Control Assignment 1

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1 Second Order Model of the Speed Control Motor

$$v = iR_a + L\frac{\mathrm{d}i}{\mathrm{d}t} + e \text{ (from KVL)}$$
$$e = NBlv = NBr\omega = K_e\omega$$
$$F = BliN; Torque = BliNr = K_T i; Torque = J\dot{\omega} = NBr\dot{\omega} = K_e\dot{\omega}$$

Combining the three equations gives the Second Order Model:

$$v = iR_a + L\frac{\mathrm{d}i}{\mathrm{d}t} + e = iR_a + L\frac{\mathrm{d}i}{\mathrm{d}t} + K_e\omega = \frac{J\dot{\omega}}{K_T}R_a + L\frac{\mathrm{d}\frac{J\dot{\omega}}{K_T}}{\mathrm{d}t} + K_e\omega$$
$$\frac{v}{K_e} = \frac{LJ}{K_TK_e}\ddot{\omega} + \frac{JR_a}{K_TK_e}\dot{\omega} + \omega$$

2 Transient Response of the System

2.1 Calculations

From the second order equation, ω_n, $zeta$ and K can also be determined according to the equation $v(k) = \frac{1}{{\omega_n}^2}\ddot{\omega} + \frac{2\zeta}{\omega_n}\dot{\omega} + \omega$, therefore, $\omega_n = \sqrt{\frac{K_TK_e}{LJ}} = 64.775$, $\zeta = \frac{JR_a\omega_n}{2K_TK_e} = 0.357$ and $k = \frac{1}{K_e} = 0.0211$. Then apply Laplace Transform to solve the ODE and obtain the open loop transfer function G(s),

$$\frac{V(s)}{K_e} = \frac{LJ}{K_TK_e}(s^2\Omega(s)) + \frac{JR_a}{K_TK_e}(s\Omega(s)) + Y(s)$$
$$V(s) = \frac{LJ}{K_T}(s^2\Omega(s)) + \frac{JR_a}{K_T}(s\Omega(s)) + K_eY(s)$$
$$G(s) = \frac{V(s)}{\Omega(s)} = \frac{LJ}{K_T}(s^2) + \frac{JR_a}{K_T}(s) + sK_e$$

Close Loop Transfer Function of the unity feedback system T(s):

$$T(s) = \frac{G(s)}{1+G(s)} = \frac{1}{1+\frac{1}{G(s)}}$$

Using the equation $G(s) = \frac{k}{\frac{1}{{\omega_n}^2}s^2 + \frac{2\zeta}{\omega_n}s + 1}$, and substitute it into T(s),

$$T(s) = \frac{1}{\frac{k + {\omega_n}^2s^2 + \frac{2\zeta}{\omega_n}s + 1}{k}} = \frac{k}{{\omega_n}^2s^2 + \frac{2\zeta}{\omega_n}s + 1 + k}$$

Substitute the calculated constant into the equation:

$$T(s) = \frac{0.0211}{4195s^2 + 0.0110s + 1.0211}$$

Transient response of the system in s domain C(s) and time domain c(t):

$$C(s) = T(s)R(s) = \frac{0.0211}{(4195s^2 + 0.0110s + 1.0211)}(\frac{1}{s})$$
$$c(t) = 1 - \frac{1}{\sqrt{1-\zeta^2}}e^{-\zeta\omega_n t}sin(\omega_n\sqrt{1-\zeta^2}t + \phi) = 1 - (1.071)e^{-23.125t}sin(60.507t + \phi)$$

Poles of the system:

$$0 = (4195s^2 + 0.0110s + 1.0211)(s)$$
$$s = 0, -1.311(10^{-6}) \pm j0.0156$$

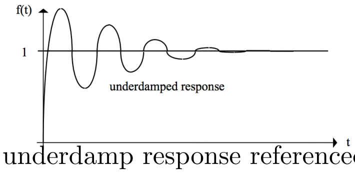

Figure 1: Typical underdamp response referenced from lecture notes

2.2 Discusssion

The position of the pole helps us examine the stability of the system. As positive complex poles means unbounded oscillation, negative complex poles means the oscillation will eventually decay, and the system can return to its steady state. From the above close loop transfer function, it shows that all poles lies on the left hand side of the s plane (i.e. non positive poles), the system is stable. Yet, one of the pole lies on the origin, meaning it is marginally stable, as any further steps will result in instability of the system.

Furthermore as the damping ratio ζ is smaller than 1, the system is underdamped, its transient response will be similar to that shown in Fig. 1. The system will response quickly and overshoot, then oscillate up and down about the steady state value before it eventually settled.

For speed control of motor that are to be used with in a lift, underdamped response is not desired. Over damped response is more preferable in a lift system. Because underdamped system tends to response quickly and, in the context of controlled the speed of a lift, a immediate stops or a sudden acceleration is not desired, it might bought safety issues for passengers within the left and brings uncomfortable to them. Furthermore, overshooting means,the speed of the lift at certain instant might be too high and we have limited control over them. This might accelerate the wearing of the lift components and raise safety issues. Over damped response is more preferred, as its raise time is relative slow, it can allow the lift to to slowing stops or start to move.

3 Improve Transient Response of the System

To improve the transient response to critically damped or over damped, the poles should be real, and damping ratio should be bigger than or equal to 1. Relating to equation $\omega_n = \sqrt{\frac{K_T K_e}{LJ}} = 64.775$, $\zeta = \frac{J R_a \omega_n}{2 K_T K_e} = 0.357$. Therefore, we can increase the damping ratio by decreasing the inductance.

dynamic modelling of systems

4 Importance of dynamic modelling of systems and transient response of the System

Modelling the system over time allow us to investigate on the response of the system over a period of time, instead of just one instant, as the response of the system in real word is often not, due to reactive component in the circuit. Understanding the response of the system is essential in building controller, as when we know the expected outcome from the system, and compare it with the desired outcome, we can build a controller to regulate the system to perform in the way we want using control loops. For example in this case, the open loop transfer function is obtained first, then we apply a unity gain feedback loop to the system, such the output of the system can now be controlled.