

# FUNCTIONS

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### Definitions

**Function** a mathematical relationship where each input has a single output. It is often written as  $f(x)$  where  $x$  is the input

**Domain** all possible  $x$  values, the input. (the domain of investigation)

**Range** possible  $y$  values, the output. (the range of outcomes)

**Coordinates** uniquely determines the position of a point, given by  $(x, y)$

### 2.1. Types of functions

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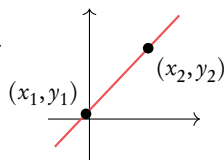
**Linear functions**  $y = mx + c$

$m$  is the *gradient*,  
 $c$  is the *y intercept*.

**Midpoint:**  $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

**Distance:**  $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

**Gradient:**  $m = \frac{y_2 - y_1}{x_2 - x_1}$



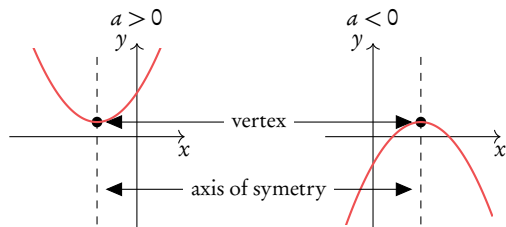
**Parallel lines:**  $m_1 = m_2$  (same gradients)

**Perpendicular lines:**  $m_1 m_2 = -1$

**Quadratic functions**  $y = ax^2 + bx + c = 0$

**Axis of symmetry:**  $x$ -coordinate of the vertex:  $x = \frac{-b}{2a}$

**Factorized form:**  $y = (x + p)(x + q)$



If  $a = 1$  use the factorization method  $(x + p) \cdot (x + q)$

If  $a \neq 1$  use the quadratic formula

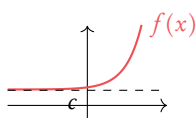
**When asked explicitly** complete the square

**Vertex form:**  $y = a(x - h)^2 + k$

**Vertex:**  $(h, k)$

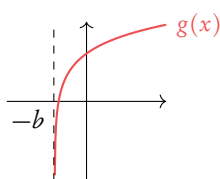
**Exponential**

$$f(x) = a^x + c$$



**Logarithmic**

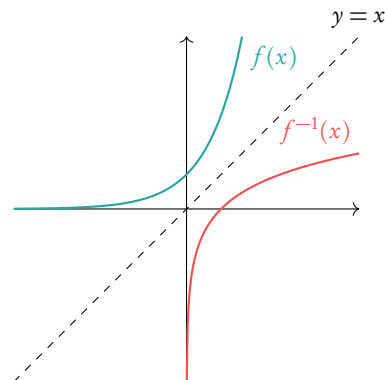
$$g(x) = \log_a(x + b)$$



### 2.2. Rearranging functions

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**Inverse function,  $f^{-1}(x)$**  reflection of  $f(x)$  in  $y = x$ .



**Composite function,  $(f \circ g)(x)$**  is the combined function  $f$  of  $g$  of  $x$ .

When  $f(x)$  and  $g(x)$  are given, replace  $x$  in  $f(x)$  by  $g(x)$ .

### Transforming functions

Change to $f(x)$	Effect
$f(x) + a$	Move graph $a$ units upwards
$f(x + a)$	Move graph $a$ units to the left
$a \cdot f(x)$	Vertical stretch by factor $a$
$f(a \cdot x)$	Horizontal stretch by factor $\frac{1}{a}$
$-f(x)$	Reflection in $x$ -axis
$f(-x)$	Reflection in $y$ -axis