quad C = [2\ 4\ 6\ 8\ 9\ 9\ 11]$$

$$\begin{array}{cccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ B = [& 0 & 0 & 1 & 1 & 2 & 2 & 3 & 3 & 4 & & 6\,] \end{array}$$

$$\begin{array}{cccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ C' = [& 0 & 2 & 4 & 6 & 8 & 9 & 10\,] \end{array}$$

$$\begin{array}{cccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \end{array}$$

$$\begin{array}{cccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ A = & [\boxed{6}, & 0, & 2, & 0, & 1, & 3, & 4, & 6, & 1, & 3, & 2] \end{array} \qquad C = [2\ 4\ 6\ 8\ 9\ 9\ 11]$$

$$\begin{array}{cccccccccccc} & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ B = & [0 & 0 & 1 & 1 & 2 & 2 & 3 & 3 & 4 & 6 & 6] \end{array}$$

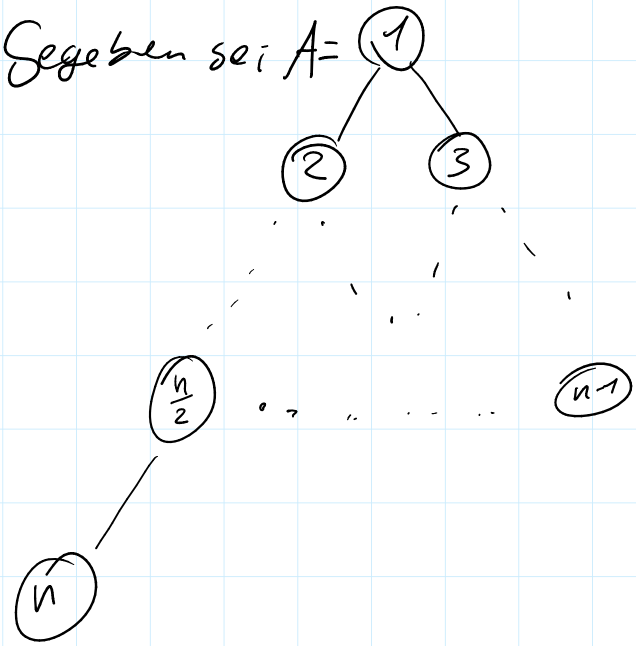
$$\begin{array}{cccccccc} & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ C' = & [0 & 2 & 4 & 6 & 8 & 9 & 9] \end{array}$$

Aufgabe 3

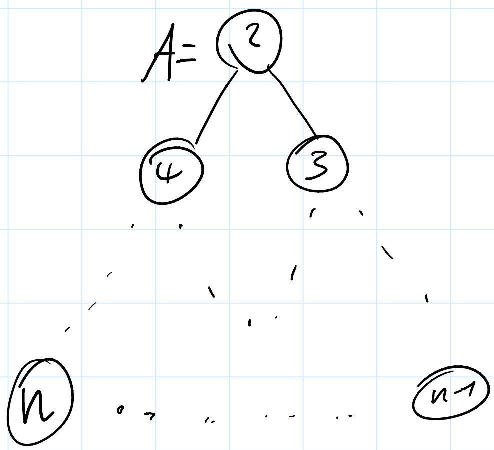
2022-05-19

Donnerstag, 19. Mai 2022 18:44

Gegeben sei A=



Dann AwesomeAlgo (A): $O(1)$



A_{davor} hat zu A_{danach} $\log(n)$ verschiedene Speicherzellen:

$\Rightarrow$ Es sind $\Omega(\log(n))$ Speicherzugriffe geschehen

$\Rightarrow$ AwesomeAlgo $\neq O(n)$ ↯

Man kann nicht

$$\underbrace{\underbrace{\text{Liste} \xrightarrow{n \times \text{Heapify}}}_{O(n\log(n))} \text{Heap} \underbrace{\xrightarrow{n \times \text{Awesome Algo}} \text{Sortierte Liste}}_{O(n \cdot 1)}}_{O(n\log(n))}$$

in $O(1)$. Hinweis ist falsch.