Natural Language Processing with Deep Learning

Lecture 5 — Text generation 1: Language models and word embeddings

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PADERBORN UNIVERSITY

Natural Language Processing Group
Paderborn University
We focus on Trustworthy Human Language Technologies

www.trusthlt.org

Finishing previous lecture: Stacking layers, non-linearity

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Language models

- Probability refresher
- ‘Classical’ language models
- Neural language models

Word embeddings

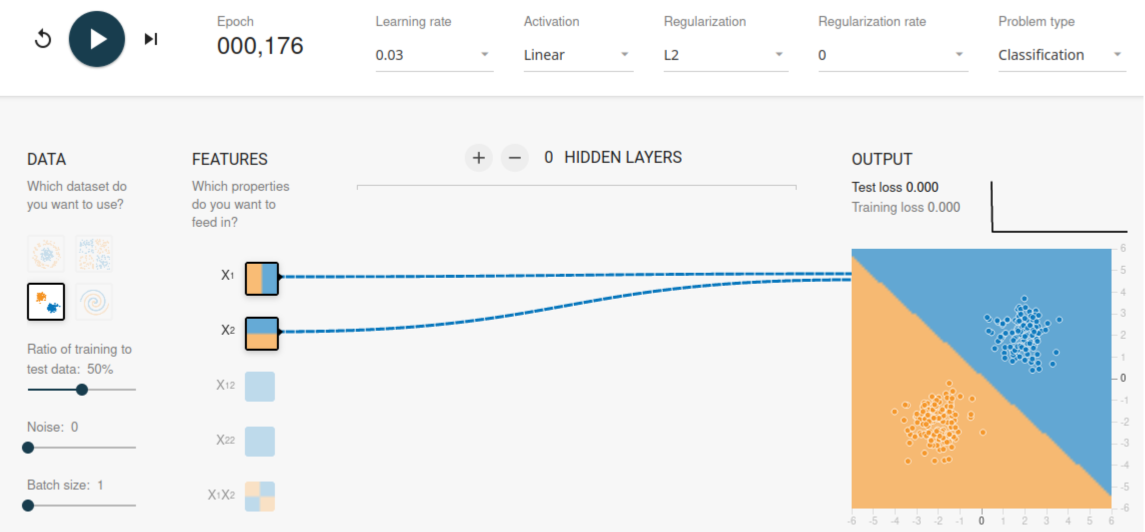

Figure 1: Linear model can tackle only linearly-separable problems (`http://playground.tensorflow.org`)

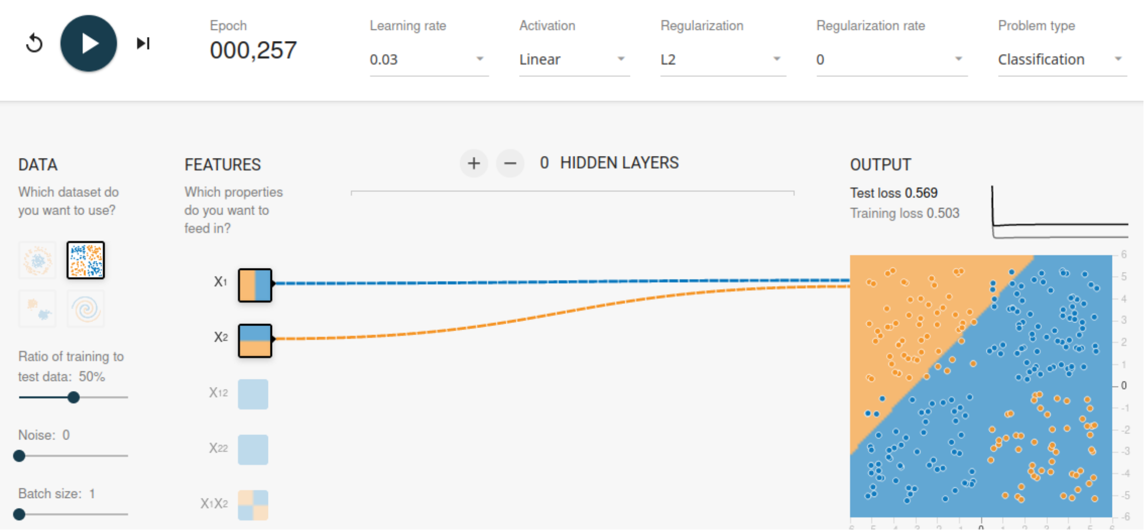

Figure 2: Linear model can tackle only linearly-separable problems (`http://playground.tensorflow.org`)

Stacking linear layers on top of each other — still linear!

$\boldsymbol{x} \in \mathbb{R}^{d_{in}} \qquad \boldsymbol{W^1} \in \mathbb{R}^{d_{in} \times d_1} \qquad \boldsymbol{b^1} \in \mathbb{R}^{d_1} \qquad \boldsymbol{W^2} \in \mathbb{R}^{d_1 \times d_{out}} \qquad \boldsymbol{b^2} \in \mathbb{R}^{d_{out}}$

$$f(\boldsymbol{x}) = \left(\boldsymbol{x}\boldsymbol{W}^1 + \boldsymbol{b}^1\right)\, \boldsymbol{W}^2 + \boldsymbol{b}^2$$

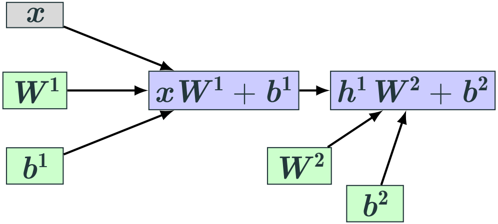

Figure 3: Computational graph; green boxes are trainable parameters, gray are constant inputs

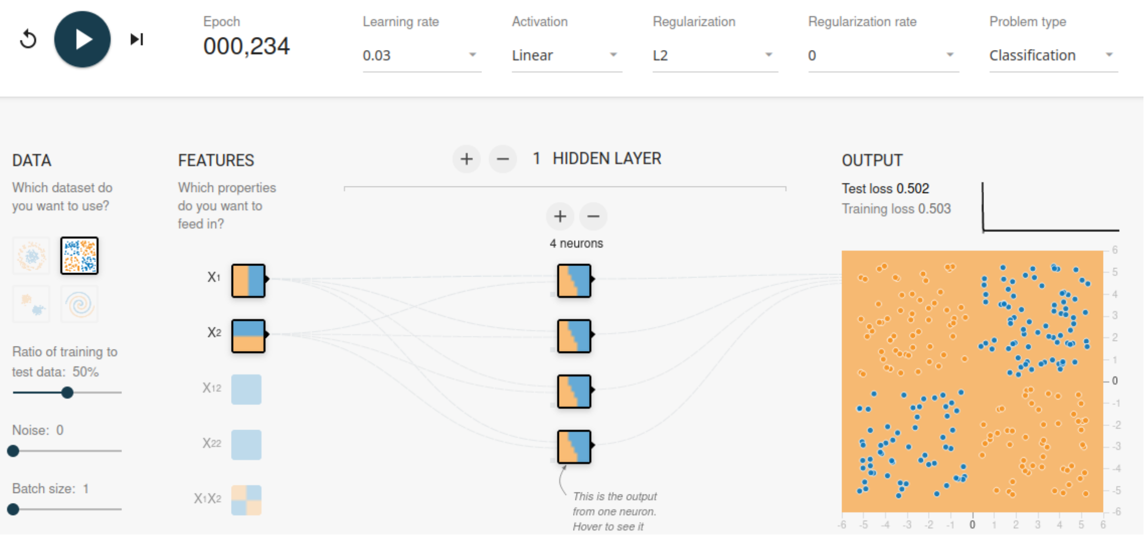

Figure 4: Linear hidden layers do not help (`http://playground.tensorflow.org`)

Adding non-linear function $g : \mathbb{R}^{d_1} \to \mathbb{R}^{d_1}$

$$f(\boldsymbol{x}) = g\left(\boldsymbol{x}\boldsymbol{W}^1 + \boldsymbol{b}^1\right)\ \boldsymbol{W}^2 + \boldsymbol{b}^2$$

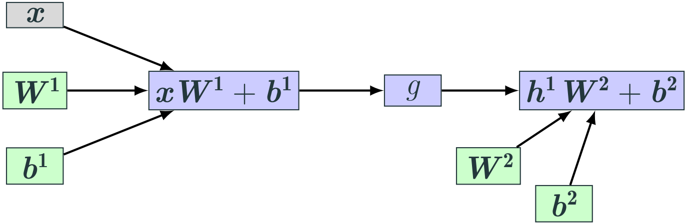

Figure 5: Computational graph; green boxes are trainable parameters, gray are constant inputs

Non-linear function g: Rectified linear unit (ReLU) activation

$$\text{ReLU}(z) = \begin{cases} 0 & \text{if } z < 0 \\ z & \text{if } z \geq 0 \end{cases}$$

or $\quad \text{ReLU}(z) = \max(0, x)$

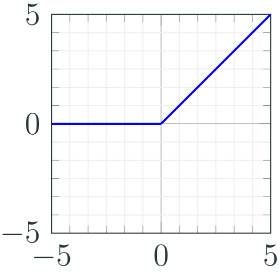

Figure 6: ReLU function

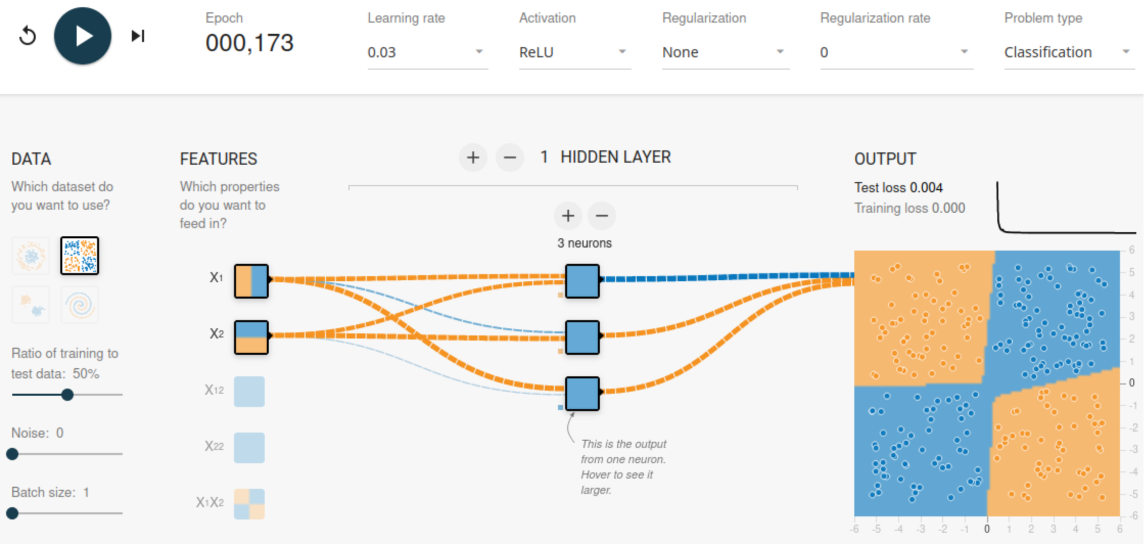

Figure 7: XOR solvable with, e.g., ReLU (`http://playground.tensorflow.org`)

XOR example in super-simplified sentiment classification

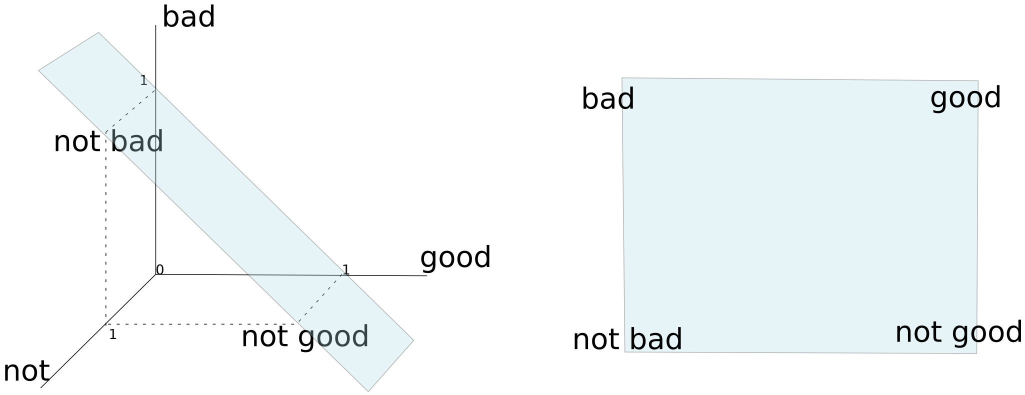

Figure 8: $V = \{\text{not}, \text{bad}, \text{good}\}$, binary features $\in \{0, 1\}$

Multi-layer perceptron (MLP)

$$f(\boldsymbol{x}) = \sigma\left(g\left(\boldsymbol{x}\boldsymbol{W}^1 + \boldsymbol{b}^1\right)\,\boldsymbol{W}^2 + \boldsymbol{b}^2\right)$$

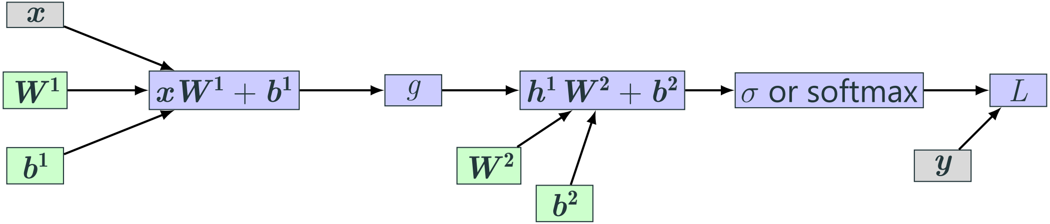

Figure 9: Computational graph; green boxes are trainable parameters, gray are constant inputs

Today: Language models and word embeddings

Finishing previous lecture: Stacking layers, non-linearity

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Probability refresher

Probability refresher 1

Categorical random variables

For example, the first word in a sentence

$W_1 \in \{\text{the}, \text{be}, \text{to}, \text{of}, \text{and}, \ldots\}$, we assume a fixed vocabulary

Probability refresher 1

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Probability distribution over random variables

For example, probability of *'the'* at position 1

$$\Pr(W_1 = w_1) = \Pr(W_1 = \text{the}) = 0.00024$$

Probability refresher 1

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$\Pr(W_1 = w_1) = \Pr(W_1 = \text{the}) = 0.00024$

Notation shortcuts: $\Pr(W_1 = w_1) \rightarrow P(W_1), P(\text{the})$, etc.

Probability refresher 2

Joint probability

For example, probability of *'the'* at position 1 and *'cat'* at position 2

$$\Pr(W_1 = \text{the} \cap W_2 = \text{cat}) = 0.0000074$$

Probability refresher 2

Joint probability

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Probability refresher 2

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Conditional probability

For example, probability of *'cat'* at position 2, **given** *'the'* at position 1

$\Pr(W_2 = \text{cat} | W_1 = \text{the}) = \frac{P(W_1, W_2)}{P(W_1)}$

Probability refresher 3

Independence

Two random variables X, Y are **independent** if and only if

$P(X, Y) = P(X) \cdot P(Y)$

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Conditional independence

Two random variables X, Y are **conditionally independent** given Z if and only if

$$P(X, Y|Z) = P(X|Z) \cdot P(Y|Z)$$

Language models

‘Classical’ language models

Goal of language modeling

Assign a probability to sentences in a language

Example

"What is the probability of seeing the sentence *the lazy dog barked loudly*?"

Assigns a probability for the likelihood of given word (or a sequence of words) to follow a sequence of words

Example

"What is the probability of seeing the word *barked* after the seeing sequence *the lazy dog*?

Language models formally

Sequence of words $w_{1:n} = w_1 w_2 w_3 \ldots w_n$ estimate

$$\Pr(w_{1:n}) = \Pr(w_1, w_2, \ldots, w_n)$$

Note: We misuse notation and usually omit the RVs

$$\Pr(W_1 = w_1, W_1 = w_2, \ldots, W_n = w_n)$$

We *factorize* the joint probability into a product

- One factorization is very useful: left-to-right

$$\Pr(w_{1:n}) = \Pr(w_1 | \text{<s>}) \Pr(w_2 | \text{<s>}, w_1) \Pr(w_3 | \text{<s>}, w_1, w_2) \cdots$$
$$\cdots \Pr(w_k | \text{<s>}, w_1, w_2, \ldots, w_{n-1})$$

Simplifications in ‘classical’ language models

Despite factorization, the last term of $\Pr(w_{1:n}) =$
$\Pr(w_1 | \text{<s>}) \Pr(w_2 | \text{<s>}, w_1) \Pr(w_3 | \text{<s>}, w_1, w_2) \cdots \Pr(w_k | \text{<s>}, w_1, w_2, \ldots, w_{n-1})$
still depends on all the previous words of the sequence

k-th order markov-assumption

The next word depends only on the last k words

$$\Pr(w_i | w_{1:i-1}) \approx \Pr(w_i | w_{i-k:i-1}) \qquad \text{(inclusive indexing!)}$$

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<s>_0 The_1 cat_2 sat_3 on_4 the_5 w_6

$i = 6, k = 2 \rightarrow \Pr(w_i|w_{1:i-1}) \approx \Pr(w_6|W_4 = \text{on}, W_5 = \text{the})$

Estimating probabilities in 'classical' language models

Maximum Likelihood Estimation (aka. counting and dividing)

$$\hat{P}_{\text{MLE}}(W_i = w | w_{i-k:i-1}) = \frac{\#(w_{i-k} \quad w_{i-k+1} \quad \ldots \quad w_{i-1} \quad w)}{\#(w_{i-k} \quad w_{i-k+1} \quad \ldots \quad w_{i-1})}$$

What if $\#(w_{i-k} \quad w_{i-k+1} \quad \ldots \quad w_{i-1}) = 0$**?**

Estimating probabilities in ‘classical’ language models

Maximum Likelihood Estimation (aka. counting and dividing)

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What if $\#(w_{i-k} \quad w_{i-k+1} \quad \ldots \quad w_{i-1}) = 0$**?**

- Add-alpha smoothing ($0 \leq \alpha \leq 1$)

$$\hat{P}_{\text{add-}\alpha}(W_i = w | w_{i-k:i-1}) = \frac{\#(w_{i-k} \ldots \quad w_{i-1} \quad w) + \alpha}{\#(w_{i-k} \quad \ldots \quad w_{i-1}) + \alpha |V|}$$

Evaluating language models: Perplexity

Recall: Trained LM tells us probability of 'sentence' s: $\Pr(s)$

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Perplexity of LM

$$2^{\text{cross-entropy}} = 2^{\left(-\frac{1}{n} \sum_{i=1}^{n} \log \Pr(s_i)\right)}$$

Shortcomings of n-gram language models

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Long-range dependencies

- To capture a dependency between the next word and the word 10 positions in the past, we need to see a relevant 11-gram in the text

Shortcomings of n-gram language models

Y. Goldberg (2017). ***Neural Network Methods for Natural Language Processing.*** Morgan & Claypool, p. 108

Long-range dependencies

- To capture a dependency between the next word and the word 10 positions in the past, we need to see a relevant 11-gram in the text

Lack of generalization across contexts

- Having observed *black car* and *blue car* does not influence our estimates of the event *red car* if we haven't see it before

Language models

Neural language models

Neural LMs

Let's build a neural network

- Input: a k-gram of words $w_{1:k}$
- Desired output: a probability distribution over the vocabulary V for the next word w_{k+1}

Embedding layer once again (recall last lecture)

If the input are symbolic **categorical features**

- e.g., words from a closed vocabulary

it is common to associate each possible feature value

- i.e., each word in the vocabulary

with a d-dimensional vector for some d

These vectors are also *parameters* of the model, and are trained jointly with the other parameters

Embedding layer: Lookup operation

The mapping from a symbolic feature values such as `word-number-48` to d-dimensional vectors is performed by an embedding layer (a lookup layer)

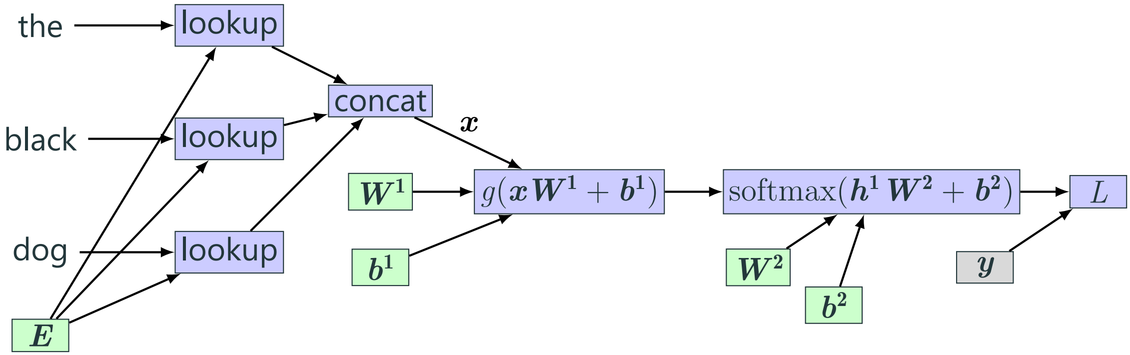
The parameters in an embedding layer are a matrix $\boldsymbol{W}^{|V| \times d}$, each row corresponds to a different word in the vocabulary

The lookup operation is then indexing $v(w)$, e.g.,

$$v(w) = v_{48} = \boldsymbol{E}_{[48,:]}$$

If the symbolic feature is encoded as a one-hot vector $\boldsymbol{x}$, the lookup operation can be implemented as the multiplication $\boldsymbol{x}\boldsymbol{E}$

Example network concatenating 3 words as embeddings ($d_w = 50$)



Each word $\in \mathbb{R}^{|V|}$ (one hot), $\boldsymbol{E} \in \mathbb{R}^{|V| \times 50}$, each lookup output $\in \mathbb{R}^{50}$, concat output $\boldsymbol{x} \in \mathbb{R}^{150}$

Neural LMs

Let's build a neural network

- Input: a k-gram of words $w_{1:k}$
- Desired output: a probability distribution over the vocabulary V for the next word w_{k+1}

Each input word w_k is associated with an embedding vector $v(w) \in \mathbb{R}^{d_w}$ (d_w — word embedding dimensionality)

Input vector $\boldsymbol{x}$ is a concatenation of k words

$$\boldsymbol{x} = [v(w_1); v(w_2); \ldots; v(w_k)]$$

Neural LMs

MLP with one (or more) hidden layers

$$
\begin{aligned}
v(w) &= \boldsymbol{E}_{w,:} \\
\boldsymbol{x} &= [v(w_1); v(w_2); \ldots; v(w_k)] \\
\boldsymbol{h} &= g(\boldsymbol{x}\boldsymbol{W}^{\mathbf{1}} + \boldsymbol{b}^{\mathbf{1}}) \\
\hat{\boldsymbol{y}} = \Pr(W_i | w_{1:k}) &= \operatorname{softmax}(\boldsymbol{h}\boldsymbol{W}^{\mathbf{2}} + \boldsymbol{b}^{\mathbf{2}})
\end{aligned}
$$

Output dimension: $\hat{\boldsymbol{y}} \in \mathbb{R}^{|V|}$

Training neural LMs

Where to get training examples?

Training examples are simply word k-grams from an unlabeled corpus

- Identities of the first $k-1$ words are used as features
- The last word is used as the target label for the classification

The model is trained using cross-entropy loss

Some advantages and limitations of neural LMs

$\approx$ linear increase in parameters with $k+1$ (better than 'classical' LMs) but

- The size of the output vocabulary affects the computation time
- The softmax at the output layer requires an expensive matrix-vector multiplication with the matrix $\boldsymbol{W^2} \in \mathbb{R}^{d_{\text{hid}} \times |V|}$, followed by $|V|$ exponentiations

Solutions: Hierarchical softmax, noise-contrastive estimation

Generating text with language models

We can generate ("sample") random sentences from the model according to their probability

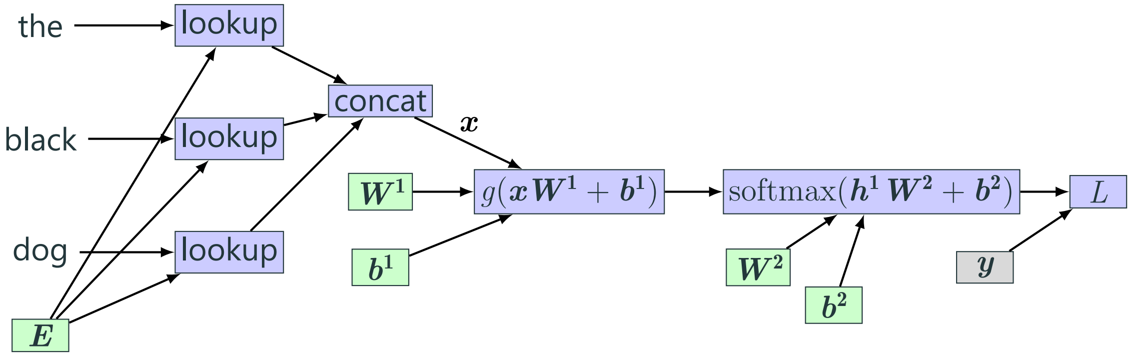
1. Predict a probability distribution over the vocabulary conditioned on the start symbol <s>
2. Draw a random word (the first word) according to the predicted distribution
3. Predict a probability distribution over the vocabulary conditioned on the start symbol and the first word
4. Draw a random word (the second word) according to the predicted distribution
5. Repeat until generated *end-of-sentence* symbol </s> (or <EOS>)

Sampling words — alternatives

Sampling (generating) the most probable word at each step might not be optimal globally

- Beam search — generate top k candidates at each step

Learned word representations as a by-product



Each row of $\boldsymbol{E}$ learns a word representation

Each column of $\boldsymbol{W}^2$ learns a word representation

Word embeddings

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Word embeddings as pre-trained word representation

Option A: We can initialize the embeddings matrix $\boldsymbol{E}$ randomly and learn during our supervised task

Option B: Use pre-trained word embeddings from task for which we have a lot of data

- Use self-supervised learning (create labeled data ‘for free’ using the next word prediction objective)
- Learned word embedding matrix plugged into our supervised task

Desired word embeddings properties: ‘Similar’ words have similar embeddings vectors

Recap

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Take aways

- Language modeling is an essential part of contemporary NLP
- Self-supervised models, unlabeled data, next word prediction
- Neural language models learn embedding of words

License and credits

Credits

Ivan Habernal