TIF345/FYM345: Project 3 (2020)

A Galton board on a rocking ship

30 Points. Due date: 2020-12-23

Questions and the requests for clarifications should be addressed to Arkady Gonoskov (arkady.gonoskov@physics.gu.se)

Description: In this project, we will study the properties of a given Galton board. The board consists of 31 rows of pegs. We are given with information that the beads possess a peculiar property of inertia that is described by a hypothesis: if a bead falls to the right/left of a peg then it starts spinning clockwise/counterclockwise and this makes it more likely that the bead falls to the same side at the next peg. We can model this by a coefficient α that quantifies the bias due to the previous move. We can assume that earlier moves (more than one peg earlier) have no effect and that the bias is zero at the first peg (see the model below). We want to determine the most likely value of α by a series of experiments, in each of which we get the distribution of beads in the bottom of the board. The difficulty is that we happen to carry out the experiments on a slowly rocking ship so that in each experiment there is an additional bias s that increases the chance that, in each hit, the beads prefer to fall in one particular direction. The bias due to the current slope of the ship is constant during a single experiment – i.e. s is the same for all events in a single experiment. The value of s quantifies the slope of the ship and varies uncontrollably from experiment to experiment. According to our model, the probability of a bead falling to the right of a peg P_+ and the probability of falling to the left of a beg P_- are described by the equation:

$$P_{\pm} = 0.5 \pm (\alpha M + s),$$

where $M = 0$ at the entrance, $M = 0.5$ if the bead bounced off the previous peg to the left (so it arrives at the present peg from the right) and $M = -0.5$ if the bead bounced off the previous peg to the right (so it arrives at the present peg from the left).

You are given with a python module “board” (upload both board.pyi and board.cpython-37m-x86_64-linux-gnu.so to the working directory) that contains a function “experiment()” for making virtual experiments. The function returns a numpy vector that contains the number of beads in each cell in the bottom of the board. The slope s is randomly assigned in each experiment. You can assume that $s \in [-0.25, 0.25]$ and $\alpha \in [0, 0.5]$. You can call the function, i.e. carry out experiments, as many times as you need.

Goal: Determine the most likely value of α and estimate the confidence interval (CI). Explain what hampers further decrease of CI in your solution. Describe possible improvements.

Methods: You are supposed to use an artificial neural network (NN) to cope with the latent variable s in the Markov Chain Monte Carlo (MCMC) procedure. Consider the following plan:

1. Write your own simulator for the described model, i.e. the function that generates the outcome of an experiment for any given values of α and s. Generate a set of simulated results for various values of parameters.
2. Train an NN to solve the inverse problem: determine the value of α and s based on the simulated result. Determine the mean square errors for α and s.
3. Consider the outcomes of the simulator. Propose a summary statistics and determine the reasonable size of the kernel for the Approximate Bayesian Computation (ABC).

4. Develop the ABC routine. The proposals for s are to be generated with help of the NN. Consider the options of using the NN also for the generation of proposals for α.
5. Run several cycles of MCMC. Discuss what factors prevent further decrease of CI.
6. There are many improvements that one can implement for this project. Try to implement the basic plan first and then describe what can be improved. The implementation of improvements is not required but could be evaluated with extra points.

Literature:

1. The Galton board: https://en.wikipedia.org/wiki/Bean_machine
2. ABC: Sisson, Fan and Beaumont, *Handbook of Approximate Bayesian Computation* (2019)

Comments:

1. This project is designed to be a toy model for the discovery of the Higgs boson at LHC.
2. The project is designed so that the runtime for the basic result should be no more than several minutes. If you want to try to achieve a better accuracy, please consider using your PC instead of the jupyter hub.
3. Please try to tackle the project as far as you can by the lab session on 2020-12-16. During the session you will have the opportunity to ask questions in case you face any difficulties.