

1 Part 1 Theory

1.1 Task 1 Divide and Conquer

a) A brief example of how divide and conquer has been used outside the scope of algorithms.

Divide and conquer in political context is a historical strategy used by empires seeking to expand their territories. The strategy involves encouraging or creating divisions among the subjects to prevent alliances that can challenge the sovereign, and aiding and promoting those who will cooperate with the sovereign. This strategy is used to empower the sovereign and control the subjects or factions with different interests.

b) Bank Security

pseudo code of problem

```
find(set, n)
if set.size == 1 and n < set.size:
    return TRUE      #yes, there is a set of more than n/2
                     #that are all equivalent

randomly select a card, C, in the set

partition the cards into two sets
    setL = cards equivalent to C
    setR = cards not equivalent to C

if len(setL) == n/2:
    return TRUE      #yes, there is a set of more than
                     #n/2 that are all equivalent

if len(setL) > n/2:
    find (setL, n)

else:
    find (setR, n - setL.size())
```

1.2 Task 2 Merge Sort

a) Visualisation of the merge sort of the array [6,4,3,5,2,1]

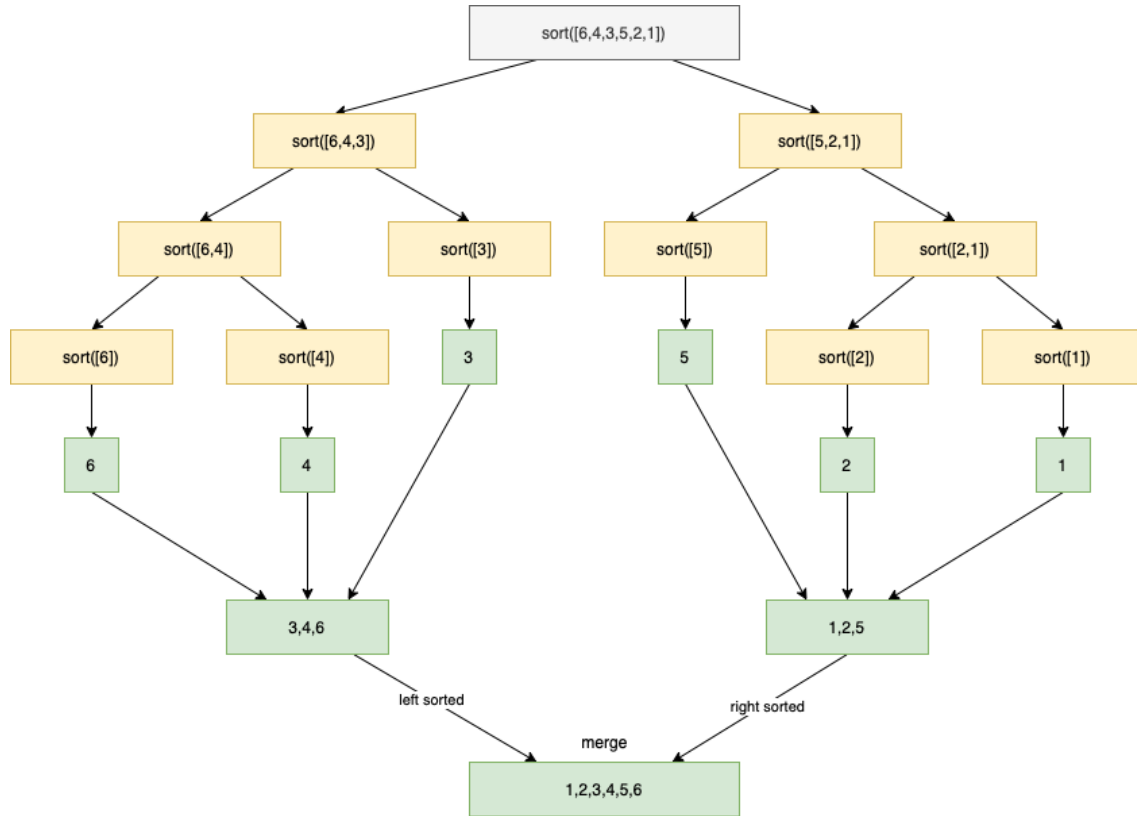


Figure 1: Visualisation of merge sort algorithm of array [6,4,3,5,2,1]

b) Number of comparisons done during this merge sort

With merge sort algorithms we have the number of comparisons given by

$$C(n) = n * \log(n) \quad (1)$$

In this case, we have $n = 6$ meaning we have the number of comparisons

$$C(6) = 6 * \log(6) \quad (2)$$

1.3 Task 3 Recurrence Relations

a) Recurrences

i)

$$\begin{aligned} T(n) &= T(n+1) - 2^n, \quad T(1) = 1 \\ T(n+1) &= T(n) + 2^n, \quad T(1) = 1 \\ T(n) &= 2^n - 1 \end{aligned} \quad (3)$$

ii)

$$\begin{aligned} T(n) &= 2 * T(\lfloor \frac{n}{2} \rfloor) + 2 \quad \text{for } n > 1, \quad T(1) = 0 \\ T(n) &= \Theta(n)_{n \rightarrow \infty} \end{aligned} \quad (4)$$

iii)

$$\begin{aligned} T(n) &= T(n-1) + \log\left(\frac{n}{n-1}\right) \quad \text{for } n > 1, \quad T(1) = 0 \\ T(n) &= 2i\pi \left\lfloor \frac{n-1}{2\pi} + \frac{1}{2} \right\rfloor + T(n-1) - \log(n-1) + \log(n) \quad \text{for } n > 1, \quad T(1) = 0 \end{aligned} \tag{5}$$