Tabular Reinforcement Learning

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Reinforcement learning

- In Dynamic Programming (planning), T and r are given
- Reinforcement learning goal: build π^* without knowing T and r
- Model-free approach: build π^* without estimating T nor r
- Actor-critic approach: special case of model-free
- Model-based approach: build a model of T and r and use it to improve the policy

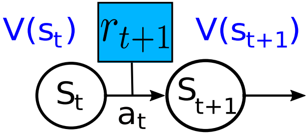
Incremental estimation

- Estimating the average immediate (stochastic) reward in a state s
- $E_k(s) = (r_1 + r_2 + ... + r_k)/k$
- $E_{k+1}(s) = (r_1 + r_2 + ... + r_k + r_{k+1})/(k+1)$
- Thus $E_{k+1}(s) = k/(k+1)E_k(s) + r_{k+1}/(k+1)$
- Or $E_{k+1}(s) = (k+1)/(k+1)E_k(s) - E_k(s)/(k+1) + r_{k+1}/(k+1)$
- Or $E_{k+1}(s) = E_k(s) + 1/(k+1)[r_{k+1} - E_k(s)]$
- Still needs to store k
- Can be approximated as

$$E_{k+1}(s) = E_k(s) + \alpha[r_{k+1} - E_k(s)] \quad (1)$$

- Converges to the true average (slower or faster depending on α) without storing anything
- Equation (1) is everywhere in reinforcement learning

Temporal Difference error



- The goal of TD methods is to estimate the value function $V(s)$
- If estimations $V(s_t)$ and $V(s_{t+1})$ were exact, we would get $V(s_t) = r_{t+1} + \gamma V(s_{t+1})$
- The approximation error is

$$\delta_t = r_{t+1} + \gamma V(s_{t+1}) - V(s_t) \tag{2}$$

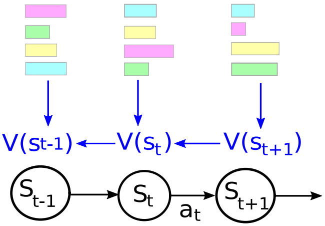
- δ_t measures the error between $V(s_t)$ and the value it should have given $r_{t+1} + \gamma V(s_{t+1})$
- If $\delta_t > 0$, $V(s_t)$ is under-evaluated, otherwise it is over-evaluated
- $V(s_t) \leftarrow V(s_t) + \alpha\delta_t$ should decrease the error (value propagation)

Temporal Difference update rule

$$E_{k+1}(s) = E_k(s) + \alpha[r_{k+1} - E_k(s)] \qquad (1)$$

$$\delta_t = r_{t+1} + \gamma V(s_{t+1}) - V(s_t) \qquad (2)$$

$$V(s_t) \leftarrow V(s_t) + \alpha[r_{t+1} + \gamma V(s_{t+1}) - V(s_t)] \qquad (3)$$



$$V(s_t) \leftarrow V(s_t) + \alpha[r_{t+1} + \gamma V(s_{t+1}) - V(s_t)] \qquad (3)$$

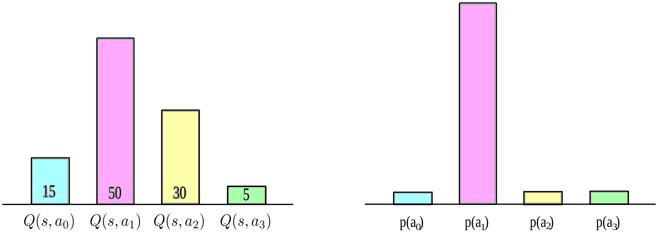
- Combines two estimation processes:
  - incremental estimation (1)
  - value propagation from $V(s_{t+1})$ to $V(s_t)$ (2)

The Policy evaluation algorithm: TD(0)

- An agent performs a sequence $s_0, a_0, r_1, \cdots, s_t, a_t, r_{t+1}, s_{t+1}, a_{t+1}, r_{t+2}, \cdots$
- Performs local Temporal Difference updates from s_t, s_{t+1} and r_{t+1}
- Proved in 1994 provided ϵ-greedy exploration
- Note: updates can be performed in any order

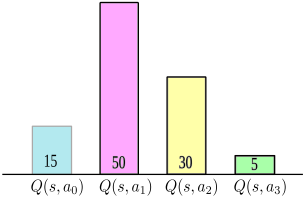
Dayan, P. & Sejnowski, T. (1994). TD(lambda) converges with probability 1. *Machine Learning*, 14(3):295–301.

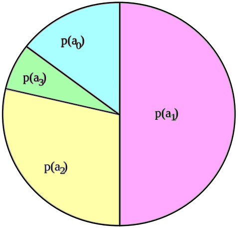
ϵ-greedy exploration



- Choose the best action with a high probability, other actions at random with low probability
- Same properties as random search
- Every state-action pair will be enough visited under an infinite horizon
- Useful for convergence proofs

Roulette wheel

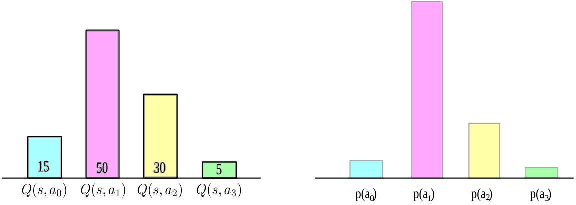




$$p(a_i) = \frac{Q(s, a_i)}{\sum_j Q(s, a_j)}$$

- The probability of choosing each action is proportional to its value

Softmax exploration



$$p(a_i) = \frac{e^{\frac{Q(s,a_i)}{\beta}}}{\sum_j e^{\frac{Q(s,a_j)}{\beta}}}$$

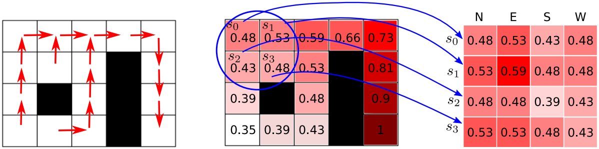
- The parameter β is called the temperature
- If $\beta \to 0$, increase contrast between values
- If $\beta \to \infty$, all actions have the same probability $\to$ random choice
- Meta-learning: tune β dynamically (exploration/exploitation)
- More used in computational neurosciences

George Velentzas, Costas Tzafestas, and Mehdi Khamassi. (2017) Bio-inspired meta-learning for active exploration during non-stationary multi-armed bandit tasks. In *2017 Intelligent Systems Conference (IntelliSys)*, pp. 661–669. IEEE

TD(0): limitation

- TD(0) evaluates $V(s)$
- One cannot infer $\pi(s)$ from $V(s)$ without knowing T: one must know which a leads to the best $V(s')$
- Three solutions:
  - Q-LEARNING, SARSA: Work with $Q(s,a)$ rather than $V(s)$.
  - ACTOR-CRITIC methods: Simultaneously learn V and update π
  - DYNA: Learn a model of T: model-based (or indirect) reinforcement learning

Value function and Action Value function



- The value function $V^\pi : S \rightarrow \mathbb{R}$ records the agregation of reward on the long run for each state (following policy π). It is a vector with one entry per state
- The action value function $Q^\pi : S \times A \rightarrow \mathbb{R}$ records the agregation of reward on the long run for doing each action in each state (and then following policy π). It is a matrix with one entry per state and per action

SARSA

- Reminder (TD): $V(s_t) \leftarrow V(s_t) + \alpha[r_{t+1} + \gamma V(s_{t+1}) - V(s_t)]$
- SARSA: For each observed $(s_t, a_t, r_{t+1}, s_{t+1}, a_{t+1})$:
  $Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r_{t+1} + \gamma Q(s_{t+1}, a_{t+1}) - Q(s_t, a_t)]$
- Policy: perform exploration (e.g. ϵ-*greedy*)
- One must know the action a_{t+1}, thus constrains exploration
- On-policy method: more complex convergence proof

Singh, S. P., Jaakkola, T., Littman, M. L., & Szepesvari, C. (2000). Convergence Results for Single-Step On-Policy Reinforcement Learning Algorithms. *Machine Learning*, 38(3):287–308.

SARSA: the algorithm

Sarsa (on-policy TD control) for estimating $Q \approx q_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
Initialize $Q(s, a)$, for all $s \in \mathcal{S}^+, a \in \mathcal{A}(s)$, arbitrarily except that $Q(terminal, \cdot) = 0$

Loop for each episode:
  Initialize S
  Choose A from S using policy derived from Q (e.g., ε-greedy)
  Loop for each step of episode:
    Take action A, observe R, S'
    Choose A' from S' using policy derived from Q (e.g., ε-greedy)
    $Q(S, A) \leftarrow Q(S, A) + \alpha\big[R + \gamma Q(S', A') - Q(S, A)\big]$
    $S \leftarrow S'$; $A \leftarrow A'$;
  until S is terminal

- Taken from Sutton & Barto, 2018

Q-LEARNING

- For each observed $(s_t, a_t, r_{t+1}, s_{t+1})$:

$$Q(s_t, a_t) \leftarrow Q(s_t, a_t) + \alpha[r_{t+1} + \gamma \max_{a \in A} Q(s_{t+1}, a) - Q(s_t, a_t)]$$

- $\max_{a \in A} Q(s_{t+1}, a)$ instead of $Q(s_{t+1}, a_{t+1})$
- Off-policy method: no more need to know a_{t+1}
- Policy: perform exploration (e.g. ϵ-*greedy*)
- Convergence proven given infinite exploration

Watkins, C. J. C. H. (1989). *Learning with Delayed Rewards*. PhD thesis, Psychology Department, University of Cambridge, England.

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Q-LEARNING: the algorithm

Q-learning (off-policy TD control) for estimating $\pi \approx \pi_*$

Algorithm parameters: step size $\alpha \in (0, 1]$, small $\varepsilon > 0$
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  until S is terminal

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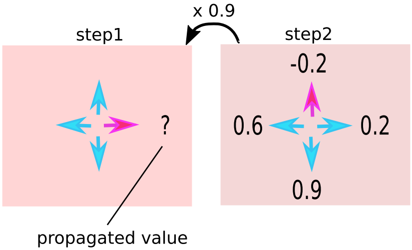
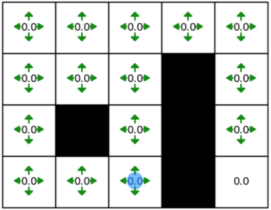
Difference between Q-LEARNING and SARSA



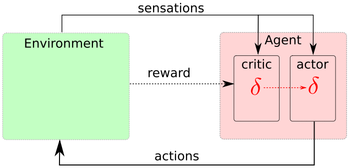
- Consider an agent taking the two pink actions
- With Q-LEARNING, the propagated value ? is $\gamma \operatorname{argmax}_a Q(s_{t+1}, a)$, thus $0.9 \times 0.9 = 0.81$
- With SARSA, it is $\gamma Q(s_{t+1}, a_{t+1})$, thus $0.9 \times -0.2 = -0.18$

Q-LEARNING in practice



- Build a states×actions table (*Q-Table*, eventually incremental)
- Initialise it (randomly or with 0 is not a good choice)
- Apply update equation after each action
- Problem: it is (very) slow

Actor-critic: Naive design

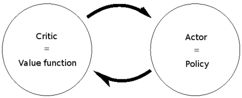


- Discrete states and actions, stochastic policy
- An update in the critic generates a local update in the actor
- Critic: compute δ and update $V(s)$ with $V_{k+1}(s) \leftarrow V_k(s) + \alpha_k \delta_k$
- Actor: $P^{\pi}_{k+1}(a|s) \leftarrow P^{\pi}_{k}(a|s) + \alpha_k \prime \delta_k$
- NB: no need for a max over actions
- NB2: one must know how to "draw" an action from a probabilistic policy (not straightforward for continuous actions)

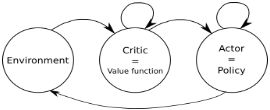
Williams, R. J. and Baird, L. (1990) A mathematical analysis of actor-critic architectures for learning optimal controls through incremental dynamic programming. In *Proceedings of the Sixth Yale Workshop on Adaptive and Learning Systems*, pages 96–101

Dynamic Programming and Actor-Critic

Policy Iteration



Actor-Critic



- In both PI and AC, the architecture contains a representation of the value function (the critic) and the policy (the actor)
- In PI, the MDP (T and r) is known
- PI alternates two stages:
  1. Policy evaluation: update $(V(s))$ or $(Q(s,a))$ given the current policy
  2. Policy improvement: follow the value gradient
- In AC, T and r are unknown and not represented (model-free)
- Information from the environment generates updates in the critic, then in the actor

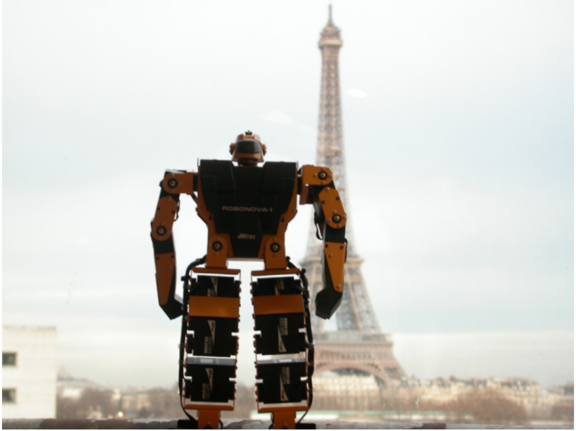
From $Q(s, a)$ to Actor-Critic

state / action	a_0	a_1	a_2	a_3
e_0	0.66	0.88*	0.81	0.73
e_1	0.73	0.63	0.9*	0.43
e_2	0.73	0.9	0.95*	0.73
e_3	0.81	0.9	1.0*	0.81
e_4	0.81	1.0*	0.81	0.9
e_5	0.9	1.0*	0.0	0.9

state	chosen action
e_0	a_1
e_1	a_2
e_2	a_2
e_3	a_2
e_4	a_1
e_5	a_1

- Given a $Q - Table$, one must determine the max at each step
- This becomes expensive if there are numerous actions
- Store the best value for each state
- Update the max by just comparing the changed value and the max
- No more maximum over actions (only in one case)
- Storing the max is equivalent to storing the policy
- Update the policy as a function of value updates

Any question?



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