Control Assignment 2

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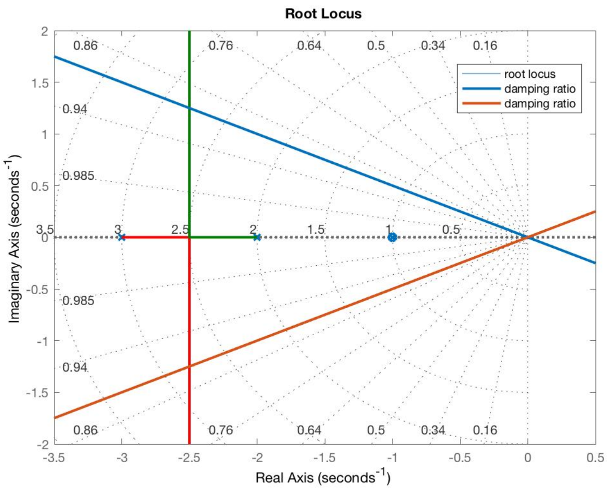
1 Uncompensated System

The two dominant poles are s=-2, -3, as s=-1 cancel out with the zero, due to pole zero cancellation.

1.1 Close Loop Poles



The close loop poles that satisfy the damping ratio requirement $\zeta = 0.5$ can be found be observing the root locus map and using trigonometric. The intersections between the root locus curve and the damping ratio line are the desired close loop poles. According to the root locus plot, the Re(s) must be -2.5 to lie on the root locus. Using trigonometric, $Img(s) = \pm 2.5cos(60) = 1.25$
Therefore, the close loop poles is at s = -2.5 ± j1.25

1.2 Gain at Close Loop Poles

The gain K is calculated by multiplying the distance between the open loop poles and the close loop poles, divided by the product of zeros

$$K = \frac{\prod poles}{\prod zeros} = \mid p_1 \mid\mid p_2 \mid = \sqrt{(2.5-2)^2 + 1.25^2 + (3-2.5)^2 + 1.25^2} = 1.813 \tag{1}$$

1.3 System Performance

As this is a lift system, the input is a step input. It is also a Type 0 system, therefore,

$$\text{Position Error Constant} K_P = \lim_{s \to 0} G(s) = \frac{Kp_1 p_2 ... p_n}{z_1 z_2 ... z_n} = \lim_{s \to 0} \frac{1.813}{(s+2)(s+3)} = 0.302 \tag{2}$$

$$\text{Steady State Error } e_{ss} = \frac{1}{1+K_P} = 0.768 \tag{3}$$

$$\text{Overshoot Percentage } \%OS = exp(\frac{-\zeta\pi}{\sqrt{1-\zeta^2}}) = 16\% \tag{4}$$

$$\text{Settling Time } T_S = \frac{4}{\zeta\omega_n} = 1.6 \tag{5}$$

2 Lead Compensator

To improve the system's transient response, lead compensator is employed to reduce the overshoot to 6%, and decrease the settling time by half to 0.8 seconds. This reduces the oscillation and oscillation duration of the lift, and hence the passage will feel more comfortable. By equation 4, $\zeta' = 0.66$, By equation 5, $(\zeta\omega_n)' = 5$. Thus, Re(s) = -5, and using trigonometric, Im(s) = 5.691. Therefore, the designed close loop s is $s = -5 \pm j5.691$

2.1 Position of the additional Pole and Zero

New pole p_c and zero z_c are added into the system to ensure the root locus pass through the designed s. The additional zero should be either at s=-2 or s=-3, such that it can cancel out one of the poles and the second order system assumption is not violated. Choosing arbitrarily z_c=-2 or -3, p_c can be found by root locus criterion.
Root Locus Criterion:

$$(z_1 + z_2 ... + z_n) - (p_1 + p_2 + ...p_n) = \pm 180 \tag{6}$$

which stated that the sum of the angle of the zeros minus the sum of the angles of the poles must equals to ±180. Upon obtaining the angle of the additional pole, we can use trigonometric to calculate whether does the pole lie on the real axis.

Table 1: The system performances when different sets of zeros and poles are chosen

	Arg(z_c)	**Arg(p_c)**	**Location of p_c on the real axis**	**K'**	K'_P	e'_{ss}
$z_c = -3$	109.362	62.206	-8	41.392	2.587	0.279
$z_c = -2$	117.794	79.638	-7	36.392	1.733	0.366

From the above table it can be shown that the pair $z_c = -3$ and $p_c = -8$ is a better choice as it yield a smaller e_{ss}. The new system is $G'(s) = \frac{K'}{(s+2)(s+8)}$

3 Lag Compensator

To further reduce the stead state error of the system, lag compensator is employed. Reduce e_{ss} ensures the lift arrived at its designated floor precisely. Again, a pair add zero and poles is added to the system, where the magnitude of zero is larger than that of the pole, as e_{ss} is proportional to $\frac{p_c}{z_c}$. Yet, the zero and pole should be place close to each other such that their angular contribution cancel out each other, and do not affect the system gain and root locus.

$$K''_P = K'_P \frac{z_c}{p_c} \tag{7}$$

Arbitrarily, setting $z_c = 0.1$ and $p_c = 0.01$. It can be shown that it satisfies the root locus criterion.

$$z_{0.1} - (p_{0.01} + p_2 + p_8) = 130.727 - (131.243 + 117.794 + 62.206) = 180$$

As the pole and zero is very close to each other, they approximately cancel out each other, hence, the second other system assumption holds. It can also be shown that it improves the system's steady state error.

$$K''_P = K'_P \frac{z_c}{p_c} = 2.587 \frac{0.1}{0.01} = 25.87$$
$$e''_{ss} = \frac{1}{1 + K''_P} = 0.0373$$

The system e_{ss} significantly reduced and is approximately $\frac{1}{7}$ of the initial e'_{ss}, which was 0.279. The final system is $G''(s) = \frac{K''(s+0.1)}{(s+0.01)(s+2)(s+8)}$

4 The overall improvements of the system

Comparing with the uncompensated system, the performance of the compensated system is more desired for a lift system. The percentage of overshoot is reduced by approximately one third, meaning the passengers will feel less oscillations when the lift is stopping. The reduction in settling time means, in general circumstance, where the lift only travels a short distance, the passengers can wait for a shorter period of time for the lift to settle and exit of the lift. This would be extremely useful when the lift is used during peak hours to facilitate the flow of people. The reduction in steady state error also means that the lift will stop at the desired floors more precisely, this enhance the safety of the lift system. If the e_{ss} is huge, the lift would stop even it has not yet reached the floor it is supposed to be at. This results in mismatch between the level of the lift floor and the building floor, causing a step at the exit of the lift, which is really undesirable, and an obvious sign of bad design. The use of compensator is hence of the interest of the passengers.