A stock Management problem - Solution proposal Part II

Master DS

Academic year 2024-2025

. . . in the company

Subjective prior specifcation !

Historical data is scarce, making it essential to leverage the expertise of mechanics by incorporating their prior beliefs into our model. To make the approach practical for real-life applications, a user-friendly interface is necessary. Specifying prior parameters directly in terms of Beta distribution parameters is not convenient for practitioners.

A straightforward solution is to express prior beliefs in terms of the expected probability of failure and the associated variability, which can be interpreted as a measure of confidence. Let mu_1, mu_2 represent the mean and variance of a $Beta(a, b)$ distribution. It is straightforward to derive that:

$$\mu_1 = \frac{a}{ab}$$
$$\mu_2 = \frac{ab}{(a+b)^2(a+b+1)}.$$

from where we get

$$a = \frac{(1-\mu_1)\mu_1^2 - \mu_1\mu_2}{\mu_2}$$
$$b = \frac{1-\mu_1}{\mu_2}\left[(1-\mu_1)\mu_1 - \mu_2\right]$$

A MUST, make it user-friendly: building a shiny app

To make it friendly one create a shiny-R app. and use 2 cursors to give prior specified from mean and variance.

http://web.maths.unsw.edu.au/~lafaye/RShiny/course.nb.html

- server.R
- ui.R (for user interface)

In this problem we need to consider one specific aspect that can be achieved using R-Shiny. We want to do reactive programming, which is a coding style that starts with reactive values that are given by the users and functional programming ahead with reactive-expressions.

In Shiny the reactive values are obtained using the input object which is pass to Shiny *server* function.

Appendix

expectation and variance

Consider two random variables (X, Y) with a joint pdf $p(x, y)$ then

$$\mathbb{E}[Y] = \mathbb{E}_X[\mathbb{E}[Y|x]]\mathbb{V}ar[Y] = \mathbb{E}_X[\mathbb{V}ar[Y|x]] + \mathbb{V}ar_X[\mathbb{E}[Y|x]]$$

Proof

$$\begin{aligned}
\mathbb{E}[Y] &= \int_Y yp(y)dy = \int_Y y \int_X p(x,y)dxdy \\
&= \int_X \left[\int_Y yp(y|x)dy\right] p(x)dx \\
&= \mathbb{E}_X[\mathbb{E}[Y|x]]
\end{aligned}$$

$$\begin{aligned}
\mathbb{V}ar[Y] &= \int_Y (y - \mathbb{E}[Y])^2 p(y)dy = \int_X \left[\int_Y (y - \mathbb{E}[Y])^2 p(y|x)dy\right] p(x)dx \\
&= \int_X \left[\int_Y (y - \mathbb{E}[Y|x] + \mathbb{E}[Y|x] - \mathbb{E}[Y])^2 p(y|x)dy\right] p(x)dx \\
&= \int_X \left[\int_Y (y - \mathbb{E}(Y|x))^2 p(y|x)dy\right] p(x)dx + \\
&\int_X (\mathbb{E}(Y|x) - \mathbb{E}(Y))^2 \left[\int_Y p(y|x)dy\right] p(x)dx \\
&= \mathbb{E}_X[\mathbb{V}ar[Y|x]] + \mathbb{V}ar_X[\mathbb{E}[Y|x]]
\end{aligned}$$