

2.2.4 Polynomial long division

When we need to divide one polynomial by another we use *polynomial long division*. The number to be divided is called the 'dividend'. The number which divides it is called 'divisor'.

Polynomial long division

Divide $3x^3 - 2x^2 + 4x - 3$ by $x^2 + 3x + 3$:

$$\begin{array}{c} \boxed{x^2 + 3x + 3} \overline{) \boxed{3x^3 - 2x^2 + 4x - 3}} \\ \text{divisor} \qquad \text{dividend} \end{array}$$

1. Divide the first term of the dividend by the first term of the divisor

$$\begin{array}{c} \text{answer} \\ \boxed{3x} \\ \hline \boxed{x^2 + 3x + 3} \overline{) \boxed{3x^3 - 2x^2 + 4x - 3}} \\ \text{first term divisor} \quad \text{first term dividend} \end{array}$$

2. Multiply the divisor by this answer and subtract this from our dividend

$$\begin{array}{c} \boxed{x^2 + 3x + 3} \times \boxed{3x} = \boxed{3x^3 + 9x^2 + 9x} \\ \hline \boxed{x^2 + 3x + 3} \overline{) \begin{array}{c} 3x^3 - 2x^2 + 4x - 3 \\ - 3x^3 - 9x^2 - 9x \\ \hline - 11x^2 - 5x - 3 \end{array}} \end{array}$$

3. Divide the result of the substitution by the first term of the divisor. Repeat the process until this is no longer possible

$$\begin{array}{c} \text{result} \\ \boxed{3x - 11} \\ \hline \boxed{x^2 + 3x + 3} \overline{) \begin{array}{c} 3x^3 - 2x^2 + 4x - 3 \\ - 3x^3 - 9x^2 - 9x \\ \hline - 11x^2 - 5x - 3 \\ 11x^2 + 33x + 33 \\ \hline \text{remainder } 28x + 30 \end{array}} \end{array}$$

4. Write the answer:
result + $\frac{\text{remainder}}{\text{divisor}}$

$$3x - 11 + \frac{28x + 30}{x^2 + 3x + 3}$$