

Integral along the component axis

Integrate[Sin[p2 * Pi * y / L] * y * Sin[p1 * Pi * y / L], {y, 0, L}]

$$\text{Out}[*]= \frac{1}{(p_1 - p_2)^2 (p_1 + p_2)^2 \pi^2} L^2 \left(-2 p_1 p_2 - p_2 (-p_1^2 + p_2^2) \pi \cos[p_2 \pi] \sin[p_1 \pi] + p_1^2 \sin[p_1 \pi] \sin[p_2 \pi] + p_2^2 \sin[p_1 \pi] \sin[p_2 \pi] + \cos[p_1 \pi] (2 p_1 p_2 \cos[p_2 \pi] + p_1 (-p_1^2 + p_2^2) \pi \sin[p_2 \pi]) \right)$$

Integrals not along the component axis

In[*]:= Integrate[Sin[p2 * Pi * y / L] * Sin[p1 * Pi * y / L], {y, 0, L}]

$$\text{Out}[*]= \frac{L p_2 \cos[p_2 \pi] \sin[p_1 \pi] - L p_1 \cos[p_1 \pi] \sin[p_2 \pi]}{p_1^2 \pi - p_2^2 \pi}$$