v	$::=$		values
		true	true value
		false	false value
		n	numeric value
		$\lambda x : T.\ t$	abstraction
nv	$::=$		non − values
		x	variable
		$t_1\ t_2$	application
		$\mathsf{olet}\ x : T = t\ \mathsf{in}\ t$	overloading let
		$t :: T$	ascription
t	$::=$		terms
		v	values
		nv	non − values
w	$::=$		labeled − values
		$< \mathsf{true}, \mathsf{Bool} >$	labeled − true value
		$< \mathsf{false}, \mathsf{Bool} >$	labeled − false value
		$< n, \mathsf{Int} >$	labeled − numeric value
		$< \lambda x : T.\ t, \mathsf{Fun} > [s]$	labeled − abstraction
w^*	$::=$		labeled − multi − values
		$\{\overline{w}\}$	

c	$::=$		configurations
		w^*	
		$nv[s]$	
		$c :: T$	
		$\mathsf{olet}\ x : T = c\ \mathsf{in}\ c$	
		$c\ c$	
		error	
T	$::=$		types
		Int	type of integers
		Bool	type of booleans
		$T \to T$	type of functions
T^*	$::=$		multi − types
		$\{\overline{T}\}$	multi − type
Γ	$::=$		typing contexts
		$\varnothing$	empty context
		$\Gamma, x : T$	term variable binding
ϕ	$::=$		multi − typing contexts
		$\varnothing$	empty context
		$\phi, x : T^*$	term variable binding
s	$::=$		explicit substitutions
		$\bullet$	empty substitution
		$x \mapsto \{(\overline{w : T})\}, s$	variable substitution

Figure 1: Syntax of the simply typed lambda-calculus with overloading.

$$\boxed{c \longrightarrow c}$$

$$w^*[s] \longrightarrow w^* \qquad \text{(Multi} - \text{Values)}$$

$$x[x \mapsto \{(\overline{w : T_1})\}, s \longrightarrow \{\overline{w}\} \qquad \text{(VarOk)}$$

$$\frac{x \neq y}{x[y \mapsto \{(\overline{w : T_1})\}, s] \longrightarrow x[s]} \qquad \text{(VarNext)}$$

$$(t :: T)[s] \longrightarrow t[s] :: T \qquad \text{(AscSub)}$$

$$\frac{\mathsf{selvt}(w^*, T) = w}{w^* :: T \longrightarrow w} \qquad \text{(Asc)}$$

$$\frac{c \longrightarrow c'}{c :: T \longrightarrow c' :: T} \qquad \text{(Asc1)}$$

$$(\mathsf{olet}\ x : T_1 = t_1\ \mathsf{in}\ t_2)[s] \longrightarrow \mathsf{olet}\ x : T_1 = t_1[s]\ \mathsf{in}\ t_2[s] \qquad \text{(LetSub)}$$

$$\frac{\mathsf{selvt}(w^*, T_1) = w}{\mathsf{olet}\ x : T_1 = w^*\ \mathsf{in}\ t_2[s] \longrightarrow t_2[x \mapsto (w : T_1) \oplus s]} \qquad \text{(Let)}$$

$$\frac{c_1 \longrightarrow c_1'}{\mathsf{olet}\ x : T_1 = c_1\ \mathsf{in}\ t_2[s] \longrightarrow \mathsf{olet}\ x : T_1 = c_1'\ \mathsf{in}\ t_2[s]} \qquad \text{(Let1)}$$

$$(t_1\ t_2)[s] \longrightarrow t_1[s]\ t_2[s] \qquad \text{(AppSub)}$$

$$\frac{T_1 = \mathsf{tag}(w)}{< (\lambda x : T_1.\ t_2)[s], \mathsf{Fun} >\ w \longrightarrow ([x \mapsto w]t_2)[s]} \qquad \text{(App)}$$

$$\frac{< (\lambda x : T_1.\ t_2)[s], \mathsf{Fun} > \in w_1^* \qquad w_{2j} \in w_2^*}{w_1^*\ w_2^* \longrightarrow < (\lambda x : T_1.\ t_2)[s], \mathsf{Fun} >\ w_{2j}} \qquad \text{(App1)}$$

$$\frac{c_1 \longrightarrow c_1'}{c_1\ c_2 \longrightarrow c_1'\ c_2} \qquad \text{(App2)}$$

$$\frac{c \longrightarrow c'}{w^*\ c \longrightarrow w^*\ c'} \qquad \text{(App3)}$$

Figure 2: Configuration reduction rules.

$$\boxed{c \longrightarrow c}$$

$$x[\ ] \longrightarrow \mathsf{Error1} \qquad \text{(ErrVarFail)}$$

$$\frac{\mathsf{selvt}(w^*, T) = \mathsf{undef}}{w^* :: T \longrightarrow \mathsf{Error2}} \qquad \text{(ErrAsc)}$$

$$\frac{\mathsf{selvt}(w^*, T_1) = \mathsf{undef}}{\mathsf{olet}\ x : T_1 = w^*\ \mathsf{in}\ t_2[s] \longrightarrow \mathsf{Error3}} \qquad \text{(ErrLet)}$$

$$\frac{T_1 \neq \mathsf{tag}(w)}{< (\lambda x : T_1.\ t_2)[s], \mathsf{Fun} >\ \ w \longrightarrow \mathsf{Error4}} \qquad \text{(ErrApp)}$$

$$\frac{\nexists < (\lambda x : T_1.\ t_2)[s], \mathsf{Fun} > \in w_1^* \qquad w_{2j} \in w_2^*}{w_1^*\ w_2^* \longrightarrow \mathsf{Error5}} \qquad \text{(ErrApp1)}$$

$$\mathsf{error}\ c \longrightarrow \mathsf{error} \qquad \text{(Error1)}$$

$$w^*\ \mathsf{error} \longrightarrow \mathsf{error} \qquad \text{(Error2)}$$

Figure 3: Configuration reduction rules.