

# Natural Language Processing with Deep Learning

## Lecture 3 — Text classification 1: Log-linear models

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We focus on Trustworthy Human Language Technologies



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# Feedback

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## Feedback

### Motivation

Towards supervised machine learning on text data

Numerical representation of natural language text

Binary text classification

- Binary classification as a function

- Finding the best model's parameters

# Thank you for your feedback!

- Slow pace

# Motivation

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# What are we going to achieve

Example task: Binary sentiment classification into positive and negative

Recall the IMDB dataset

We will learn a simple yet powerful supervised machine learning model

- Known as logistic regression, maximum entropy classifier
- In fact, it is a single-layer neural network
- An essential important building block of deep neural networks

# Towards supervised machine learning on text data

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## Scalars, vectors, matrices

Lowercase letters represent scalars:  $x, y, b$

Bold lowercase letters represent vectors:  $\mathbf{w}, \mathbf{x}, \mathbf{b}$

Bold uppercase letters represent matrices:  $\mathbf{W}, \mathbf{X}$

## Indexing

$[\cdot]$  as the index operator of vectors and matrices

$b_{[i]}$  is the  $i$ -th element of vector  $\mathbf{b}$

$\mathbf{W}_{[i,j]}$  is the  $i$ -th row,  $j$ -th column of matrix  $\mathbf{W}$

## Sequences

$\mathbf{x}_{1:n}$  is a sequence of vectors  $\mathbf{x}_1, \dots, \mathbf{x}_n$

$[\mathbf{v}_1; \mathbf{v}_2]$  is vector concatenation

**Note! We use vectors as *row* vectors**

$$\mathbf{x} \in \mathbb{R}^d$$

Example  $d = 5$ :

$$\mathbf{x} = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \end{pmatrix}$$

which is simply a list (1-d array) of numbers  $(1, 2, 3, 4, 5)$



# Multiplication example

$$\mathbf{x} \in \mathbb{R}^{d_{in}} \quad \mathbf{W} \in \mathbb{R}^{d_{in} \times d_{out}} \quad \mathbf{b} \in \mathbb{R}^{d_{out}}$$

**Example:**  $\mathbf{y} = \mathbf{x}\mathbf{W} + \mathbf{b}$ ,  $d_{in} = 3$ ,  $d_{out} = 2$

$$\begin{pmatrix} x_{[1]} & x_{[2]} & x_{[3]} \end{pmatrix} \begin{pmatrix} w_{[1,1]} & w_{[1,2]} \\ w_{[2,1]} & w_{[2,2]} \\ w_{[3,1]} & w_{[3,2]} \end{pmatrix} + \begin{pmatrix} b_{[1]} & b_{[2]} \end{pmatrix} = \begin{pmatrix} y_{[1]} & y_{[2]} \end{pmatrix}$$

# Mult. simplified with dot product $u \cdot v = \sum_i u_{[i]} v_{[i]}$

**Example:**  $y = xW + b$ ,  $d_{in} = 3$ ,  $d_{out} = 1$

$$\mathbf{x} \in \mathbb{R}^{d_{in}} \quad \mathbf{W} \in \mathbb{R}^{d_{in} \times d_{out}} \quad \mathbf{b} \in \mathbb{R}^{d_{out}}$$

$$\begin{pmatrix} \mathbf{x}_{[1]} & \mathbf{x}_{[2]} & \mathbf{x}_{[3]} \end{pmatrix} \begin{pmatrix} \mathbf{W}_{[1,1]} \\ \mathbf{W}_{[2,1]} \\ \mathbf{W}_{[3,1]} \end{pmatrix} + b = y$$

**Equivalent dot product:**  $y = \mathbf{x} \cdot \mathbf{w} + b$ ,  $d_{in} = 3$ ,  $d_{out} = 1$

$$\mathbf{x} \in \mathbb{R}^{d_{in}} \quad \mathbf{w} \in \mathbb{R}^{d_{out}} \quad b \in \mathbb{R}$$

$$\begin{pmatrix} \mathbf{x}_{[1]} & \mathbf{x}_{[2]} & \mathbf{x}_{[3]} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{w}_{[1]} & \mathbf{w}_{[2]} & \mathbf{w}_{[3]} \end{pmatrix} + b = y$$

We often restrict ourselves to search over specific families of functions, e.g., the space of all linear functions with  $d_{in}$  inputs and  $d_{out}$  outputs

- By restricting to a specific hypothesis class, we are injecting the learner with **inductive bias** (a set of assumptions about the form of the desired solution)

Function  $f(\mathbf{x}) : \mathbb{R}^{d_{in}} \rightarrow \mathbb{R}^{d_{out}}$

$$f(\mathbf{x}) \text{ or } f(\mathbf{x}; \underbrace{\mathbf{W}, \mathbf{b}}_{\text{Explicit parameters}}) = \mathbf{x}\mathbf{W} + \mathbf{b}$$

where  $\mathbf{x} \in \mathbb{R}^{d_{in}}$        $\mathbf{W} \in \mathbb{R}^{d_{in} \times d_{out}}$        $\mathbf{b} \in \mathbb{R}^{d_{out}}$

Vector  $\mathbf{x}$  is the **input**, matrix  $\mathbf{W}$  and vector  $\mathbf{b}$  are the **parameters** — typically denoted  $\Theta = \mathbf{W}, \mathbf{b}$

## Goal of learning

Set the values of the parameters  $\mathbf{W}$  and  $\mathbf{b}$  such that the function behaves as intended on a collection of input values  $\mathbf{x}_{1:k} = \mathbf{x}_1, \dots, \mathbf{x}_k$  and the corresponding desired outputs  $\mathbf{y}_{1:k} = \mathbf{y}_1, \dots, \mathbf{y}_k$

Function  $f(\mathbf{x}) : \mathbb{R}^{d_{in}} \rightarrow \mathbb{R}$

$$f(\mathbf{x}) \text{ or } f(\mathbf{x}; \underbrace{\mathbf{w}, \mathbf{b}}_{\text{Explicit parameters}}) = \mathbf{x} \cdot \mathbf{w} + b$$

However, for binary text classification

- Our input is in the form of a natural language text
- Our labels are two categories, e.g., positive and negative

## Let's start with the labels

Very easy: Just arbitrarily map the categories into 0 and 1  
(e.g., negative = 0, positive = 1)

# Numerical representation of natural language text

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# Goal: Transform text into a fixed-size vector of real numbers

What's our setup:

$$f(\mathbf{x}) : \mathbb{R}^{d_{in}} \rightarrow \mathbb{R} \quad f(\mathbf{x}) = \mathbf{x} \cdot \mathbf{w} + b$$

What we need:

$$\mathbf{x} \in \mathbb{R}^{d_{in}}$$

What we have:

*One of my favorite movies ever, The Shawshank Redemption is a modern day classic as it tells the story of two inmates who become friends and find solace over the years in which this movie takes place. Based on a Stephen King novel, ...*

# What is a “word”?

Y. Goldberg (2017). *Neural Network Methods for Natural Language Processing*. Morgan & Claypool

A matter of debate among linguists, answer not always clear

Very simplistic definition: words are sequences of letters separated by whitespace

But: dog, dog?, dog., and dog ) would be different words

Better: words separated by whitespace or punctuation

A process called **tokenization** splits text into tokens based on whitespace and punctuation

- English: the job of the tokenizer is quite simple
- Hebrew, Arabic: sometimes without whitespace
- Chinese: no whitespaces at all



Symbols cat and Cat have the same meaning, but are they the same word?

Something like New York, is it two words, or one?

- We distinguish between words and tokens
- We refer to the output of a tokenizer as a token, and to the meaning-bearing units as words

## Keep in mind

We use the term **word** very loosely, and take it to be interchangeable with **token**.

In reality, the story is more complex than that.

We build a fix-sized static **vocabulary** (e.g., by tokenizing training data)

- Typical sizes: 20,000 – 100,000 words

Each word has a unique fixed index

$$V = \left( a_1 \quad \text{abandon}_2 \quad \dots \quad \text{cat}_{852} \quad \dots \quad \text{zone}_{2,999} \quad \text{zoo}_{3,000} \right)$$

# (Averaged) Bag-of-words

$$\mathbf{x} = \frac{1}{|D|} \sum_{i=1}^{|D|} \mathbf{x}^{D[i]}$$

$D[i]$  – word in doc  $D$  at position  $i$ ,  $\mathbf{x}^{D[i]}$  – one-hot vector

**Example: a cat sat**  $\rightarrow$  a, cat, sat

$$V = \begin{pmatrix} a_1 & abandon_2 & \dots & cat_{852} & \dots & zone_{2,999} & zoo_{3,000} \end{pmatrix}$$

$$\mathbf{a} = \mathbf{x}^{D[1]} = \begin{pmatrix} 1_1 & 0_2 & 0_3 & \dots & 0_{2,999} & 0_{3,000} \end{pmatrix}$$

$$\mathbf{cat} = \mathbf{x}^{D[2]} = \begin{pmatrix} 0_1 & \dots & 1_{852} & \dots & 0_{2,999} & 0_{3,000} \end{pmatrix}$$

$$\mathbf{sat} = \mathbf{x}^{D[3]} = \begin{pmatrix} 0_1 & \dots & 1_{2,179} & \dots & 0_{2,999} & 0_{3,000} \end{pmatrix}$$

## Averaged bag-of-words example: $\mathbf{x} \in \mathbb{R}^{3,000}$

**Example: a cat sat  $\rightarrow$  a, cat, sat**

$$\mathbf{a} = \mathbf{x}^{D[1]} = \begin{pmatrix} 1_1 & 0_2 & 0_3 & \dots & 0_{2,999} & 0_{3,000} \end{pmatrix}$$

$$\text{cat} = \mathbf{x}^{D[2]} = \begin{pmatrix} 0_1 & \dots & 1_{852} & \dots & 0_{2,999} & 0_{3,000} \end{pmatrix}$$

$$\text{sat} = \mathbf{x}^{D[3]} = \begin{pmatrix} 0_1 & \dots & 1_{2,179} & \dots & 0_{2,999} & 0_{3,000} \end{pmatrix}$$

$$\mathbf{x} = \frac{1}{|D|} \sum_{i=1}^{|D|} \mathbf{x}^{D[i]}$$

$$= \begin{pmatrix} 0.33_1 & 0_2 & \dots & 0_{851} & 0.33_{852} & 0_{853} & \dots & 0.33_{2,179} & \dots & 0_{3,000} \end{pmatrix}$$

# Out-of-vocabulary (UNK) tokens

P. Koehn (2020). **Neural Machine Translation**. (not freely available). Cambridge University Press

Words in a language are very unevenly distributed (Zipf's law)

- There is always a large 'tail' of rare words

When building the vocabulary, use the most frequent words, all others represented by an unknown token (UNK or OOV)

**Example vocabulary, most common 3,000 words and UNK**

$$V = \left( a_1 \quad \text{abandon}_2 \quad \dots \quad \text{zone}_{2,999} \quad \text{zoo}_{3,000} \quad \text{UNK}_{3,001} \right)$$

- In machine translation, how to translate the UNK word?

# Subword units: Byte-pair encoding

1. The words in the corpus are split into characters (marking original spaces with a special space character) — this is the initial vocabulary  $V$
2. The most frequent pair of characters is merged and added to  $V$
3. Repeat 2 for a fixed given number of times
4. Each of these steps increases  $V$  by one, beyond the original inventory of single characters

When done over large corpora with multiple languages and writing systems, BPE prevents OOV!

## Byte-pair encoding example on a toy corpus (part 1)

t h i s \_ f a t \_ c a t \_ w i t h \_ t h e \_  
h a t \_ i s \_ i n \_ t h e \_ c a v e \_ o f \_  
t h e \_ t h i n \_ b a t

Most frequent: t h (6 times), merge into a single token

th i s \_ f a t \_ c a t \_ w i t h \_ t h e \_ h a  
t \_ i s \_ i n \_ t h e \_ c a v e \_ o f \_ t h e  
\_ t h i n \_ b a t

Most frequent: a t (4 times), merge into a single token

th i s \_ f a t \_ c a t \_ w i t h \_ t h e \_ h a t  
\_ i s \_ i n \_ t h e \_ c a v e \_ o f \_ t h e \_  
t h i n \_ b a t

## Byte-pair encoding example on a toy corpus (part 2)

th i s \_ f at \_ c at \_ w i th \_ th e \_ h at  
\_ i s \_ i n \_ th e \_ c a v e \_ o f \_ th e \_  
th i n \_ b at

At the end of this process,  
the most frequent words  
will emerge as single tokens,  
while rare words consist of  
still unmerged subwords

Most frequent: th e (3 times), merge into a single token

th i s \_ f at \_ c at \_ w i th \_ the \_ h at \_  
i s \_ i n \_ the \_ c a v e \_ o f \_ the \_ th i  
n \_ b at

$V =$

{t, h, i, s, \_, f, a, c, w, e, n, v, o, f, b, th, at, the}



# SentencePiece: A variant of byte pair encoding

## Byte-pair example. Word splits indicated with @.

```
[the] [relationship] [between] [Obama]  
[and] [Net@@] [any@@] [ahu] [is] [not]  
[exactly] [friendly] [.]
```

SentencePiece escapes the whitespace with \_ and tokenizes the input into an arbitrary subword sequence

## SentencePiece example of "Hello world."

```
[Hello] [_wor] [ld] [.]
```

Lossless tokenization — all the information to reproduce the normalized text is preserved

T. Kudo and J. Richardson (2018). **"SentencePiece: A simple and language independent subword tokenizer and detokenizer for Neural Text Processing"**. In: *Proceedings of the 2018 Conference on Empirical Methods in Natural Language Processing: System Demonstrations*. Brussels, Belgium: Association for Computational Linguistics, pp. 66–71

# Recap: Transform text into a fixed-size vector of real numbers

What's our setup:

$$f(\mathbf{x}) : \mathbb{R}^{d_{in}} \rightarrow \mathbb{R} \quad f(\mathbf{x}) = \mathbf{x} \cdot \mathbf{w} + b$$

What we need:

$$\mathbf{x} \in \mathbb{R}^{d_{in}}$$

What we have:

*One of my favorite movies ever, The Shawshank Redemption is a modern day classic ...*

Simple solution:

- Bag-of-words (tokenized),  $d_{in} = |V|$

# Binary text classification

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# Binary text classification

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## Binary classification as a function

We have this linear function

$$f(\mathbf{x}) : \mathbb{R}^{d_{in}} \rightarrow \mathbb{R} \quad f(\mathbf{x}) = \mathbf{x} \cdot \mathbf{w} + b = \mathbf{x}_{[1]} \mathbf{w}_{[1]} + \dots + \mathbf{x}_{[d_{in}]} \mathbf{w}_{[d_{in}]} + b$$

**Derivatives wrt. parameters  $w$  and  $b$**

$$\frac{df}{d\mathbf{w}_{[i]}} = \mathbf{x}_{[i]} \quad \frac{df}{db} = 1$$

# Non-linear mapping to $[0, 1]$

We have this linear function

$$f(\mathbf{x}) : \mathbb{R}^{d_{in}} \rightarrow \mathbb{R} \quad f(\mathbf{x}) = \mathbf{x} \cdot \mathbf{w} + b$$

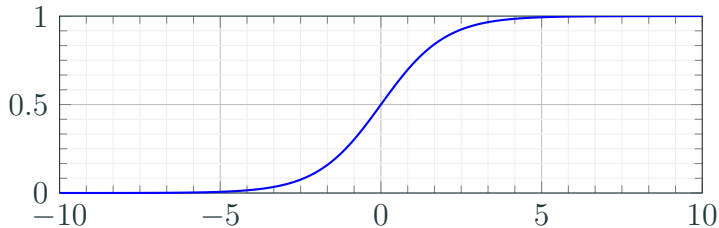
which has an unbounded range  $(-\infty, +\infty)$

However, each example's label is  $y \in \{0, 1\}$

# Sigmoid (logistic) function

**Sigmoid function**  $\sigma(t) : \mathbb{R} \rightarrow \mathbb{R}$

$$\sigma(t) = \frac{\exp(t)}{\exp(t) + 1} = \frac{1}{1 + \exp(-t)}$$



Symmetric function, range of  $\sigma(t) \in [0, 1]$ ,

# Sigmoid $\sigma(t) = \frac{1}{1+\exp(-t)}$

## Derivative of sigmoid wrt. its input

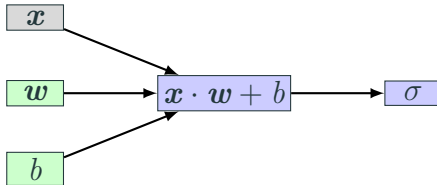
$$\begin{aligned}\frac{d\sigma}{dt} &= \frac{\exp(t) \cdot (1 + \exp(t)) - \exp(t) \cdot \exp(t)}{(1 + \exp(t))^2} \\ &= \dots \\ &= \sigma(t) \cdot (1 - \sigma(t))\end{aligned}$$



# Our binary text classification function

Linear function through sigmoid — log-linear model

$$\hat{y} = \sigma(f(\mathbf{x})) = \frac{1}{1 + \exp(-(\mathbf{x} \cdot \mathbf{w} + b))}$$



**Figure 1:** Computational graph; green circles are trainable parameters, gray are inputs

# Decision rule of log-linear model

Log-linear model  $\hat{y} = \sigma(f(\mathbf{x})) = \frac{1}{1 + \exp(-( \mathbf{x} \cdot \mathbf{w} + b ))}$

- Prediction = 1 if  $\hat{y} > 0.5$
- Prediction = 0 if  $\hat{y} < 0.5$

Natural interpretation: Conditional probability of prediction  
= 1 given the input  $\mathbf{x}$

$$\sigma(f(\mathbf{x})) = \Pr(\text{prediction} = 1 | \mathbf{x})$$

$$1 - \sigma(f(\mathbf{x})) = \Pr(\text{prediction} = 0 | \mathbf{x})$$

# Binary text classification

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**Finding the best model's parameters**

Loss function: Quantifies the loss suffered when predicting  $\hat{y}$  while the true label is  $y$  for a single example. In binary classification:

$$L(\hat{y}, y) : \mathbb{R}^2 \rightarrow \mathbb{R}$$

Given a labeled training set  $(\mathbf{x}_{1:n}, \mathbf{y}_{1:n})$ , a per-instance loss function  $L$  and a parameterized function  $f(\mathbf{x}; \Theta)$  we define the corpus-wide loss with respect to the parameters  $\Theta$  as the average loss over all training examples

$$\mathcal{L}(\Theta) = \frac{1}{n} \sum_{i=1}^n L(f(\mathbf{x}_i; \Theta), y_i)$$

$$\mathcal{L}(\Theta) = \frac{1}{n} \sum_{i=1}^n L(f(\mathbf{x}_i; \Theta), y_i)$$

The training examples are fixed, and the values of the parameters determine the loss

The goal of the training algorithm is to set the values of the parameters  $\Theta$ , such that the value of  $\mathcal{L}$  is minimized

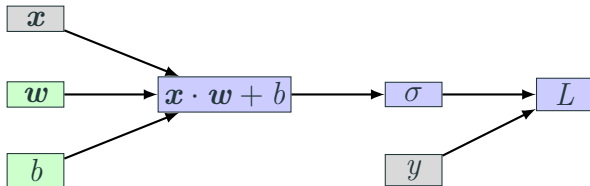
$$\hat{\Theta} = \underset{\Theta}{\operatorname{argmin}} \mathcal{L}(\Theta) = \underset{\Theta}{\operatorname{argmin}} \frac{1}{n} \sum_{i=1}^n L(f(\mathbf{x}_i; \Theta), y_i)$$

# Binary cross-entropy loss (logistic loss)

$$L_{\text{logistic}} = -y \log \hat{y} - (1 - y) \log(1 - \hat{y})$$

**Partial derivative wrt. input  $\hat{y}$**

$$\frac{dL_{\text{Logistic}}}{d\hat{y}} = - \left( \frac{y}{\hat{y}} - \frac{1-y}{1-\hat{y}} \right) = - \frac{y - \hat{y}}{\hat{y}(1 - \hat{y})}$$



**Figure 2:** Computational graph; green nodes are trainable parameters, gray are constant inputs

How can we minimize this function?

- Recall Lecture 2: (a) Gradient descent and (b) backpropagation

- 1: **function** SGD( $f(\mathbf{x}; \Theta)$ ,  $(\mathbf{x}_1, \dots, \mathbf{x}_n)$ ,  $(\mathbf{y}_1, \dots, \mathbf{y}_n)$ ,  $L$ )
- 2:     **while** stopping criteria not met **do**
- 3:         Sample a training example  $\mathbf{x}_i, \mathbf{y}_i$
- 4:         Compute the loss  $L(f(\mathbf{x}_i; \Theta), \mathbf{y}_i)$
- 5:          $\hat{\mathbf{g}} \leftarrow$  gradient of  $L(f(\mathbf{x}_i; \Theta), \mathbf{y}_i)$  wrt.  $\Theta$
- 6:          $\Theta \leftarrow \Theta - \eta_t \hat{\mathbf{g}}$
- 7:     **return**  $\Theta$

Loss in line 4 is based on a **single training example**  $\rightarrow$  a rough estimate of the corpus loss  $\mathcal{L}$  we aim to minimize

The noise in the loss computation may result in inaccurate gradients



```
1: function MBSGD( $f(\mathbf{x}; \Theta)$ ,  $(\mathbf{x}_1, \dots, \mathbf{x}_n)$ ,  $(\mathbf{y}_1, \dots, \mathbf{y}_n)$ ,  $L$ )
2:   while stopping criteria not met do
3:     Sample  $m$  examples  $\{(\mathbf{x}_1, \mathbf{y}_1), \dots (\mathbf{x}_m, \mathbf{y}_m)\}$ 
4:      $\hat{\mathbf{g}} \leftarrow 0$ 
5:     for  $i = 1$  to  $m$  do
6:       Compute the loss  $L(f(\mathbf{x}_i; \Theta), \mathbf{y}_i)$ 
7:        $\hat{\mathbf{g}} \leftarrow \hat{\mathbf{g}} + \text{gradient of } \frac{1}{m}L(f(\mathbf{x}_i; \Theta), \mathbf{y}_i) \text{ wrt. } \Theta$ 
8:      $\Theta \leftarrow \Theta - \eta_t \hat{\mathbf{g}}$ 
9:   return  $\Theta$ 
```

The minibatch size can vary in size from  $m = 1$  to  $m = n$

Higher values provide better estimates of the corpus-wide gradients, while smaller values allow more updates and in turn faster convergence

Lines 6+7: May be easily parallelized

# Recap

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- Tokenization is tricky
- Simplest representation of text as bag-of-word features
- Binary classification as a linear function of words and a sigmoid
- Binary cross-entropy (logistic) loss
- Training as minimizing the loss using minibatch SGD and backpropagation

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