1 Languages

1.1 Flexible Language

Flexible Language:

- Non deterministic semantic, with branches that reduce to a value and other ending in error.
- Name error detection.
- Type error detection.
- Without any type information in the syntax of the language.
- Uses the explicit substitution. In the case of the mlet is used the explicit substitution because the implicit substitution of a variable by a value would eliminate the overloading.

Characterization of the errors for Flexible Language:

- Name error is detected if a variable is evaluated in the empty environment.
- Type error is detected if the operators not or add1 are applied with parameters that are not boolean or numeric value, respectively. Also, if the left side of the function application is not a lambda.

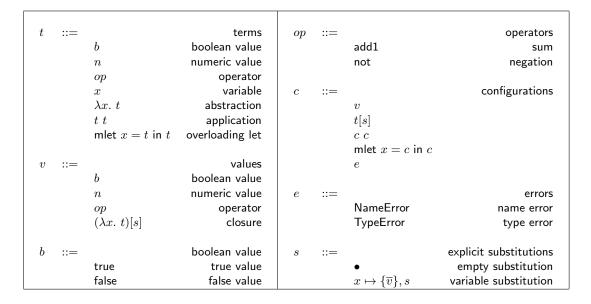


Figure 1: Syntax of the Flexible Language.

$$\begin{array}{c} \boxed{c \longrightarrow c} \\ \\ b[s] \longrightarrow b \\ \\ n[s] \longrightarrow n \\ \\ op[s] \longrightarrow op \\ \\ x[x \mapsto \{\overline{v}\}, s] \longrightarrow v_i \\ \\ \hline x[x \mapsto \{\overline{v}\}, s] \longrightarrow v_i \\ \\ \hline x[y \mapsto \{\overline{v}\}, s] \longrightarrow x[s] \\ \\ \text{mlet } x = v \text{ in } t_2[s] \longrightarrow t_2[x \mapsto v \oplus s] \\ \\ (\lambda x. \ t_2)[s] \ v \longrightarrow t_2[x \mapsto v \oplus s] \\ \\ \text{add1} \ n \longrightarrow n+1 \\ \\ \text{not } b \longrightarrow \neg \ b \\ \\ \text{(Negation)} \\ \end{array}$$

Figure 2: Reduction rules for Flexible Language.

$$(\mathsf{mlet}\ x = t_1\ \mathsf{in}\ t_2)[s] \longrightarrow \mathsf{mlet}\ x = t_1[s]\ \mathsf{in}\ t_2[s] \qquad (\mathsf{LetSub})$$

$$(t_1\ t_2)[s] \longrightarrow t_1[s]\ t_2[s] \qquad (\mathsf{AppSub})$$

Figure 3: Substitution rules for Flexible Language.

Definition 1 (\oplus). Given an environment s and a variable binding $x \mapsto v_1$, the operator \oplus is defined as follows:

$$s \oplus x \mapsto v_1 = \begin{cases} x \mapsto \{v_1\} & s = \emptyset \\ x \mapsto \{\overline{v}\} \cup \{v_1\}, s' & s = x \mapsto \{\overline{v}\}, s' \\ y \mapsto \{\overline{v}\}, s' \oplus x \mapsto v_1 & s = y \mapsto \{\overline{v}\}, s' \end{cases}$$

1.2 Tag Driven Language

Tag Driven Language:

- Non deterministic semantic, with branches that reduce to a value and other ending in error.
- name error detection.
- Type error detection.
- Dispatch error detection.
- Without any type information in the syntax of the language.
- Semantic "tag driven", introducing flat tag.

Characterization of the errors for Tag Driven Language:

Figure 4: Congruence rules for Flexible Language.

Figure 5: Propagation error rules for Flexible Language.

- Name and type error are detected in the same cases that the Flexible Language.
- Dispatch error is detected if the operators not or add1 are applied with overloaded parameters that do not have instance with tag Bool or Int in the environment, respectively. Also, if the left side of the function application is an overloaded variable, but does not exist an instance with tag Fun in the environment.

Definition 2 (lookup). The relation lookup is defined as follows:

$$\mathsf{lookup} = \{(x, s, S, v) \mid x \mapsto v \in \mathsf{flat}(s) \land \mathsf{tag}(v) = S\}$$

Definition 3 (flat). The function flat is defined as follows:

$$\mathsf{flat}(s) = \begin{cases} \varnothing & s = \varnothing \\ x \mapsto v_1 \cdots, x \mapsto v_n, \mathsf{flat}(s') & s = x \mapsto \{\overline{v}\}, s' \end{cases}$$

Definition 4 (tag). The function tag is defined as follows:

$$\mathsf{tag}(v) = \begin{cases} \mathsf{Int} & v = n \\ \mathsf{Bool} & v = b \\ \mathsf{Fun} & v = \lambda x. \ t \vee v = op \end{cases}$$

1.3 Tag Driven Language with ascription

1.4 Strict Language

Strict Language:

Deterministic semantic. With the use of multi-values a program can reduce to a set of value.

$$x[\] \longrightarrow \mathsf{NameError} \qquad \qquad (\mathsf{NameError})$$

$$\underbrace{\begin{array}{c} v_1 \neq \lambda x. \ t \lor v_1 \neq op \\ \hline v_1 \ v_2 \longrightarrow \mathsf{TypeError} \end{array}}_{\mathsf{add1} \ v \longrightarrow \mathsf{TypeError}} \qquad (\mathsf{TypeErrSum})$$

$$\underbrace{\begin{array}{c} v \neq n \\ \hline \mathsf{add1} \ v \longrightarrow \mathsf{TypeError} \end{array}}_{\mathsf{not} \ v \longrightarrow \mathsf{TypeError}} \qquad (\mathsf{TypeErrNegation})$$

Figure 6: Error rules for Flexible Language.

S	::=		tags
		Int	integer tag
		Bool	boolean tag
		Fun	function tag
e	::=		errors
		NameError	name error
		TypeError	type error
		${\sf DispatchError}$	dispatch error

Figure 7: Syntax of the Tag Driven Language.

- Name error detection.
- Type error detection.
- Dispatch error detection.
- Ambiguity error detection.

Characterization of the errors for Strict Language:

- Free variable and type error are detected in the same cases that the Tag Driven Language with ascription.
- Ambiguity error is detected if the left side of the function application have more than one instance with Fun tag or in the right side have more than one value for the application. Strict detection of ambiguity.

The only rule that change is:
$$\frac{c \longrightarrow c'}{w \ c \longrightarrow w \ c'}$$
 (App2)

Definition 5 (\oplus). Given an environment s and a variable binding $x \mapsto v_1$, the operator \oplus is defined as follows:

$$s \oplus_1 x \mapsto w = \begin{cases} x \mapsto w & s = \varnothing \\ x \mapsto w' \cup w, s' & s = x \mapsto w', s' \\ y \mapsto w', s' \oplus x \mapsto w & s = y \mapsto w', s' \end{cases}$$
$$s \oplus_2 x \mapsto w = \begin{cases} x \mapsto w & s = \varnothing \\ x \mapsto w' \cup w, s' & s = x \mapsto w', s' \\ y \mapsto w', s' \oplus_2 x \mapsto w & s = y \mapsto w', s' \end{cases}$$

$$\begin{array}{c} \cdots \\ \hline \frac{\mathsf{lookup}(x_1,[s_1],\mathsf{Fun},v_1)}{x_1[s_1] \ v_2 \longrightarrow v_1 \ v_2} \\ \hline \frac{\mathsf{lookup}(x,[s],\mathsf{Int},n)}{\mathsf{add1} \ x[s] \longrightarrow \mathsf{add1} \ n} \\ \hline \frac{\mathsf{lookup}(x,[s],\mathsf{Bool},b)}{\mathsf{not} \ x[s] \longrightarrow \mathsf{not} \ b} \end{array} \tag{\mathsf{NegationVar}}$$

Figure 8: Reduction rules for Tag Driven Language.

Figure 9: Congruence rules for Tag Driven Language.

Definition 6 (filter (\cdot, \cdot)). The function filter is defined as follows:

$$\mathsf{filter}(w, S) = \{ v \mid v \in w \land \mathsf{tag}(v) = S \}$$

1.5 Overloading Language

- Deterministic semantic. With the use of multi-values a program can reduce to a set of value.
- Type error detection.
- Dispatch error detection.
- Ambiguity error detection.
- Type annotation in lambda functions, mlet and ascription.
- More expressive than Strict Language, with the use structural tags.
- Do not support context-dependent overloading.
- Como no esta verificacda la información de tipo, no se puede decir nada acerca de la semantica.

Definition 7 (tag). The function tag is defined as follows:

$$\label{eq:tag} \mathsf{tag}(v) = \begin{cases} \mathsf{Int} & v = n \\ \mathsf{Bool} & v = b \\ T_1 \to T_2 & v = ((\lambda x.\ t_2)^{T_1 \to T_2})[s] \end{cases}$$

Figure 10: Error rules for Tag Driven Language.

Figure 11: Syntax of the Tag Driven Language with ascriptions.

Definition 8 (filter (\cdot, \cdot)). The function filter is defined as follows:

$$filter(w,T) = \{v \mid v \in w \mid tag(v) = T\}$$

Definition 9 (lookup). The function lookup is defined as follows:

$$\mathsf{lookup}(w_1, w_2) = \{(v_1, v_2) \mid v_1 \in w_1 \land v_2 \in w_2 \land \mathsf{tag}(v_1) = T_1 \to T_2 \land \mathsf{tag}(v_2) = T_1\}$$

1.6 Static semantic for Overloading Language

Definition 10 (\oplus). Given a multi-type context ϕ and a pair (x : T), the operator \oplus is defined as follows:

$$\phi \oplus (x:T) = \begin{cases} x:\{T\} & \phi = \varnothing \\ \phi', x:(T^* \cup \{T\}) & \phi = \phi', x:T^* \\ \phi' \oplus (x:T), y:T^* & \phi = \phi', y:T^* \end{cases}$$

Definition 11 ($\Gamma(s)$). The typing context built from a substitution s, writing $\Gamma(s)$, it is defined as follows:

$$\Gamma(s) = \begin{cases} \varnothing & s = \bullet \\ \Gamma(s'), x : T & s = (x, v) : s' \land \Gamma \vdash_c v : T \end{cases}$$

$$\begin{array}{c} \dots \\ \frac{\operatorname{tag}(v) = S}{v :: S \longrightarrow v} \\ \\ (t :: S)[s] \longrightarrow t[s] :: S \\ \\ \frac{\operatorname{lookup}(x, [s], S, v)}{x[s] :: S \longrightarrow v} \\ \\ \frac{c \longrightarrow c' \quad \operatorname{notVal_Var}(c)}{c :: S \longrightarrow c' :: S} \\ \\ \frac{\operatorname{error} :: S \longrightarrow \operatorname{error}}{x[s] :: S \longrightarrow \operatorname{NameError}} \\ \\ \frac{\operatorname{tag}(v) \neq S}{v :: S \longrightarrow \operatorname{TypeError}} \\ \\ \frac{-\operatorname{lookup}(x, [s], S, v)}{x[s] :: S \longrightarrow v} \\ \end{array} \right. \tag{Asc1}$$

Figure 12: Rules for Tag Driven Language with ascriptions.

Figure 13: Syntax of the Strict Language.

$$\begin{array}{c} c \longrightarrow c \\ \\ b[s] \longrightarrow b \\ \\ n[s] \longrightarrow n \\ \\ op[s] \longrightarrow op \\ \\ (Op) \\ \{\overline{v}\}[s] \longrightarrow \{\overline{v}\} \\ \\ (MultiValue) \\ \\ x[x \mapsto w, s] \longrightarrow w \\ \\ (VarOk) \\ \\ \hline \frac{x \neq y}{x[y \mapsto w, s] \longrightarrow x[s]} \\ \\ \hline \frac{filter(w, S) = w' \quad w' \neq \emptyset}{w :: S \longrightarrow w'} \\ \\ mlet \ x = w \ in \ t_2[s] \longrightarrow t_2[x \mapsto w \oplus s] \\ \\ (\lambda x. \ t_2)[s] \ w \longrightarrow t_2[x \mapsto w \oplus s] \\ \hline (\lambda x. \ t_2)[s] \ w \longrightarrow t_2[x \mapsto w \oplus s] \\ \hline \frac{filter(\{\overline{v}\}, \operatorname{Fun}) = \{v_1\}}{\{\overline{v}\} \ w_2 \longrightarrow v_1 \ w_2} \\ \hline \frac{filter(w, \operatorname{Int}) = \{\overline{n}\}}{\operatorname{add} 1 \ w \longrightarrow \{\overline{n}+1\}} \\ \hline \frac{filter(w, \operatorname{Bool}) = \{\overline{b}\}}{\operatorname{not} \ w \longrightarrow \{\overline{n}, \overline{b}\}} \\ \hline \end{array} \qquad \text{(Negation)}$$

Figure 14: Reduction rules for Strict Language.

Figure 15: Reduction rules for Strict Language.

Figure 16: Error rules for Strict Language.

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values
                                               terms
                                     boolean value
                                                                         b
                                                                                                   boolean value
                                     numeric value
                                                                                                   numeric value
                                            operator
                                                                                                         operator
                                                                         ((\lambda x.\ t)^{T\to T})[s]
                                         abstraction
                                                                                                           closure
                                            variable
                                         application
                                                                 ::=
                                                                                                   configurations
            \mathsf{mlet}\ x = t\ \mathsf{in}\ t
                                    overloading let
                                                                         w
                                          ascription
                                                                         t[s]
            t :: T
T
     ::=
                                               types
                                                                         \mathsf{mlet}\ x:T=c\ \mathsf{in}\ c
                                   type of integers
            Int
                                  type of booleans
                                                                         c :: T
            Bool
            T \to T
                                  type of functions
```

Figure 17: Syntax of the Overloading Language.

$$w[s] \longrightarrow w \qquad (\text{MultiValue})$$

$$x[x \mapsto w, s] \longrightarrow w \qquad (\text{VarOk})$$

$$\frac{x \neq y}{x[y \mapsto w, s] \longrightarrow x[s]} \qquad (\text{VarNext})$$

$$s \mapsto t[s] \longrightarrow t[s] :: T \qquad (\text{AscSub})$$

$$(\text{Inlet } x : T_1 = t_1 \text{ in } t_2)[s] \longrightarrow \text{mlet } x : T_1 = t_1[s] \text{ in } t_2[s] \qquad (\text{LetSub})$$

$$(t_1 t_2)[s] \longrightarrow t_1[s] t_2[s] \qquad (\text{AppSub})$$

$$\frac{\text{filter}(w, S) = \{v\}}{w :: T \longrightarrow \{v\}} \qquad (\text{Asc})$$

$$\frac{\text{filter}(w, T_1) = \{v\}}{w :: T \longrightarrow \{v\}} \qquad (\text{Let})$$

$$\frac{((((\lambda x. t_2)^{T_1 \to T_2})[s], v_2) = \text{lookup}(w_1, w_2)}{w_1 w_2 \longrightarrow ([x \mapsto v_2]t_2)[s]} \qquad (\text{App})$$

$$\frac{\text{filter}(w, \text{Int}) = \{n\}}{\text{add1 } w \longrightarrow \{n+1\}} \qquad (\text{Sum})$$

$$\frac{\text{filter}(w, \text{Bool}) = \{b\}}{\text{not } w \longrightarrow \{\neg b\}} \qquad (\text{Negation})$$

$$\frac{c \longrightarrow c'}{c :: T \longrightarrow c' :: T} \qquad (\text{Asc}1)$$

$$\frac{c_1 \longrightarrow c'_1}{c_1 c_2 \longrightarrow c'_1 c_2} \qquad (\text{App}1)$$

$$\frac{c \longrightarrow c'}{v c \longrightarrow v c'} \qquad (\text{App}2)$$

Figure 18: Reduction rules for Overloading Language.

Figure 19: Syntax for Overloading Language with static semantic.

$$\Gamma; \phi \vdash b : \{\mathsf{Bool}\} \qquad (\mathsf{TBool})$$

$$\Gamma; \phi \vdash n : \{\mathsf{Int}\} \qquad (\mathsf{TInt})$$

$$\Gamma; \phi \vdash \mathsf{not} : \{\mathsf{Bool} \to \mathsf{Bool}\} \qquad (\mathsf{TNegation})$$

$$\Gamma; \phi \vdash \mathsf{add1} : \{\mathsf{Int} \to \mathsf{Int}\} \qquad (\mathsf{TVar}\Gamma)$$

$$\frac{x : T \in \Gamma}{\Gamma; \phi \vdash x : \{T\}} \qquad (\mathsf{TVar}\Gamma)$$

$$\frac{x : T^* \in \phi}{\Gamma; \phi \vdash x : T^*} \qquad (\mathsf{TVar}\phi)$$

$$\frac{x \notin \mathsf{dom}(\Gamma \cup \phi)}{\Gamma, x : T_1; \phi \vdash t_2 : T_2^*} \qquad T_2 \in T_2^*}{\Gamma \vdash_c (\lambda x. \ t_2)^{T_1 \to T_2} : \{T_1 \to T_2\}} \qquad (\mathsf{TAbs})$$

$$\frac{\Gamma; \phi \vdash t : T^* \qquad T \in T^*}{\Gamma; \phi \vdash t : T : \{T\}} \qquad (\mathsf{TAsc})$$

$$\frac{x \notin \mathsf{dom}(\Gamma) \qquad \Gamma; \phi \vdash t_1 : T_1^*}{\Gamma; \phi \vdash \mathsf{mlet} \ x : T_1 = t_1 \ \mathsf{in} \ t_2 : T_2^*} \qquad (\mathsf{TLet})$$

$$\Gamma; \phi \vdash \mathsf{tot} : T_1^* \qquad \Gamma; \phi \vdash \mathsf{tot} : T_2^* \qquad (\mathsf{TApp})$$

Figure 20: Term typing rules.

$$\Gamma \vdash_c b : \{\mathsf{Bool}\} \qquad (\mathsf{CTBool})$$

$$\Gamma \vdash_c b[s] : \{\mathsf{Bool}\} \qquad (\mathsf{CSTBool})$$

$$\Gamma \vdash_c n : \{\mathsf{Int}\} \qquad (\mathsf{CTInt})$$

$$\Gamma \vdash_c n[s] : \{\mathsf{Int}\} \qquad (\mathsf{CSTInt})$$

$$\Gamma \vdash_c \mathsf{not} : \{\mathsf{Bool} \to \mathsf{Bool}\} \qquad (\mathsf{CTNegation})$$

$$\Gamma \vdash_c \mathsf{not} : \{\mathsf{Bool} \to \mathsf{Bool}\} \qquad (\mathsf{CSTNegation})$$

$$\Gamma \vdash_c \mathsf{not} [s] : \{\mathsf{Bool} \to \mathsf{Bool}\} \qquad (\mathsf{CSTNegation})$$

$$\Gamma \vdash_c \mathsf{not} [s] : \{\mathsf{Bool} \to \mathsf{Int}\} \qquad (\mathsf{CTSum})$$

$$\Gamma \vdash_c \mathsf{not} [s] : \{\mathsf{Int} \to \mathsf{Int}\} \qquad (\mathsf{CTSum})$$

$$\Gamma \vdash_c \mathsf{not} [s] : \{\mathsf{Int} \to \mathsf{Int}\} \qquad (\mathsf{CTSum})$$

$$\Gamma \vdash_c \mathsf{not} [s] : \{\mathsf{Int} \to \mathsf{Int}\} \qquad (\mathsf{CTSum})$$

$$\frac{x : T \in \Gamma}{\Gamma \vdash_c x[s] : T^*} \qquad (\mathsf{CTVar}\Gamma)$$

$$\frac{x : T \in \varphi(s)}{\Gamma \vdash_c x[s] : T^*} \qquad (\mathsf{CTVar}\Gamma)$$

$$\frac{x : T \in \varphi(s)}{\Gamma \vdash_c x[s] : T^*} \qquad (\mathsf{CTVar}\varphi)$$

$$\frac{x \notin \mathsf{dom}(\Gamma \cup \varphi(s))}{\Gamma \vdash_c x[s] : T^*} \qquad (\mathsf{CTAbs})$$

$$\frac{\Gamma \vdash_c t[s] : T : T^*}{\Gamma \vdash_c (t : T)[s] : T^*} \qquad (\mathsf{CTAsc})$$

$$\frac{\Gamma \vdash_c t[s] : T : T^*}{\Gamma \vdash_c c : T : \{T\}} \qquad (\mathsf{CTAsc})$$

$$\frac{\Gamma \vdash_c \mathsf{mlet} x : T_1 = t_1[s] \; \mathsf{in} \; t_2[s] : T^*}{\Gamma \vdash_c (\mathsf{mlet} x : T_1 = t_1[s] \; \mathsf{in} \; t_2[s] : T^*} \qquad (\mathsf{CTLet})$$

$$\frac{\Gamma \vdash_c \mathsf{mlet} x : T_1 = t_1[s] \; \mathsf{in} \; t_2[s] : T^*}{\Gamma \vdash_c \mathsf{mlet} x : T_1 = c_1 \; \mathsf{in} \; t_2[s] : T^*} \qquad (\mathsf{CTLet})$$

$$\frac{\Gamma \vdash_c \mathsf{mlet} x : T_1 = c_1 \; \mathsf{in} \; t_2[s] : T^*}{\Gamma \vdash_c \mathsf{mlet} x : T_1 = c_1 \; \mathsf{in} \; t_2[s] : T^*} \qquad (\mathsf{CTCApp})$$

$$\frac{\Gamma \vdash_c c_1 : T_1^*}{\Gamma \vdash_c c_1 : t_1^*} \qquad \Gamma \vdash_c c_2 : T_2^*}{\Gamma \vdash_c c_1 : t_1^*} \qquad (\mathsf{CTCApp})$$

$$\frac{\Gamma \vdash_c c_1 : T_1^*}{\Gamma \vdash_c c_1 : t_2 : t_1^*} \qquad \Gamma \vdash_c c_2 : T_2^*}{\Gamma \vdash_c c_1 : c_1 : t_2 : t_1^*} \qquad (\mathsf{CSTApp})$$

Figure 21: Configuration typing rules.