Mais Exemplos:

1.DNA

$$A \quad C \quad G \quad T$$

$$p_A = \frac{1}{2}, \quad p_C = \frac{1}{4}, \quad p_G = \frac{1}{8}, \quad p_T = \frac{1}{8},$$

$$u_A = 1 \text{ bit}, \quad u_C = 2 \text{ bits}, \quad u_G = 3 \text{ bits}, \quad u_T = 3 \text{ bits},$$

$$H = \frac{1}{2} \cdot 1 + \frac{1}{4} \cdot 2 + \frac{1}{8} \cdot 3 + \frac{1}{8} \cdot 3 = 1.75$$

$$\boxed{\begin{array}{lcl} A & = & 1 \\ C & = & 01 \\ G & = & 000 \\ T & = & 001 \end{array}}$$

$$ACATGAAC \qquad \longrightarrow \qquad 10110010001101.$$

$$p_A = \frac{1}{2}, \quad p_C = \frac{1}{6}, \quad p_G = \frac{1}{6}, \quad p_T = \frac{1}{6},$$

2. alafabeto {A a H }

p_A = 1/2
p_B = 1/4
p_C = 1/8
p_D = 1/16
p_E = 1/32
p_F = 1/64
p_G = 1/128
p_H = 1/128

Evento	*Representação*
A	0
B	10
C	110
D	1110
E	11110
F	111110
G	1111110
H	1111111

$$\sum_{n=A}^{H} p_n \times \ \textit{número de bits na representação}$$

$$= \frac{1}{2}\times 1 + \frac{1}{4}\times 2 + \frac{1}{8}\times 3 + \frac{1}{16}\times 4 + \frac{1}{32}\times 5 + \frac{1}{64}\times 16 + \frac{1}{128}\times 7 + \frac{1}{128}\times 7 = \boxed{2{,}14 \text{ bits}}$$

$$H(X) = \sum_{n=1}^{5} - p_{x_i} \log_2 p_{x_i}$$
$$= \frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 4 + \frac{1}{8}\log_2 8 + \frac{1}{16}\log_2 16 + \frac{1}{32}\log_2 32 + \frac{1}{64}\log_2 64 + \frac{1}{128}\log_2 128 + \frac{1}{128}\log_2 128$$
$$= \frac{1}{2}1 + \frac{1}{4}2 + \frac{1}{8}3 + \frac{1}{16}4 + \frac{1}{32}5 + \frac{1}{64}6 + \frac{1}{128}7 + \frac{1}{128}7$$
$$= 0{,}5 + 0{,}5 + 0{,}375 + 0{,}25 + 0{,}15625 + 0{,}09375 + 0{,}0546875 + 0{,}0546875 = 1{,}984375 \ \textit{bits}$$

3. com 5 símbolos não–equiprováveis:

Resultado	***Probab.***	***Repres. A***	***Repres. B***	***Repres. C***
A	1/8	1111	11	110
B	1/8	1110	10	111
C	1/4	110	011	10
D	1/4	10	010	01
E	1/4	0	001	00

Representação A

$$\frac{1}{8}\times 4 + \frac{1}{8}\times 4 + \frac{1}{4}\times 3 + \frac{1}{4}\times 2 + \frac{1}{4}\times 1 = 2{,}5 \ \textit{bits}$$

Representação B

$$\frac{1}{8}\times 2 + \frac{1}{8}\times 2 + \frac{1}{4}\times 3 + \frac{1}{4}\times 3 + \frac{1}{4}\times 3 = 2{,}75 \ \textit{bits}$$

Representação C

$$\frac{1}{8}\times 3 + \frac{1}{8}\times 3 + \frac{1}{4}\times 2 + \frac{1}{4}\times 2 + \frac{1}{4}\times 2 = 2{,}25 \ \textit{bits}$$

Entropia Conjunta e Entropia Condicional (Wells)

2 fontes A e B, estatísticamente independentes

entropia total → H(A,B)= H(A) + H(B)

Pi,j : probabilidade conjunta de que A envia ai e B envie bj

P i,j = Pr (ai,bj)

Sendo estatísticamente independentes: Pi,j = Pr(ai) * Pr(bj) = pi*pj

A emissão combinada dos símbolos ai e bj considerada como um símbolo
composto C i,j ≡ <ai,bj>

$$H(C) = \sum_{c_{i,j} \in C} p_{i,j} \log_2(1/p_{i,j}) = \sum_{i=0}^{M_A-1} \sum_{j=0}^{M_B-1} p_{i,j} \log_2(1/p_{i,j}).$$

H(C) = H(A,B) = H(A) + H(B|A) *entropia* ***condicional***
= H(B) + H(A|B)

se estatísticamente independente:

H(B|A) = $\sum_i p_i \sum_j p_{j|i} \log_2(1/p_{j|i}) = \sum_i p_i \sum_j p_j \log_2(1/p_j) = H(B).$

$$H(C) = H(A,B) = H(A) + H(B|A).$$

Mas se B é dependente de A

então H(B|A) < H(B) → H(A,B) ≤ H(A) + H(B)

Many computer backplanes and memory systems employ a *parity bit* as a simple means of error detection. Let A be an information source with alphabet $A = \{0, 1, 2, 3\}$. Let each symbol a be equally probable and let $B = \{0, 1\}$ be a parity generator with

$$b_j = \begin{cases} 0 & if \quad a = 0 \; or \; a = 3 \\ 1 & if \quad a = 1 \; or \; a = 2 \end{cases}.$$

What are $H(A)$, $H(B)$, and $H(A, B)$?

Solution: From the definition of entropy, $H(A) = 4 \cdot \frac{1}{4} \cdot \log_2(4) = 2$. Likewise, the two symbols in B each have probability 0.5, so $H(B) = 1$. However, the conditional probabilities $\Pr(b|a)$ are

$$\begin{aligned} \Pr(0|0) &= 1, \quad \Pr(1|0) = 0, \\ \Pr(0|1) &= 0, \quad \Pr(1|1) = 1, \\ \Pr(0|2) &= 0, \quad \Pr(1|2) = 1, \\ \Pr(0|3) &= 1, \quad \Pr(1|3) = 0. \end{aligned}$$

Therefore,

$$H(B|A) = \sum_{i=0}^{3} p_i \sum_{j=0}^{1} p_{j|i} \log_2(1/p_{j|i}).$$

Since $\lim_{x \to 0} x \log(x) = 0$, this expression evaluates to $H(B|A) = 4 \cdot 0.25 \cdot (1 \cdot \log_2(1) - 0 \cdot \log_2(0)) = 0$ which simply says that B is *completely determined* by A. Therefore,

$$H(A,B) = H(A) + H(B|A) = 2 + 0 = 2.$$

Source B contributes no information to the compound signal. (Source B is said to be "redundant.")