Natural Language Processing with Deep Learning

Lecture 4 — Text classification 2: Deep neural networks

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Natural Language Processing Group
Paderborn University
We focus on Trustworthy Human Language Technologies

PADERBORN UNIVERSITY

www.trusthlt.org

Where we finished last time

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Log-linear multi-class classification

- Representations
- From multi-dimensional linear transformation to probabilities

Loss function for softmax

Stacking transformations and non-linearity

Our binary text classification function

Linear function through sigmoid — log-linear model

$$\hat{y} = \sigma(f(\boldsymbol{x})) = \frac{1}{1 + \exp(-(\boldsymbol{x} \cdot \boldsymbol{w} + b))}$$

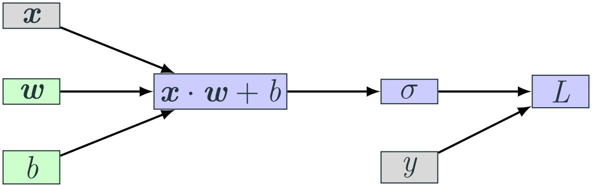

Figure 1: Computational graph; green nodes are trainable parameters, gray are constant inputs

How can we minimize this function?

(Online) Stochastic Gradient Descent

function SGD($f(\boldsymbol{x}; \Theta)$, $(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n)$, $(\boldsymbol{y}_1, \ldots, \boldsymbol{y}_n)$, L)
  while stopping criteria not met **do**
    Sample a training example $\boldsymbol{x}_i, \boldsymbol{y}_i$
    Compute the loss $L(f(\boldsymbol{x}_i; \Theta), \boldsymbol{y}_i)$
    $\hat{\boldsymbol{g}} \leftarrow$ gradient of $L(f(\boldsymbol{x}_i; \Theta), \boldsymbol{y}_i)$ wrt. Θ
    $\Theta \leftarrow \Theta - \eta_t \hat{\boldsymbol{g}}$
  return Θ

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Loss in line 4 is based on a **single training example** $\rightarrow$ a rough estimate of the corpus loss $\mathcal{L}$ we aim to minimize

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Loss in line 4 is based on a **single training example** $\rightarrow$ a rough estimate of the corpus loss $\mathcal{L}$ we aim to minimize

The noise in the loss computation may result in inaccurate gradients

Minibatch Stochastic Gradient Descent

function MBSGD($f(\boldsymbol{x};\Theta)$, $(\boldsymbol{x}_1, \ldots, \boldsymbol{x}_n)$, $(\boldsymbol{y}_1, \ldots, \boldsymbol{y}_n)$, L)

  while stopping criteria not met **do**

    Sample m examples $\{(\boldsymbol{x}_1, \boldsymbol{y}_1), \ldots (\boldsymbol{x}_m, \boldsymbol{y}_m)\}$

    $\hat{\boldsymbol{g}} \leftarrow 0$

    for $i = 1$ to m **do**

      Compute the loss $L(f(\boldsymbol{x}_i;\Theta), \boldsymbol{y}_i)$

      $\hat{\boldsymbol{g}} \leftarrow \hat{\boldsymbol{g}} +$ gradient of $\frac{1}{m} L(f(\boldsymbol{x}_i;\Theta), \boldsymbol{y}_i)$ wrt. Θ

    $\Theta \leftarrow \Theta - \eta_t \hat{\boldsymbol{g}}$

  return Θ

Properties of Minibatch Stochastic Gradient Descent

The minibatch size can vary in size from $m = 1$ to $m = n$

Higher values provide better estimates of the corpus-wide gradients, while smaller values allow more updates and in turn faster convergence

Lines 6+7: May be easily parallelized

Log-linear multi-class classification

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From binary to multi-class labels

So far we mapped our gold label $y \in \{0, 1\}$

What if we classify into distinct categorical classes?

- Categorical: There is no 'ordering'
- Example: Classify the language of a document into 6 languages (En, Fr, De, It, Es, Other)

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One-hot encoding of labels

$$\text{En} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad \text{Fr} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$\text{De} = \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} \qquad \ldots$$

$\boldsymbol{y} \in \mathbb{R}^{d_{out}}$ where d_{out} is the number of classes

Possible solution: Six weight vectors and biases

Consider for each language $\ell \in \{\text{En}, \text{Fr}, \text{De}, \text{It}, \text{Es}, \text{Other}\}$

- Weight vector $\boldsymbol{w}^{\ell}$ (e.g., $\boldsymbol{w}^{\text{Fr}}$)
- Bias b^{ℓ} (e.g., b^{Fr})

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We can predict the language resulting in the highest score

$$\hat{y} = f(\boldsymbol{x}) = \underset{\ell \in \{\text{En},\text{Fr},\text{De},\text{It},\text{Es},\text{Other}\}}{\operatorname{argmax}} \boldsymbol{x} \cdot \boldsymbol{w}^{\ell} + b^{\ell}$$

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But we can re-arrange the $\boldsymbol{w} \in \mathbb{R}^{d_{in}}$ vectors into columns of a matrix $\boldsymbol{W} \in \mathbb{R}^{d_{in} \times 6}$ and $\boldsymbol{b} \in \mathbb{R}^{6}$, to get

$$f(\boldsymbol{x}) = \boldsymbol{x}\boldsymbol{W} + \boldsymbol{b}$$

Projecting input vector to output vector $f(x) : \mathbb{R}^{d_{in}} \to \mathbb{R}^{d_{out}}$

Projecting input vector to output vector $f(\boldsymbol{x}) : \mathbb{R}^{d_{in}} \to \mathbb{R}^{d_{out}}$

Recall from lecture 3: High-dimensional linear functions

Function $f(\boldsymbol{x}) : \mathbb{R}^{d_{in}} \to \mathbb{R}^{d_{out}}$

$$f(\boldsymbol{x}) = \boldsymbol{x}\boldsymbol{W} + \boldsymbol{b}$$

where $\boldsymbol{x} \in \mathbb{R}^{d_{in}}$ $\quad \boldsymbol{W} \in \mathbb{R}^{d_{in} \times d_{out}}$ $\quad \boldsymbol{b} \in \mathbb{R}^{d_{out}}$

The simplest neural network — a perceptron (simply a linear model)

- How to find the prediction $\hat{y}$?

Prediction of multi-class classifier

Project the input $\boldsymbol{x}$ to an output $\boldsymbol{y}$

$$\hat{\boldsymbol{y}} = f(\boldsymbol{x}) = \boldsymbol{x}\boldsymbol{W} + \boldsymbol{b}$$

and pick the element of $\hat{\boldsymbol{y}}$ with the highest value

$$\text{prediction} = \hat{y} = \operatorname*{argmax}_i \hat{\boldsymbol{y}}_{[i]}$$

Sanity check

What is $\hat{y}$?

Prediction of multi-class classifier

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What is $\hat{y}$?

Index of 1 in the one-hot

For example, if $\hat{y} = 3$, then the document is in German
De $= \begin{pmatrix} 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$

Log-linear multi-class classification

Representations

Two representations of the input document

$$\hat{\boldsymbol{y}} = \boldsymbol{x}\boldsymbol{W} + \boldsymbol{b}$$

Vector $\boldsymbol{x}$ is a document representation

- Bag of words, for example ($d_{in} = |V|$ dimensions, sparse)

Vector $\hat{\boldsymbol{y}}$ is **also** a document representation

- More compact (only 6 dimensions)
- More specialized for the language prediction task

Matrix W as learned representation — columns

$\hat{y} = xW + b \quad \rightarrow$ two views of W, as rows or as columns

	En	Fr	De	It	Es	Ot
a	•	•	•	•	•	•
at	•	•	•	•	•	•
...						
zoo	•	•	•	•	•	•

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...						
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Each of the 6 columns (corresponding to a language) is a d_{in}-dimensional vector representation of this language in terms of its characteristic word unigram patterns (e.g., we can then cluster the 6 language vectors according to their similarity)

Matrix W as learned representation — rows

$$\hat{\boldsymbol{y}} = \boldsymbol{x}\boldsymbol{W} + \boldsymbol{b}$$

	En	Fr	De	It	Es	Ot
a	•	•	•	•	•	•
at	•	•	•	•	•	•
...						
zoo	•	•	•	•	•	•

Each of the d_{in} rows corresponds to a particular unigram, and provides a 6-dimensional vector representation of that unigram in terms of the languages it prompts

From bag-of-words to continuous bag-of-words

Recall from lecture 3 — Averaged bag of words

$$\boldsymbol{x} = \frac{1}{|D|} \sum_{i=1}^{|D|} \boldsymbol{x}^{D_{[i]}}$$

$D_{[i]}$ — word in doc D at position i, $\boldsymbol{x}^{D_{[i]}}$ — one-hot vector

$$\hat{\boldsymbol{y}} = \boldsymbol{x}\boldsymbol{W} =$$

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$$\hat{\boldsymbol{y}} = \boldsymbol{x}\boldsymbol{W} = \left(\frac{1}{|D|} \sum_{i=1}^{|D|} \boldsymbol{x}^{D_{[i]}} \right) \boldsymbol{W}$$

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$$=$$

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$$= \frac{1}{|D|} \sum_{i=1}^{|D|} \boldsymbol{W}^{D_{[i]}}$$

(we ignore the bias $\boldsymbol{b}$ here)

From bag-of-words to continuous bag-of-words (CBOW)

Two equivalent views; $W^{D_{[i]}}$ is the $D_{[i]}$-th row of matrix $\boldsymbol{W}$

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The continuous-bag-of-words (CBOW) representation

- Either by summing word-representation vectors
- Or by multiplying a bag-of-words vector by a matrix in which each row corresponds to a dense word representation (also called **embedding matrix**)

Learned representations — central to deep learning

Representations are central to deep learning

One could argue that the main power of deep-learning is the ability to learn good representations

Log-linear multi-class classification

From multi-dimensional linear transformation to probabilities

Turning output vector into probabilities of classes

Recap: Categorical probability distribution

Categorical random variable X is defined over K categories, typically mapped to natural numbers $1, 2, \ldots, K$, for example En = 1, De = 2, $\ldots$

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How to turn an **unbounded** vector in $\mathbb{R}^K$ into a categorical probability distribution?

The softmax function $\mathrm{softmax}(\boldsymbol{x}) : \mathbb{R}^K \to \mathbb{R}^K$

Softmax

Applied element-wise, for each element $\boldsymbol{x}_{[i]}$ we have

$$\mathrm{softmax}(\boldsymbol{x}_{[i]}) = \frac{\exp\left(\boldsymbol{x}_{[i]}\right)}{\sum_{k=1}^{K} \exp\left(\boldsymbol{x}_{[k]}\right)}$$

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- Nominator: Non-linear bijection from $\mathbb{R}$ to $(0; \infty)$
- Denominator: Normalizing constant to ensure $\sum_{j=1}^{K} \mathrm{softmax}(\boldsymbol{x}_{[j]}) = 1$

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We also need to know how to compute the partial derivative of $\mathrm{softmax}(\boldsymbol{x}_{[i]})$ wrt. each argument $\boldsymbol{x}_{[k]}$: $\frac{\partial \, \mathrm{softmax}(\boldsymbol{x}_{[i]})}{\partial \boldsymbol{x}_{[k]}}$

Softmax can be smoothed with a 'temperature' T

$$\text{softmax}(\boldsymbol{x}_{[i]}; T) = \frac{\exp\left(\frac{\boldsymbol{x}_{[i]}}{T}\right)}{\sum_{k=1}^{K} \exp\left(\frac{\boldsymbol{x}_{[k]}}{T}\right)}$$

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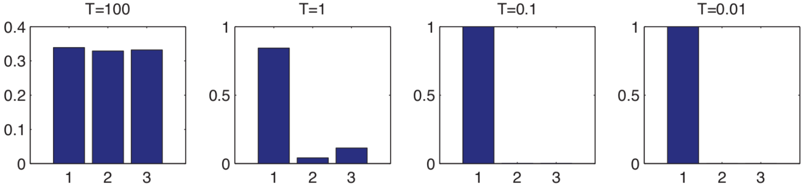
Figure from K. Murphy (2012). ***Machine Learning: a Probabilistic Perspective.*** MIT Press

Example: Softmax of $x = (3, 0, 1)$ at different T



High temperature $\rightarrow$ uniform distribution

Low temperature $\rightarrow$ ‘spiky’ distribution, all mass on the largest element

Loss function for softmax

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Categorical cross-entropy loss (aka. negative log likelihood)

Vector representing the gold-standard categorical distribution over the classes/labels $1, \ldots, K$:

$$\boldsymbol{y} = (\boldsymbol{y}_{[1]}, \boldsymbol{y}_{[2]}, \ldots, \boldsymbol{y}_{[K]})$$

Output from softmax:

$$\hat{\boldsymbol{y}} = (\hat{\boldsymbol{y}}_{[1]}, \hat{\boldsymbol{y}}_{[2]}, \ldots, \hat{\boldsymbol{y}}_{[K]})$$

which is in fact $\hat{\boldsymbol{y}}_{[\boldsymbol{i}]} = \Pr(y = i | \boldsymbol{x})$

Cross entropy loss

$$L_{\text{cross-entropy}}(\hat{\boldsymbol{y}}, \boldsymbol{y}) = - \sum_{k=1}^{K} \boldsymbol{y}_{[k]} \log \left(\hat{\boldsymbol{y}}_{[k]} \right)$$

Background: K-L divergence (also known as *relative entropy*)

Let Y and $\hat{Y}$ be categorical random variables over same categories, with probability distributions $P(Y)$ and $Q(\hat{Y})$

$$\begin{aligned}
\mathbb{D}(P(Y)||Q(\hat{Y})) &= \mathbb{E}_{P(Y)}\left[\log \frac{P(Y)}{Q(\hat{Y})}\right] \\
&= \mathbb{E}_{P(Y)}\left[\log P(Y) - \log Q(\hat{Y})\right] \\
&= \mathbb{E}_{P(Y)}\left[\log P(Y)\right] - \mathbb{E}_{P(Y)}\left[\log Q(\hat{Y})\right] \\
&= -\mathbb{E}_{P(Y)}\left[\log \frac{1}{P(Y)}\right] - \mathbb{E}_{P(Y)}\left[\log Q(\hat{Y})\right] \\
&= -\mathbb{H}_P(Y) - \mathbb{E}_{P(Y)}\left[\log Q(\hat{Y})\right]
\end{aligned}$$

Stacking transformations and non-linearity

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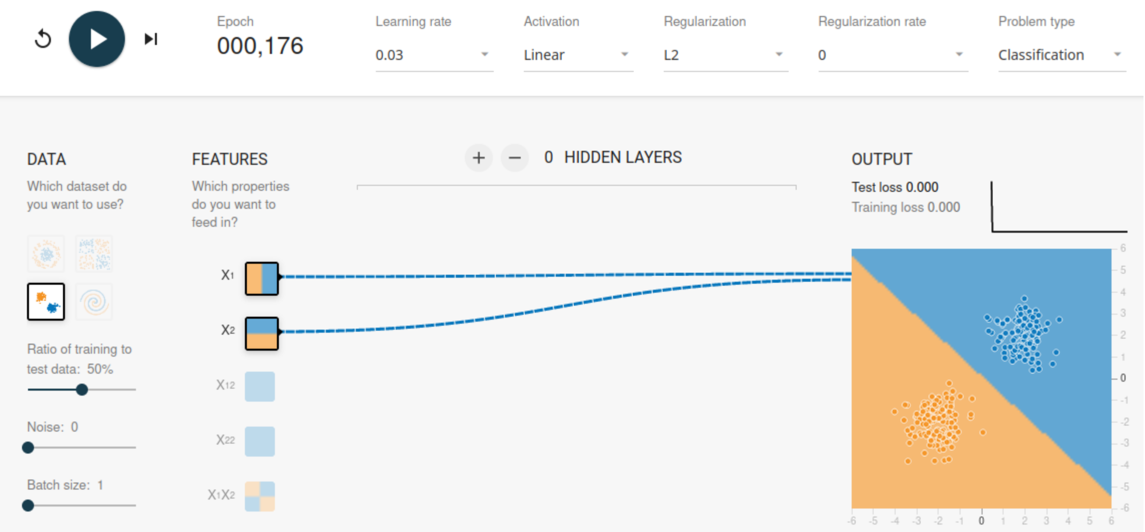

Figure 2: Linear model can tackle only linearly-separable problems (`http://playground.tensorflow.org`)

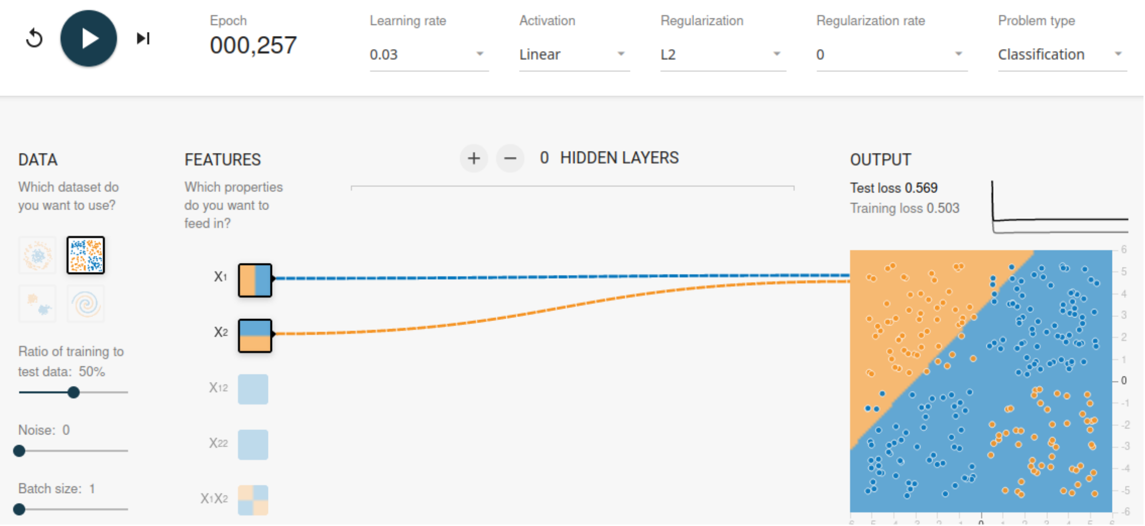

Figure 3: Linear model can tackle only linearly-separable problems (`http://playground.tensorflow.org`)

Stacking linear layers on top of each other — still linear!

$$\boldsymbol{x} \in \mathbb{R}^{d_{in}} \qquad \boldsymbol{W^1} \in \mathbb{R}^{d_{in} \times d_1} \qquad \boldsymbol{b^1} \in \mathbb{R}^{d_1} \qquad \boldsymbol{W^2} \in \mathbb{R}^{d_1 \times d_{out}} \qquad \boldsymbol{b^2} \in \mathbb{R}^{d_{out}}$$

$$f(\boldsymbol{x}) = \left(\boldsymbol{x}\boldsymbol{W^1} + \boldsymbol{b^1}\right) \boldsymbol{W^2} + \boldsymbol{b^2}$$

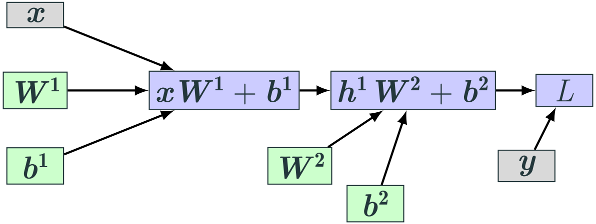

Figure 4: Computational graph; green circles are trainable parameters, gray are constant inputs

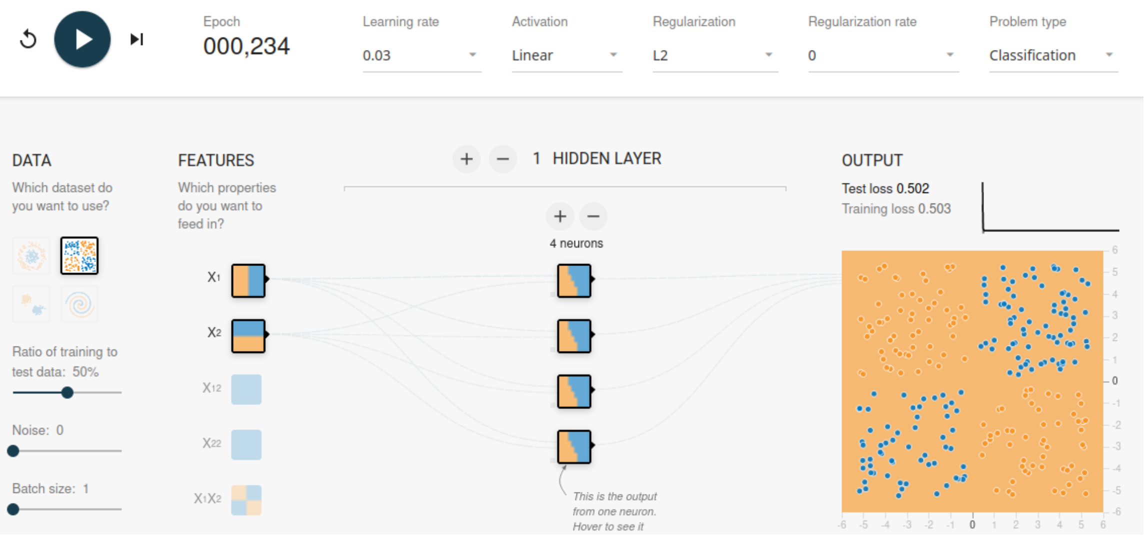

Figure 5: Linear hidden layers do not help (`http://playground.tensorflow.org`)

Adding non-linear function $g : \mathbb{R}^{d_1} \rightarrow \mathbb{R}^{d_1}$

$$f(\boldsymbol{x}) = g\left(\boldsymbol{x}\boldsymbol{W}^1 + \boldsymbol{b}^1\right)\boldsymbol{W}^2 + \boldsymbol{b}^2$$

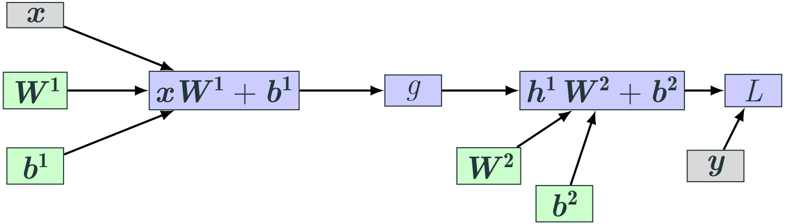

Figure 6: Computational graph; green circles are trainable parameters, gray are constant inputs

Non-linear function g: Rectified linear unit (ReLU) activation

$$\mathrm{ReLU}(z) = \begin{cases} 0 & \text{if } z < 0 \\ z & \text{if } z \geq 0 \end{cases}$$

or $\mathrm{ReLU}(z) = \max(0, x)$

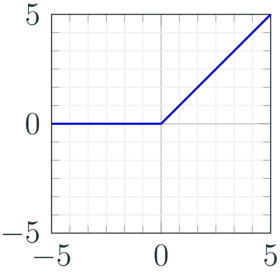

Figure 7: ReLU function

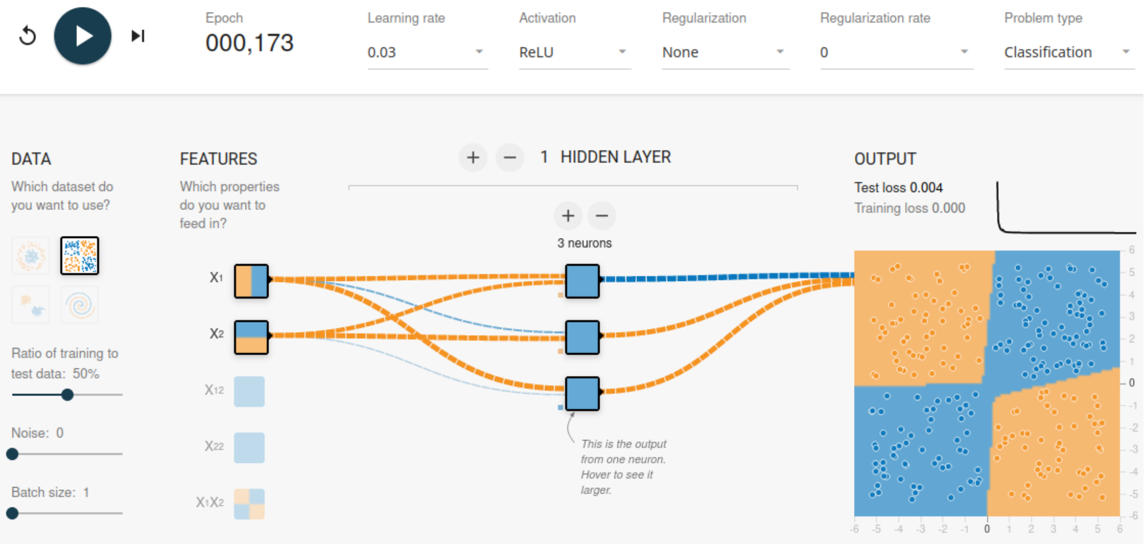

Figure 8: XOR solvable with, e.g., ReLU (`http://playground.tensorflow.org`)

XOR example in super-simplified sentiment classification

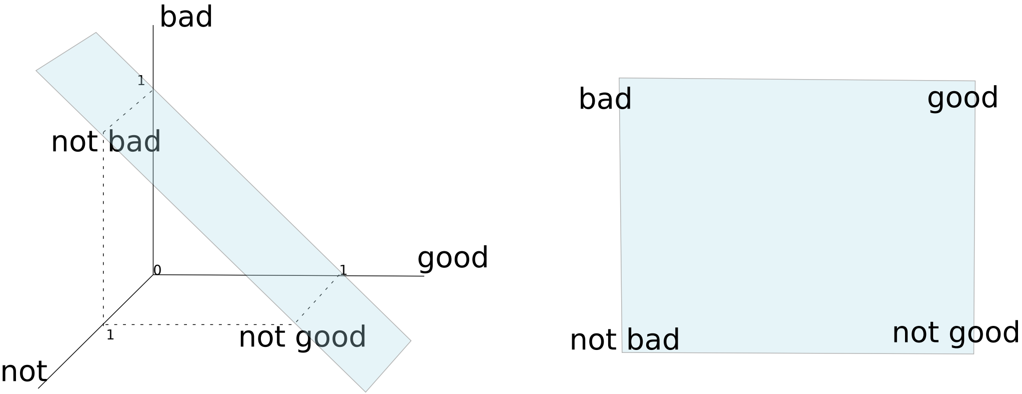

Figure 9: $V = \{\text{not}, \text{bad}, \text{good}\}$, binary features $\in \{0, 1\}$

Recap

Where we finished last time

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Take aways

- Binary classification as a linear function of words and a sigmoid
- Binary cross-entropy (logistic) loss
- Training as minimizing the loss using minibatch SGD and backpropagation
- Stacking layers and non-linear functions: MLP (Multi-Layer Perceptron)
- ReLU as a go-to activation function in NLP

License and credits

Credits

Ivan Habernal