MIT Sloan School of Management

Analytics of Finance 15.457
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Problem Set 5

(Due: 2:30 PM, Thursday, April 23)

1. **Modeling inflation.**

   (a) Download the file "CPI.csv", which contains the data of seasonally adjusted monthly Consumer Price Index for all urban consumers from the U.S. Bureau of Labor Statistics. Compute the monthly inflation rate as $\pi_t = \ln(P_t/P_{t-1})$, where P_t is the price index level in quarter t. Plot the series of π_t. Also plot the autocorrelation function ρ_k from 1 to 12 lags. You can either use the definition below or the function *acf* in R to produce the autocorrelation function.

$$\rho_k = \frac{Cov(x_t, x_{t-k})}{\sqrt{Var(x_t)}\sqrt{Var(x_{t-k})}}$$

   (b) Use AIC and BIC to select the optimal AR(p) model for monthly inflation. Code your own version of model selection, and compare the results with those from an auto-selection function such as auto.arima in R.

2. **Forecasting corporate earnings:** It is the beginning of 2014. You are an equity analyst in charge of building a model to forecast the quarterly earnings of Wal-Mart Stores, Inc. Download the data file "WMT.csv", which contains quarterly earnings (based on net income) for Wal-Mart from 1980Q1 to 2018Q4.

   (a) Denote the log quarterly earnings by x_t. Plot the time series of the first difference $\Delta x_t = x_t - x_{t-1}$, and the seasonal difference $\Delta_4 x_t = x_t - x_{t-4}$. Explain their economic meanings and comment on their statistical properties.

   (b) As a first attempt, build an ARIMA(0,1,1) for x_t,

$$x_{t+1} - x_t = a_0 + \varepsilon_{t+1} - \theta\varepsilon_t.$$

   Use data from 1980Q1 to 2014Q4 to estimate this model. (Hint: In R, you can use the function *arima* to do the estimation.)

   (c) Next, generate your one-quarter ahead forecast[1] for Wal-Mart's earnings in fiscal quarter Q1 of 2015 through Q4 of 2018 based on the estimated model. (Hint: You can use the function *predict* in R to produce the forecasts.)

[1] One-quarter ahead forecast means predicting x_{t+1} while taking all the observations up to x_t as given.

(d) Next, build an "airline model" for x_t,

$$(x_t - x_{t-1}) - (x_{t-4} - x_{t-5}) = (\varepsilon_t - \theta_1 \varepsilon_{t-1}) - \theta_4(\varepsilon_{t-4} - \theta_1 \varepsilon_{t-5}).$$

Again, use data from 1980Q1 to 2014Q4 to estimate this model, and generate your one-quarter ahead forecasts for Wal-Mart's earnings in fiscal quarter Q1 of 2015 through Q4 of 2018 based on this model. Explain the economic meanings of θ_1 and θ_4.

(e) Summarize the forecasting errors of the two models using the mean-squared error (MSE), defined as

$$MSE = \frac{1}{n} \sum_{i=1}^{n} (x_{t+i} - E_{t+i-1}[x_{t+i}])^2,$$

where x_{t+i} is the realized earnings in quarter $t+i$, and $E_{t+i-1}[x_{t+i}]$ is your prediction for that quarter. Which model performs better? In addition, comment on the predictive accuracy of the two models towards the end of the sample (2017-2018).

3. **Portfolio Risk Model and CPPI.** You have been managing an all-equity portfolio from 2001 to 2016, and you decided that it is time to update the risk model. The spreadsheet NAV.csv contains the historical daily net asset value (NAV) for your portfolio.

(a) Analyze the performance of your portfolio based on annualized Sharpe ratio, CAPM alpha, and information ratio (obtain any additional data on Ken French's web site if needed).

(b) Compute the monthly realized volatility for the portfolio. Plot the realized volatility series, and comment on its properties.

(c) Suppose that the log portfolio return series $r_1, \cdots, r_n$ follows the AR(1)-GARCH(1,1) model,

$$\begin{aligned} r_t &= \mu + \phi_1 r_{t-1} + x_t, \\ x_t &= \sigma_t \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0,1) \\ \sigma_t^2 &= a_0 + a_1 x_{t-1}^2 + b_1 \sigma_{t-1}^2. \end{aligned}$$

Derive the conditional log-likelihood function of the data.

(d) Estimate the AR(1)-GARCH(1,1) model using the portfolio returns. (Hint: This can be done using garchFit in R.)

(e) Based on the estimated AR(1)-GARCH(1,1) model, compute the 1-day and 10-day 1% VaR of your portfolio every day from 2002 to 2016. Plot the two time series.

(f) (**Optional**) Notice that your NAV at some point was as low as \$63.46. Suppose your investor demanded from the start that the portfolio be liquidated whenever

its NAV falls below 90. Without changing the portfolio composition (so that the return on equity investment remains the same) but instead simply by adjusting the fraction in equity and in cash each period, could you have avoided liquidation? What would your NAV have been at the end of 2016 based on this strategy? Assume that the annualized risk-free rate on cash holding is 1%.

(g) (**Optional**) Redo (c)-(e) under the assumption that ε_t follows the Student's t distribution. (Hint: Again, this is easy to do using garchFit in R.)