Methods for data-driven Modelling : Tutorial 2

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Bayesian inference and Quantum non Demolition (QND) Photon Counting

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Serge Haroche has been awarded in 2012 by the nobel prize for "for ground-breaking experimental methods that enable measuring and manipulation of individual quantum systems." Haroche and his collaborators have designed experiments to study quantum phenomena when matter and light interact.

They have been able to capture photons using a cavity with two mirrors which they can bounce between. This device allowed them to count the photons trapped in the cavity by passing atoms through the trap. Here we will re-analyse the data of this work [1] (see also the notes of the lectures given in 2008 at the College de France [2]), kindly given to us by Igor Dotsenko (College de France).

The goal of this tutorial is to use Bayes theorem to measure the number of photons in the cavity from repeated measures of the spin state of atoms passing through the cavity (see Fig.1).

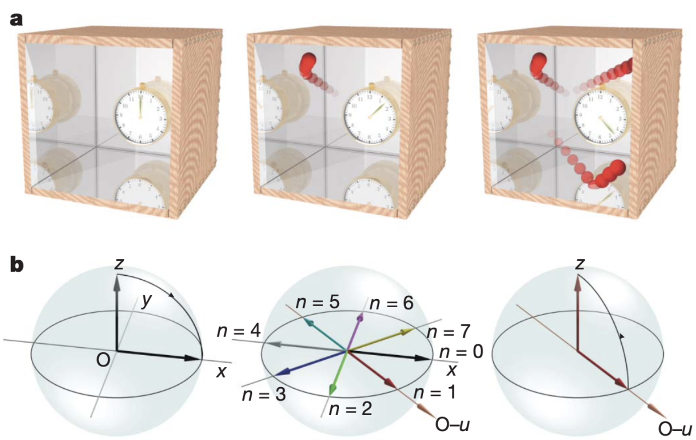

Figure 1 | Principle of QND photon counting. **a**, Thought experiment with a clock in a box containing n photons. The hand of the clock undergoes a $\pi/4$ phase-advance per photon ($n = 0, 1, 3$ represented). **b**, Evolution of the atomic spin on the Bloch sphere in a real experiment: an initial pulse R_1 rotates the spin from O–z to O–x (left). Light shift produces a $\pi/4$ phase shift per photon of the spin's precession in the equatorial plane. Directions associated with $n = 0$ to 7 end up regularly distributed over 360° (centre). Pulse R_2 maps the direction O–u onto O–z, before the atomic state is read out (right).

Figure 1: Sketch and description of the experiment from [1]

Problem

We consider a Rydenberg atom passing through a cavity, its dipole is measured by a Ramsey Interferometer, each photon it encounters produces a phase-shift Φ (see Fig 1). Trajectories of repeated detections of the spin states of the atoms passing in the cavity will be used to infer the number of photons in the field stored there. During the evolution of the measures we will observe the creation or the destructions of photons in the cavity up to the collapse in a Fock state.

The probability to detect the spin in the ground state or in the excited state depends on the number of photons n in the cavity:

$$\begin{aligned} P(g|\phi_k, n) &= A - B/2\ \cos(n\Phi + \phi_k) \\ P(e|\phi_k, n) &= (1 - A) + B/2\ \cos(n\Phi + \phi_k) \end{aligned} \tag{1}$$

where the parameters $A = 0.551$ (offset) and $B = 0.698$ (contrast) depends on the interferometer; the phase $\Phi = 0.233\ \pi(\approx \pi/4)$ is fixed to detect a number of photons in the cavity ranging from 0 to 7, and the initial state of the atom is prepared with four initial phases $\phi_k = -0.836, 0.033, 0.905, 1.442$, calibrated for optimal detections, such phases are changed cyclically during the experiments.

Data: Two files resuming the quantum detections in spin state g and e can be downloaded from the webpage, (Tutorial 8) . Each file contains a $N_{ex} \times T$ matrix of detections $\mathbf{D}$.The rows of the matrix are N_{ex} independent repetitions of the experiment. Each row is a 'quantum trajectory' of T measures. Its entries $d(ex, t)$ indicate the number of atoms detected in the state g, e for the two files respectively, at the detection t, and with Ramsey Phases $k(i)$. The 4 possible Ramsey phases are cyclically changed to $\phi_0, \phi_1, \phi_2, \phi_3, \phi_0 \ldots$along each 'quantum trajectory., so $k(i) = 0, 1, 2, 3..\ldots$. Atoms are sent in packets, containing at most 5 atoms, in the cavity, each atom is independent from the others. $d(ex, t)$ ranges therefore then between 0 (no detected atoms) to 5.

Questions:

1. (a) Write the likelihood of a set of T detections along a quantum trajectory, given a number n of photons in the cavity and the protocol of Ramsey phases $k(i)$ used in the experiment. Deduce, using Bayes rule, the probability of the number n of photons in the cavity given the detections along a quantum trajectory. Use an uniform prior in the interval $n = 0 \ldots 7$.

   (b) Re-express the log-probability of the number n of photons in the cavity, using the counts c_g^k for detections in the state g and c_e^k in the state e for each Ramsey phases ($k = 0 \ldots 3$).

2. Focus on the trajectory $ex = 1163$, and by the detection interval starting in $t_i = 400$ and ending in $t_f = t_{in} + M$ with $M = 400$. Count the number of atoms detected during this interval in the state e and g for each Ramsey phases.

3. Focus on the trajectory $ex = 1163$ Plot the probability of n after M detection starting from $t_{in} = 400$ and for $M = 0, 12, 100, 200, 400$. Describe how the posterior evolves as a function of the number of measures M. Compare qualitatively to Fig. 2 of [1].

4. Compute the entropy of the posterior after M detections. What is its expected value with no measurement ($M = 0$)?. Plots it as a function of the number of measures M, how it decays ? Deduce how the posterior behaves as a function of M.

   Give the most probable value of n (n_{opt}) after $M = 400$ measures

5. Start by trajectory 1163 (and repeat for trajectories 1293,1146,479,407). Plot the most probable value of n as a function of t_{in} starting from $t_{in} = 0$ and in sliding windows (sliding at each step by 4 detections to take into account the 4 phases) of $M = 400$ detections. Convert initial number of detections in time using a detection every $8.33\,10^{-5}$ s. Plot the sequences of n_{opt} as a function of t_{in}. Observe the quantum jumps in n_{opt} and the collapse to the vacuum state $n = 0$. Compare with the Fig. 4 of [1] (the order of the trajectories given above corresponds to the ones of the panels in fig 4). What is the peculiarity of trajectory 407?

6. Plot the distribution of n_{opt} over the N_{ex} independent repetitions of the experiments from $M = 400$ detections starting at $t_{in} = 16, 400, 800, 1600$. How the average values evolves? Compare with the distribution given in Fig.2 of [1] and the value $n_{av} = 3.46$ given in the paper .

7. Compute the mutual information between the data $\mathbf{d_0}, \mathbf{d_1}$ and the number of photons in the cavity, compute it for different values of the phase shift (not only $\Phi = \pi/4$) : $0.4, 0.78, 1.2, 1.6, 2$ and for different value of the number of acquired data M, starting from M =1. Plot it at a function of M for different values of the phase. How it behaves as a function of M, which is the value of Φ maximising it?
   a) To compute the mutual information you should average over all possible trajectories, as it this is analytically too costly you can replace the exact average over the possible trajectories with the empirical average on the samlped trajectory
   b) Just consider a simpler setting in which all the $\phi_k = 1.442$, in this setting you can sum over the all possible trajectories analitically without using the empirical average.

8. Optional: compute the probability for a single detection of the g and e spin for $n = 0 \dots 7$. For the ideal setting A=0.5,B=1 $\Phi = 0.233\ \pi (\approx \pi/4)$, and Ramsey phases $\phi_k = q\pi/4$ with $q = -2, -1, 0, 1$ and the real one, determined from the experimental limitation of the interferometer.

References

[1] C. Guerlin et al., *Progressive field-state collapse and quantum non-demolition photon counting* , Nature 448,889 (2007).

[2] S. Haroche , *Cours 2007-2008: Sixième Leçon 3 Mars 2008:Comptage QND de plusieurs photons et génération d'états de Fock de la lumière* (2007).