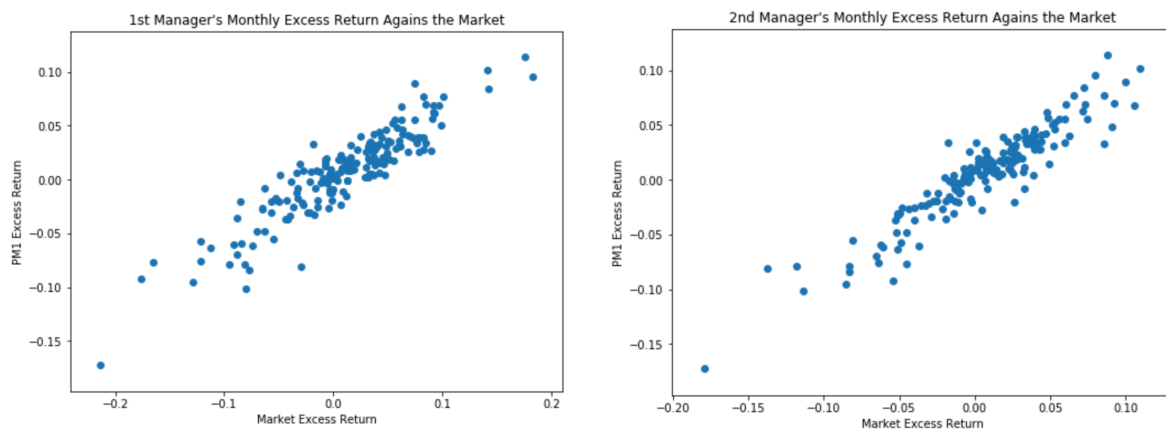


Advanced Analytics of Finance
Problem Set 3
Elina Harutyunyan

Question 3

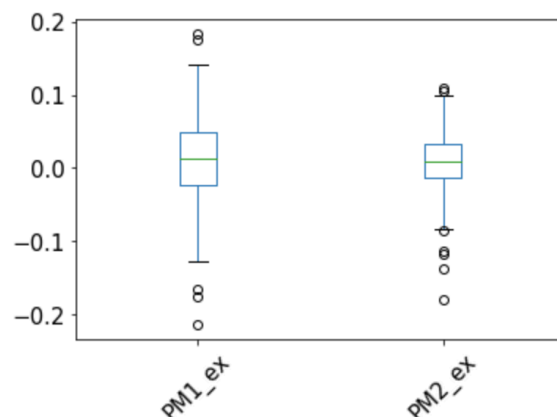
Part A

Below are the scatterplots of managers' monthly excess returns against the market excess returns:



As we can see, both managers' excess returns are positively correlated with market excess returns, however for the second manager, the excess monthly returns are more concentrated towards being higher compared with the first manager.

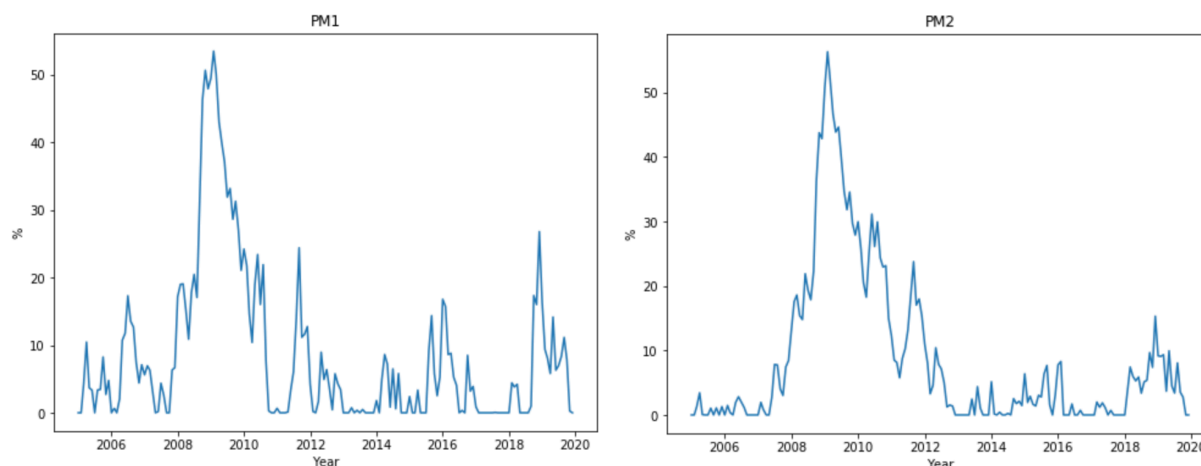
To explore even further, we plot the boxplots of managers' monthly returns:



We can observe, that the median returns of both managers are very close to each other and there are some outlier excess returns for both of them, however for the second manager the outliers seem to be more on the negative side. In general, we can deal with these outliers by winsorizing them,

in other words those excess returns in that are higher in magnitude than $\text{median} \pm 3.5 * \text{Interquartile Range}$ would be change to equal that amount.

	Mean, %	St. Dev, %	Sharpe Ratio	Information Ratio	Maximum Drawdown
Manager 1	12.309	21.088	0.584	0.377	53.489
Manager 2	8.553	15.398	0.555	-0.006	56.322



The table above reports annualized mean, standard deviation Sharpe Ratio and Information Ratio of both managers, as well as the maximum drawdown during the period from 2005 to 2019. As we can observe, the first manager has both higher average excess returns and higher volatility, however the volatility is not as high compared to the excess returns hence the Sharpe ratio of the first manager is higher as well. In addition, the information ratio is higher for the first manager and maximum drawdown is lower, meaning that the first manager outperforms the market for the given amount of tracking error, however the second manager underperforms the market.

Part B

In order to understand if any manager has the ability to outperform the market and generate alpha that is significant for our sample time period we perform regressions for each of the managers excess returns:

$$R_{i,t}^e = \alpha_i + \beta_i R_{m,t}^e + \gamma_i R_{hml,t} + \delta_i R_{smb,t} + \epsilon_{i,t}$$

The result of this regression for the first manager is reported below. As can be seen, the alpha is negative but close to zero and not significant, whereas all the market and Fama French factors are significant at a 1% level. The coefficients on the market and SMB factor are positive, whereas it is negative for the HML factor, suggesting that HML is not part of this managers strategy.

```

=====
                        OLS Regression Results
=====
Dep. Variable:          PM1_ex      R-squared:                0.992
Model:                  OLS          Adj. R-squared:            0.992
Method:                  Least Squares  F-statistic:              6983.
Date:                    Sun, 15 Mar 2020  Prob (F-statistic):      1.11e-182
Time:                    16:11:01      Log-Likelihood:           679.79
No. Observations:        180          AIC:                     -1352.
Df Residuals:            176          BIC:                     -1339.
Df Model:                 3
Covariance Type:         nonrobust
=====

```

	coef	std err	t	P> t	[0.025	0.975]
const	-7.841e-06	0.000	-0.018	0.985	-0.001	0.001
Mkt-RF	1.2961	0.011	115.753	0.000	1.274	1.318
SMB	0.8003	0.020	40.441	0.000	0.761	0.839
HML	-0.6925	0.017	-41.443	0.000	-0.725	-0.660

```

=====
Omnibus:                0.320      Durbin-Watson:            1.986
Prob(Omnibus):           0.852      Jarque-Bera (JB):         0.180
Skew:                    -0.074      Prob(JB):                 0.914
Kurtosis:                3.044      Cond. No.                 48.6
=====

```

For the second manager we have the following result:

```

=====
                        OLS Regression Results
=====
Dep. Variable:          PM2_ex      R-squared:                0.963
Model:                  OLS          Adj. R-squared:            0.962
Method:                  Least Squares  F-statistic:              1511.
Date:                    Sun, 15 Mar 2020  Prob (F-statistic):      2.59e-125
Time:                    16:11:01      Log-Likelihood:           601.29
No. Observations:        180          AIC:                     -1195.
Df Residuals:            176          BIC:                     -1182.
Df Model:                 3
Covariance Type:         nonrobust
=====

```

	coef	std err	t	P> t	[0.025	0.975]
const	0.0010	0.001	1.468	0.144	-0.000	0.002
Mkt-RF	0.9518	0.017	54.962	0.000	0.918	0.986
SMB	-0.1182	0.031	-3.861	0.000	-0.179	-0.058
HML	0.5333	0.026	20.634	0.000	0.482	0.584

```

=====
Omnibus:                0.180      Durbin-Watson:            2.008
Prob(Omnibus):           0.914      Jarque-Bera (JB):         0.029
Skew:                    -0.006      Prob(JB):                 0.986
Kurtosis:                3.061      Cond. No.                 48.6
=====

```

Here we can see that again the alpha is very small and insignificant, suggesting that market excess returns and the factors explain the excess returns produced by this manager. We can also observe that the R-squared is very high 96.3% and was 99.2% for the previous manager, suggesting that Fama-French model can almost completely explain the excess returns produced by these managers. In this case, we can see that the loading on the SMB factor is negative, which means that this manager does not include SMB in his/her strategy.

Part C

To understand which PM is better at beating the market, we perform a difference in means test, to understand if their expected excess returns are significantly different from each other or not.

$$\begin{aligned}H_0: \mu_1 &= \mu_2 \\H_1: \mu_1 &\neq \mu_2\end{aligned}$$

In this case, the variance of the mean excess returns is taken to be the unbiased variance of the excess returns. Using the central limit theorem, we would have that the test statistic for this hypothesis is normally distributed which we calculate by the following formula

$$\text{Test Statistic} = \frac{\bar{\mu}_1 - \bar{\mu}_2}{\sqrt{\frac{\sigma_1^2}{T} + \frac{\sigma_2^2}{T}}}$$

We obtain that our test statistic is 0.557 which is less than 1.96. Hence, we would fail to reject at 5% significance level that the two mean excess returns are not equal. This result is expected since we saw from the above regressions that alphas for both managers were very close to zero and insignificant.

In order to understand, whether the Sharpe ratios of both managers are different from each other and whether one is higher than the other we have the following hypothesis:

$$\begin{aligned}H_0: SR_1 &= SR_2 \\H_1: SR_1 &\neq SR_2\end{aligned}$$

We define

$$h(\theta) = \frac{\mu_1}{\sigma_1} - \frac{\mu_2}{\sigma_2}$$

To be the difference in Sharpe Ratio estimates for both managers, hence according to the standard GMM estimation, the asymptotic variance-covariance matrix of $h(\hat{\theta})$ would be $\hat{\Omega} = \frac{\hat{V}}{T}$ where

$$\hat{V} = (\hat{d}'\hat{S}^{-1}\hat{d})^{-1}$$

And \hat{d} is

$$\frac{\partial \hat{E}(h(\theta))}{\partial \theta'} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & -2\hat{\sigma}_1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -2\hat{\sigma}_2 \end{bmatrix}$$

We have that

$$\hat{S} = E(h(\theta)h(\theta)')$$

From the above we obtain that

$$\frac{h(\hat{\theta}) - h(\theta_0)}{\sqrt{\widehat{\text{Var}}[h(\hat{\theta})]}} \sim \mathcal{N}(0, 1)$$

Hence, our test statistic with these calculations is 0.011 and we fail to reject that the Sharpe Ratios of these managers are different from each other.

We further perform a test to determine whether the Information Ratios are the same for both managers:

$$\begin{aligned} H_0: IR_1 &= IR_2 \\ H_1: IR_1 &\neq IR_2 \end{aligned}$$

We perform the same two sample t-test as we did above for the means, using the fact that the variance of the information ratios can be used as the standard error of the alphas from the regressions, which is due to the fact that we can treat the standard errors of the residuals as constant. We have the following test statistic

$$Test\ Statistic = \frac{\widehat{IR}_1 - \widehat{IR}_2}{\sqrt{\frac{\sigma_1^2}{T} + \frac{\sigma_2^2}{T}}}$$

And obtain that it is equal to -14.10, hence we can reject the null hypothesis with 1% significance level that the information ratios of both managers are not equal to each other.